Ex/SC/MATH/UG/CORE/TH/01/2023

B. Sc. Mathematics (Hons.) Examination, 2023

(1st Year, 1st Semester)

Algebra, Geometry & Calculus

Paper – Core-01

Time : Two hours | Full Marks : 40

Use separate Answer script for each Part.

Symbols / Notations have their usual meanings.

Part – I

1. Answer ***any three*** questions. 3×4=12

a) If a, b, c are the sides of a triangle, then prove that $A \leq \frac{s^2}{3\sqrt{3}}$, where A is the area of the triangle and $2s$ is the perimeter of the triangle. 4

b) i) Find the sum of the 33^{th} powers of roots of the equation $x^5 - 1 = 0$.

ii) Find the values of μ and λ such that the rank of the matrix $\begin{bmatrix} 1 & 4 & 2 & 1 \\ 2 & 7 & 5 & 2\lambda \\ 4 & \mu & 0 & 2\lambda+1 \end{bmatrix}$ is 2. 2+2

c) Solve the equation by using Cardan's method: $x^3 - 27x - 54 = 0$ 4

d) i) Prove that the number of primes is infinite.

ii) Find the integers m and n satisfying

$d = 858m + 325n$, where d is the g.c.d. of 858 and 325. 2+2

e) Let $AX = B$ and $RX = S$ be two equivalent systems and α be a solution of $AX = B$. Show that α is also a solution of $RX = S$. 4

2. Answer ***any three*** questions. 3×4=12

a) Show that the point of inflexion of the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x + a = 4b$. 4

b) i) Explain mathematically how to measure the bending of a planar curve at a given point.

ii) If $y = 2\cos x(\sin x - \cos x)$, show that $(y_{10})_{x=0} = 2^{10}$, where y_n denotes the n^{th} order derivative of y with respect to x. 2+2

c) Find the area of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$. Also find the area included between the curve and its asymptotes. 4

d) Find a reduction formula for $\int \sin^m x \cos^n x dx$, where m and n are positive integers. Use the above formula to evaluate $\int_0^{\frac{\pi}{2}} \sin^8 x \cos^6 x dx$. 3+1

e) Find the volume and the surface area of the solid generated by revolving the cardiode $r = a(1 - \cos\theta)$ about the initial line. 4

Part – II

Answer ***any four*** from the following: 4×4=16

1. Show that the equation of the circle which passes through the focus of the conic $(l/r) = (1 - e\cos\theta)$ and touches it at the point $\theta = \alpha$ is given by $r(1 - e\cos\theta)^2 = l\cos(\theta - \alpha) - el\cos(\theta - 2\alpha)$.

2. A plane passes through a fixed point (a, b, c) and cuts the co-ordinate axes at A, B, C respectively. Show that the locus of the center of the sphere OABC is $(a/x) + (b/y) + (c/z) = 2$.

3. Prove that the two circles each of which passes through the points $(0, k)$ and $(0, -k)$ and touches the line $y = mx + c$ will cut orthogonally if $c^2 = k^2(2 + m^2)$.

4. The plane $(x/a) + (y/b) + (z/c) = 1$ cuts the axes at A, B, C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC.

5. Show that the locus of the pole with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of any tangent to the director circle of the ellipse $\frac{x^2}{c^2} + \frac{y^2}{d^2} = 1$ is $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{(c^2 + d^2)}$.