B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Bio Mathematics

Paper – DSE-4B

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Part – I (Marks: 24)

Answer ***any three*** questions.

1. Write a single species time-delay model. Investigate the qualitative behavior of the model and give your comments on the results. Prove that a single species population growth without delay cannot exhibit limit cycle behavior. 8

2. Two animal species are lying in a certain territory. The population densities are P, $Q \geq 0$ and both the species follow the law of logistic growth. It is also assumed that a species with density Q is not actually predating on the other P, but encounters between these two species are always in the advantage of y species.

   a) Write a suitable mathematical model based on the above observations.

   b) Discuss the stability behaviour around the biological feasible equilibria.

c) Justify your answer in ecological meanings. 8

3. Define functional response and numerical response in prey-predator interactions. Describe different Holling type functional responses in mathematical form and draw the sketches of the response curves. 8

4. A generalised Pre-Predator interaction is taken in the form:

$$\frac{dx}{dt} = xg(x) - yp(x),$$
$$\frac{dy}{dt} = y\{kp(x) - \mu\}.$$

Discuss the steady states and their qualitative behavior. 8

5. Write the growth equations of two mutualistic populations. In absence of the other both the populations grow logistically. Determine the steady states and discuss the stability properties of the interior equilibrium. 8

Part – II (Marks: 16)

Answer ***any two*** questions.

6. a) If λ_1 and λ_2 are two eigenvalues of the matrix A for 2D linear autonomous system $\dot{X} = AX$. Describe the stability and sketch phase portrait of the system when λ_1 and λ_2 are complex conjugate.

b) Determine the nature of the point (0, 0, 0) of the nonlinear system

$$\dot{x}_1 = -2x_2 + x_2x_3,$$
$$\dot{x}_2 = x_1 - x_1x_3,$$
$$\dot{x}_3 = x_1x_2$$ 5+3

7. a) Characterise the bifurcation with respect to the parameter $\mu \in R$ for the system

$$\dot{x} = \mu x - x^3,$$
$$\dot{y} = -y.$$

Sketch phase and bifurcation diagram.

b) Define positive semi-definite function. 7+1

8. a) Define phase portrait for the system $\dot{x} = f(x),\ x \in R^2$.

b) Determine the nature of the difference equation $x_{n+1} = -x_n^3 - x_n$ at the point $x = 0$.

c) Consider the discrete predator-prey model

$$X_{t+1} = X_t(a - X_t - Y_t),\ a > 0$$
$$Y_{t+1} = Y_t(b + X_t),\ 0 < b < 1,$$

where X_t, Y_t are the number of prey and predator individual at time t.

Find all fixed points of the model. Determine the stability criteria of model around the interior fixed point. 1+2+5