B. Sc. 1st Year, 2nd Semester Examination, 2023

Mathematics – I

Paper – GE-2

Time : Two hours | Full Marks : 40

Use separate Answer script for each Part.

Symbols / Notations have their usual meanings.

Part – I (Marks: 16)

Answer ***any four*** of the following five questions.

1. Show that the equation of the chord joining the points $\theta = \theta_1$ and $\theta = \theta_2$ on the circle $r = 2a\cos\theta$ is $r\cos(\theta - \theta_1 - \theta_2) = 2a\cos\theta_1\cos\theta_2$ and hence, find the equation of the tangent to the circle at the point $\theta = \theta_1$. 4

2. If a point lies on the ellipse $\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1$, prove that its polar with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the ellipse $\frac{a_1^2}{a^4}x^2 + \frac{b_1^2}{b^4}y^2 = 1$. 4

3. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and also, find the point of contact. 4

4. A plane passes through a fixed point (a,b,c) and cuts the axes on P, Q and R. Show that the locus of the centre of the sphere $OPQR$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$. 4

5. Find the locus of the poles with respect to the circle $x^2+y^2=a^2$ of the tangents to the circle $x^2+y^2-2ax=0$. 4

Part – II (Marks: 12)

Answer ***any three*** questions.

1. a) Find $D^n\left\{e^{ax}\sin(bx+c)\right\}$. 4

 b) Show that the points of inflexion of the curve $y^2=(x-a)^2(x-b)$ lie on the line $3x+a=4b$. 4

 c) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that $xu_x+yu_y=\sin 2u$. 4

 d) Show that the function

$$f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

 is continuous at (0, 0). 4

 e) i) Is Rolle's theorem applicable to the function $f(x)=\tan x$ in $[0,\pi]$? Justify your answer.

 ii) Find $\underset{x\to 0}{Lt}(\cos x)^{\cot^2 x}$. 2+2

Part – III (Marks: 12)

Answer ***any three*** questions.

1. a) Define Riemann theory of Integration.

 b) State and prove the First Mean Value theorem of Integral Calculus. 1+3

2. If α and φ are positive acute angles, then show that

$$\varphi<\int_0^{\varphi}\frac{dx}{\sqrt{1-\sin^2\alpha\cdot\sin^2 x}}<\frac{\varphi}{\sqrt{1-\sin^2\alpha\cdot\sin^2\varphi}}$$ 4

3. Show that

 a) $B(m,n)=B(m+1,n)+B(m,n+1)$

 b) $\int_0^{\frac{\pi}{2}}(\sin\theta)^4(\cos\theta)^5\,d\theta=\frac{8}{315}$ 2+2

4. a) Show that $\int_{-\infty}^{0}e^{2x}dx$ converges to $\frac{1}{2}$.

 b) Evaluate, if possible, $\int_0^{\infty}\frac{dx}{1+x^2}$. 2+2

5. Evaluate $\iint x^2y^3dxdy$ over the region enclosed by $x\geq 0$, $y\geq 0$, $x^2+y^2\leq 1$. 4