B. Sc. Mathematics (Hons.) Examination, 2023

(1st Year, 2nd Semester)

Differential Equation

Paper – Core-04

Time : Two hours Full Marks : 40

Use separate Answer script for each Part.

Symbols / Notations have their usual meanings.

Part – I (Marks: 20)

Answer ***any two*** questions.

1. a) Define 'integrating factor' of the differential equation $M(x,y)dx + N(x,y)dy = 0$.

   If an integrating factor does exist, is it unique? Justify your answer.

   b) Show that the solutions y_1, y_2 of the 2nd order linear differential equation

   $$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$$

   are linearly independent on [c, d] iff $w(y_1, y_2) \neq 0$ for some x on the interval [c, d].

   c) Show that the system of confocal conics

   $$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$$

   is self orthogonal, where λ is a parameter and a, b are given constants. 4+3+3

[Turn over

2. a) If y_1, y_2 are two linearly independent solutions of $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$ then find the general solution of $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=X(x)$ by using the method of variation of parameters, where P, Q and X are functions of x only.

b) Find general and singular solution (if any) of the differential equation $y=2px+p^2\ \left(p\equiv\frac{dy}{dx}\right)$. 5+5

3. a) Does unique solution of $\frac{dy}{dx}=\sqrt{y}$, $y(0)=0$, exist? Justify your answer.

b) Solve: $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$

c) Reduce the differential equation

$$(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$$

to Euler-Cauchy form and hence solve it. 2+3+5

Part – II (Marks: 20)

Answer ***any two*** questions.

4. Show that $x=0$ is a regular singular point of $(2x+x^3)\frac{d^2y}{dx^2}-\frac{dy}{dx}-6xy=0$ and find it's solution about $x=0$. 10

5. Solve the Legendre's equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0 \text{ about } x=0.$$ 10

6. Find the general solution of the following linear system

$$\begin{pmatrix}\frac{dx_1}{dt}\\ \frac{dx_2}{dt}\\ \frac{dx_3}{dt}\end{pmatrix}=\begin{pmatrix}7 & 4 & 4\\ -6 & -4 & -7\\ -2 & -1 & 2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$ 10