small radius, the axis of the pulley being horizontal. Masses of 40 lbs and 35 lbs are attached to the ends of the string and the motion takes place. Show that the time taken by the smaller mass to reach the pulley is $\frac{\sqrt{15}}{4}\ln\left(9+4\sqrt{5}\right)$ seconds.

B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Mechanics

Paper – Core-14

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meaning)

Group – A

Answer ***any two*** questions. 2×7=14

1. A force F acts along the axis of Z, and another force $2F$ along a straight line intersecting the axis of x at the point (1, 0, 0) and parallel to the plane of yz. Show that as this straight line turns round the axis of x, the central axis of the force generates the surface
$$\left\{4z^2+3y^2\right\}(1-x)^2=x^2z^2.$$
2. Find the position of the Centre of gravity of the area enclosed by the curves $x^2+y^2-6x=0$ and $x^2+y^2-4x=0$ on the positive side of the axis of x (and lying above the x-axis).
3. Six equal heavy rods freely hinged at the ends, form a regular hexagon ABCDEF, which when hung up by a point A, is kept from altering the shape by two light rods BF and CE. Prove that the thrust of the rods are in ratio $5:1$, and hence find their magnitude.

Group – B

Answer ***any two*** questions. 2×7=14

4. A particle projected with a velocity u, is acted upon by a force which produces a constant acceleration f in the plane of motion inclined at a constant angle α with the direction of motion. Let A be the point of projection. If the arctual distance of the particle at any time is measured from A and fixed straight line in the plane of motion of the particle is parallel to the tangent at A, then find the intrinsic equation of the path of the particle and show that the particle will be moving in the opposite direction to that of projection at time $\frac{u}{f\cos\alpha}\left(e^{\pi\cot\alpha}-1\right)$.

5. A particle moves with a central acceleration F in a medium whose resistance is k(velocity)^2 per unit mass. Show that the differential $\left(\frac{d^2u}{d\theta^2}+\mathrm{u}\right)=\frac{F}{h_0^2u^2}e^{2ks}$, $\mathrm{u}=\frac{1}{r}$, where s is the arctual distance of the particle at time t and h_0 is the initial moment of momentum of the particle per unit mass about the centre of force when $s=0$. (h_0 is the initial moment of momentum of the particle per unit mass when $s=0$).

6. i) A particle moves under the action of central force $\mu\mathrm{u}^2+\lambda u^3$ per unit mass, where $u=1/r$. The velocity of projection at a distance R is V. Show that the particle will ultimately go off to inifinity if $V^2>\frac{2\mu}{R}+\frac{\lambda}{R^2}$.

   ii) If the path of the particle of the nearly circular orbit is $p^2\left(a^{m-2}-r^{m-2}\right)=b^m$ with $m>0$, then show that the path of the particle is stable. Find the apsidal angle for the path.

Group – C

Answer ***any two*** questions. 2×6=12

7. Find the condition for which a given straight line will be a principal axis of a material system at a particular point. Also determine the other two principal axes. Further, prove that, if the given straight line passes through the centre of gravity of the material then it will be a principal axis at any point of its length.

8. A plank of mass M is initially at rest along a line of greatest slope of a smooth plank inclined at an angle 'α' to the horizon and a man of mass M', starting from the upper end walk down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$, where 'a' is the length of the plank.

9. A uniform string of length 20 feet and 40 lbs. hangs in equal length over a circular pulley, of mass 10 lbs, and