B. Sc. Mathematics (Hons.) Examination, 2023

(2nd Year, 2nd Semester)

Statistics - II

Paper – GE-4

Time : Two hours Full Marks : 40

Symbols / Notations have their usual meanings.

Answer ***any four*** questions. 10×4

All questions carry equal marks.

1. a) Define unbiased estimate of a population parameter. Show that, if $x_1, x_2, ..., x_n$ are the random sample of size n from a population with variance σ^2 (σ^2 known), then $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$ is an unbiased estimate of σ^2 where $\overline{x}$ is the sample mean.

   b) $x_1, x_2, ..., x_n$ are random observations on a Bernoulli variable taking the value 1 with probability θ and the value 0 with probability $(1-\theta)$. Show that $\frac{\tau(\tau-1)}{n(n-1)}$ is an unbiased estimate of θ^2 where $\tau = \sum_{i=1}^{n} x_i$. 5+5

2. a) Define the term 'consistancy' of the estimators. Prove that for Cauchy's distribution, not sample mean, but sample median is a consistant estimator of the population median.

b) Let $x_1, x_2, ..., x_n$ be a random sample from a uniform population on $[0, \theta]$, find a sufficient estimator for θ. 6+4

3. a) State and prove Rao-Blackwell Theorem in statistical theory.

b) For a random sampling from a normal population $N(m, \sigma^2)$, find the maximum likelihood estimators (MLE) for

i) μ when σ^2 is known,

ii) σ^2 when μ is known. 6+4

4. a) Find the MLE for the parameter λ of a Poisson distribution on the basis of sample of size n. Also find its variance.

b) Show that the sample mean $\bar{x}$ is sufficient for estimating the parameter λ of the Poisson distribution. 7+3

5. Explain the following terms :

i) Type I and type II errors,

ii) The best critical region,

iii) Power function of a test,

iv) Level of significance,

v) Simple and composite hypotheses. 5×2

6. Use the Neyman-Pearson Lemma to obtain the region for testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$ in the case of a normal population $N(\theta, \sigma^2)$ where σ^2 is known. Hence find the power of the test. 10