Ref. No.: SC/MATH/UG/CORE/TH/11/2023

Bachelor Of Science Examination - 2023

(Third Year, First Semester)

Mathematics

Core 11

(Partial Differential Equation)

Full Marks : 40 Time : 2 Hours

Symbols/Notations have their usual meaning

Use separate answerscripts for each Group

Group A

(20 Marks)

Answer any FOUR questions

1. Determine the integral surface of the equation $x(y^2+u)u_x - y(x^2+u)u_y = (x^2-y^2)u$ with the data $x+y=0,\ u=1$. 5

2. Reduce the equation $yu_x + u_y = x$ to canonical form and hence solve for u(x,y). 5

3. Use Charpit's method to solve $pxy + pq + qy - yz = 0$. 5

4. Using the method of separation of variables suitably, show that the solution of the Cauchy problem $y^2{u_x}^2 + x^2{u_y}^2 = (xyu)^2,\quad u(x,0) = e^{x^2}$ is given by $u(x,y) = e^{x^2 + \frac{i\sqrt{3}}{2}y^2}$. 5

5. Find the solution of the equation $u(x+y)u_x + u(x-y)u_y = x^2+y^2$, with the Cauchy data $u=0$ on $y=2x$. 5

6. Find the general solution of the linear equation $x^2u_x + y^2u_y = (x+y)u$. 5

[Turn over

Group – B
(20 Marks)

Answer **any two** questions

1. (a) If $u(x, y, z, t)$ denotes the temperature at a point P(x, y, z) at time t in some domain D and κ is the thermal diffusivity of the substance, show that $u(x, y, z, t)$ satisfies (5)

$$u_t = \kappa \nabla^2 u, \quad t > 0, \quad (x, y, z) \in D.$$

(b) Prove that the canonical form of the Tricomi equation $u_{xx} + x u_{yy} = 0$ is $u_{\alpha\alpha} + u_{\beta\beta} + \frac{u_\beta}{3\beta} = 0$ for $x > 0$. (5)

2. Find solution of the problem (10)

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, \quad 0 < x < \infty, \ t > 0 \\ u(x,0) &= f(x), \quad 0 \le x < \infty \\ u_t(x,0) &= g(x), \quad 0 \le x < \infty \\ u_x(0,t) &= 0, \quad 0 \le t < \infty. \end{aligned}$$

Derive the conditions on $f(x)$ and $g(x)$ necessary for existence of the solution.

3. Use separation of variables method to solve (10)

$$\begin{aligned} u_t &= u_{xx}, \quad 0 < x < \pi, \ t > 0, \\ u(x,0) &= 2\cos^2 x, \ 0 \le x \le \pi \\ u_x(0,t) &= 0, \quad u_x(\pi, t) = 0, \quad t \ge 0. \end{aligned}$$