B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 1st Semester)

Probability & Statistics

Paper – DSE-2A

Time : Two hours Full Marks : 40

Answer **Q. No. 1** and ***any six*** from rest.

1. a) Let the random variable X be uniformly distributed on (1, 100). Find the probability of the event $\left(X+\frac{49}{X}<50\right)$.

   b) Let the random variable X be normally distributed with $E(\mathrm{X})=\mu$ and Var(X)=4. We are going to test the hypothesis $H_0:\mu=-1$ against the alternative hypothesis $H_1:\mu=1$ on the basis of a sample of size $n: x_1, x_2, \ldots x_n$. If the critical region

$$W=\left\{\left(x_1, x_2, \ldots, x_n\right): x_1+2x_2+\ldots.+nx_n \geq 0\right\}$$

   and the power of the test is $0{\cdot}5+\Phi(a)$, then show that $a=\sqrt{\frac{3}{8}\cdot\frac{n(n+1)}{2n+1}}$, where $\Phi(a)=\frac{1}{\sqrt{2\pi}}\int_0^a e^{-\frac{u^2}{2}}du$.

5+5

2. The joint probability density function of the random variables X and Y is given by

$$f(x,y)=\begin{cases} k(1-x-y), & x>0,\ y>0,\ x+y<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the value of k and $P\left(X<\frac{1}{2},\ Y>\frac{1}{4}\right)$. 5

3. Prove that $\alpha_k = (-1)^k i^k k!\ a_k$

   where $\alpha_k = k^{\text{th}}$ order moment of the random variable X about the origin,

   a_k = coefficient of t^k in the series expansion of $\chi(t)$.

   $\chi(t)$ = characteristic function of any random variable X, $i=\sqrt{-1}$. 5

4. If $E(X^2)$, $E(Y^2)$, $E(XY)$ exists finitely for two random variables X and Y, then prove that $\{E(XY)\}^2 \le E(X^2)E(Y^2)$ and hence show that $|\rho(X,Y)| \le 1$ 5

5. State and prove Bernoulli's theorem. Use this theorem to prove that

   $$f(A) \xrightarrow[\text{in } p]{} P(A) \text{ as } n \to \infty,$$

   where $f(A)=\frac{n(A)}{n}$ and

   $n(A)$ = number of occurrence of the event A connected with a random experiment E out of n repetition of E. 5

6. Find the approximate $100(1-\varepsilon)\%(0<\varepsilon<1)$ confidence interval of the parameter μ of Poisson variate for large size of the sample drawn from the Poisson-μ variate. 5

7. Use the maximum likelihood method to estimate the parameter μ of a population of X having probability mass function $P(X=i)=\frac{1}{1+\mu}\left(\frac{\mu}{1+\mu}\right)^i,\quad i=0,1,2,\ldots,\mu>0$

   Use a sample of size $n: x_1, x_2, \ldots, x_n$. 5

8. Let $x_1, x_2, \ldots, x_n$ be a sample of size n drawn from the population of the random variable X. If var(X) exists finitely, then determine the value of d such that the statistic $\sum_{i=1}^{n} d(x_1-\bar{x})^2$ with $\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i$ is an unbiased estimate of the population variate $\sigma^2 = \text{var}(X)$. 5

9. Let $X = N(m, \sigma)$. Then the statistic $\frac{ns^2}{\sigma^2}$ has a χ^2-distribution with $(n-1)$ degrees of freedom. 5

10. If X is a standard Normal variable then find then probability density function of $7X^2$. 5