B. Sc. [redacted] Examination, 2023

(2nd Year, 1st Semester)

Mathematics-II

Paper – GE3

Time : Two hours Full Marks : 40

Use separate Answer script for each Part.

Symbols / Notations have their usual meanings.

Part – I (12 Marks)

Answer ***any three*** questions.

1. Find mod Z and principal amplitude of

   $z = 1 - \sin\theta(\sin\theta + i\cos\theta)$, $\frac{\pi}{2} < \theta < \pi$. 4

2. Show that the root of the following equation are all real

   $\frac{1}{x+a_1} + \frac{1}{x+a_2} + \cdots + \frac{1}{x+a_n} = \frac{1}{x+b}$, where $a_i, b \in \mathbb{R}^+$ and $b > a_i \forall i$. 4

3. Solve the equation $x^4 - 2x^2 + 8x - 3 = 0$ by Ferrai's method. 4

4. Find the rank of A, where $A = \begin{pmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{pmatrix}$. 4

5. a) Find the eigenvalue(s) and eigenvector of $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$.

b) If the roots of the equation $x^3 + px^2 + qx + r = 0 \ (r \neq 0)$ are α, β, γ. Find the equation whose roots are $\frac{\alpha+\beta}{\gamma}, \frac{\beta+\gamma}{\alpha}, \frac{\gamma+\alpha}{\beta}$. 3+1

Part – II (16 Marks)

Answer **Q. No. 1** and ***any three*** from the rest four questions. 4×4=16

1. Reduce the equation $xp^2 - 2yp + x + 2y = 0$ to Clairaut's form by using the substitution $x^2 = u$ and $y - x = v$ and then solve it.

Solve the following differential equations (***any three***) :

2. $xy\frac{dy}{dx} = \frac{\left(1+y^2\right)\left(1+x+x^2\right)}{\left(1+x^2\right)}$
3. $\frac{dy}{dx} + \frac{ay}{x} = \frac{b}{x^n}$
4. $y\left(2xy + e^x\right)dx - e^x dy = 0$
5. $\left(D_x^3 - 7D_xD_y^2 - 6D_y^3\right)z = \sin\left(x+2y\right) + e^{3x+y}$

Part – III (12 Marks)

Answer ***any three*** questions.

1. Find by iteration method the real root of the equation $3x - \cos x - 1 = 0$ which lies between 0 and $\frac{\pi}{2}$, correct to three significant figures. 4

2. Find the real positive root of the equation $x^3 - x - 1 = 0$ by Newton-Raphson method, correct to three decimal places. 4
3. Calculate the approximate value of $\int_{-3}^{3} x^4 dx$, by Simpson's one-third rule and Trapezoidal rule, taking six equal sub-intervals. 4
4. Solve the following system of equations $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$ by Gauss-Jordan method. 4
5. Use Euler's method to find the value of y at $x = 0{\cdot}05$ from $\frac{dy}{dx} = x + y + xy$, $y(0) = 1$, taking step size $h = 0{\cdot}025$ 4