Ex/SC/MATH/UG/CORE/TH/07/2023

B. Sc. Mathematics (Hons.) Examination, 2023

(2nd Year, 1st Semester)

Numerical Methods

Paper – Core-07 (Theory)

Time : Two hours — Full Marks : 24

Use separate Answer script for each Part.

Symbols / Notations have their usual meanings.

Part – I (12 Marks)

Answer ***any three*** questions.

1. Show that for a system of linear equations $Ax = b$, the Gauss-Seidal iteration scheme converges for any initial starting vector if A is strictly diagonally dominant. 4

2. Using a proper numerical scheme, find the equation of a catenery of the form $y = c\cosh\frac{(x-a)}{c}$ passing through the points (1, 1) and (2, 3), correct to 4 decimal places. 4

3. i) What is an ill-conditioned system?

   ii) Let $A(\alpha) = \begin{bmatrix} 0.1\alpha & 0.1\alpha \\ 1.0 & 1.5 \end{bmatrix}$. Using the maximum norm, determine α so that $cond\left[A(\alpha)\right]$ is minimized.

   2+2

4. The iteration defined by $x_{k+1} = \frac{1}{2}\left(x_k^2 + c\right)$, where $0 < c < 1$, has two fixed points ξ_1 and ξ_2, where

$0 < \xi_1 < 1 < \xi_2$. Show that $x_{k+1} - \xi_1 = \frac{1}{2}\left(x_k + \xi_1\right)\left(x_k - \xi_1\right)$, $k = 0,1,2,\ldots$ and deduce that $\lim_{k\to\infty} x_k = \xi_1$ if $0 \le x_0 < \xi_2$. 4

5. i) Give a brief description of round-off error and truncation error with proper examples.

   ii) Show that for the bisection method, the convergence rate is linear. 2+2

Part – II (12 Marks)

Answer ***any two*** questions. 2×6=12

1. Evaluate y at $x = 0{\cdot}37$ using the following table.

x	0·00	0·10	0·20	0·30	0·40
y	1·0000	1·2214	1·4918	1·8221	2·2255

   The result should be correct upto four decimal places. 6

2. Find the quadrature formula

$$\int_0^1 \frac{f(x)\,dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$$

   which is exact for polynomials of highest possible degree. Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x - x^3}}$. 6

3. Show that Euler method applied to $y' = \alpha y$, $y(0) = 1$, $\alpha < 0$ is stable for stepsizes satisfying $-2 < \alpha h < 0$. 6