# Model Reference Two-Degree-of-Freedom Adaptive Control for Non-minimum Phase System

Thesis submitted By Mita Pal

**Doctor of Philosophy (Engineering)** 

Electrical Engineering Department Faculty Council of Engineering & Technology Jadavpur University Kolkata, India

2021

# JADAVPUR UNIVERSITY

# Kolkata-700032

### **INDEX NO: 22/16/E**

### 1. Title of the Thesis:

Model Reference Two-Degree-of-Freedom Adaptive Control for Non-minimum Phase System

### 2. Name, Department & Institution of the supervisors:

Dr. Gautam Sarkar	Dr. Ranjit Kumar Barai
Electrical Engineering Department	Electrical Engineering Department
Jadavpur university	Jadavpur University

Dr. Tamal Roy

Electrical Engineering Department MCKV Institute of Engineering

### 3. List of Publication

#### (a) International Referred Journal:

- M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Two-Degree-of Freedom based Model Reference Adaptive Control of Non-minimum Phase System", *IETE Journal of Research*, June 2021, vol.67, no. 2, pp. 1-13.
- ii) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Model Reference Adaptive Feed-Forward Control for Non- Minimum Phase System in Two- Degree-of-Freedom Framework", *Lectures on Modelling and Simulation, Selection at the AMSE Conference Calcutta/India*, November 2017, pp. 86-97.
- iii) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Design of Different reference model based model reference adaptive controller for inversed model non-minimum phase system", *Mathematical Modelling of Engineering Problems*, 2017, vol. 2, no. 4, pp. 75-79.

#### (b) International Conference:

- M. Pal, G. Sarkar, R. K. Barai and T. Roy, "Design of adaptive two-degree-offreedom controller for inversion based reference input tracking of non-minimum phase system", *Michael Faraday IET International Summit.* 2015, pp. 78-83.
- M. Pal, G. Sarkar, R. K. Barai and T. Roy, "Reference input tracking of inversionbased non-minimum phase system using adaptive two-degree-of-freedom control", IEEE *First International Conference on Control, Measurement and Instrumentation (CMI)*, 2016, pp. 508-513.
- iii) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Model Reference Adaptive Control of unstable system with different reference models", *International Conference on Advances In Science And Technology*, Howrah, India, March 2017, pp. 252-256.
- iv) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Two-Degree-of-Freedom Control of Non-minimum Phase Mechanical System", Association for the Advancement of Modeling and Simulation Techniques in Enterprises, Processing of MS-17, Kolkata, India, November 2017, pp. 365-378.
- v) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Model Reference Adaptive Control of Wheel Mobile Robot", *Michael Faraday IET International Summit*, Kolkata, India, 2020, pp. 236-241.
- vi) M. Pal, R. K. Barai, G. Sarkar and T. Roy, "Elimination of Initial undershoot of Non-minimum Phase System using Model Reference Adaptive Control", *IEEE Second International Conference on Control, Measurement and Instrumentation* (CMI), 2021, pp. 41-45.

### 4. List of Patents: Nil

#### 5. List of presentation in National / International Conference:

- M. Pal, G. Sarkar, R. K. Barai and T. Roy, "Design of adaptive two-degree-of-freedom controller for inversion based reference input tracking of non-minimum phase system", *Michael Faraday IET International Summit*, 2015, pp. 78-83.
- M. Pal, G. Sarkar, R. K. Barai and T. Roy, "Reference input tracking of inversion-based non-minimum phase system using adaptive two-degree-of-freedom control", *IEEE First International Conference on Control, Measurement and Instrumentation (CMI)*, 2016, pp. 5080-5013.

- iii) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Model Reference Adaptive Control of unstable system with different reference models", *International Conference on Advances In Science And Technology*, Howrah, India, March 2017, pp. 252-256.
- iv) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Two-Degree-of -Freedom Control of Nonminimum Phase Mechanical System", Association for the Advancement of Modeling and Simulation Techniques in Enterprises, Processing of MS-17, Kolkata, India, November 2017, pp. 365-378.
- v) M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Model Reference Adaptive Control of Wheel Mobile Robot", *Michael* Faraday IET International Summit, 2020, Online Conference, Kolkata, India, pp. 236-241.
- vi) M. Pal, R. K. Barai, G. Sarkar and T. Roy, "Elimination of Initial undershoot of Nonminimum Phase System using Model Reference Adaptive Control", 2021 IEEE Second International Conference on Control, Measurement and Instrumentation (CMI).

# "Statement of Originality"

1 Mita Pal registered on 27.09.2016 do hereby declare that this thesis entitled "Model Reference Two-Degree-of-Freedom Adaptive Control for Non-minimum Phase System", contains literature survey and original research work done by the undersigned candidate as part of Doctoral studies.

All information in this thesis have been obtained and presented in accordance with existing academic rules and ethical conduct. I declare that, as required by these rules and conduct, I have fully cited and referred all materials and results that are not original to this work.

I also declare that I have checked this thesis as per the "Policy on Anti Plagiarism, Jadavpur University, 2019", and the level of similarity as checked by iThenticate software is 6 %.

nitolal Signature of Candidate:

Date: 20, 12, 2021

Certified by Supervisor(s): (Signature with date, seal)

1. Charton Antelectrostrof work Department Kalka - 700 032 2. Electrical finglatering Department JADAUPI IN PUIVERSITY he anu 100 032 3. 12 21 HEAD OF THE DEPARTMENT ELECTRICAL ENGINEERING MCKV INSTITUTE OF ENGINEERING

# CERTIFICATE FROM THE SUPERVISORS

Index no. 22/16/E

This is to certify that this thesis titled "Model Reference Two-Degree-of-Freedom Adaptive Control for Non-minimum Phase System" submitted by Smt. MITA PAL, who got her name registered on 27.09.2016 for the award of Ph. D. (Engg.) degree of Jadavpur University, is absolutely based on her own work under the supervision of DR. GAUTAM SARKAR, DR. RANJIT KUMAR BARAI and DR. TAMAL ROY, and that neither her thesis nor any part of the thesis has been submitted for any degree/diploma or any other academic award anywhere before.

Electrical Engineering Department ~h~ped1421212.

DR. GAUTAM SARKAR

Signature of the Supervisor and date with official seal

Professor

DR. RANJIT KUMAR BARAI

Signature of the Supervisor and date with official seal

Kolkata-700 032

Canal Ro 120/12/21

DR. TAMAL ROY Signature of the Supervisor and date with official seal

HEAD OF THE DEPARTMENT ELECTRICAL ENGINEERING MCKV INSTITUTE OF ENGINEERING

# **ACKNOWLEDGEMENT**

I would like to convey my deepest gratitude to my supervisors *Dr. Gautam Sarkar*, *Dr. Ranjit Kumar Barai* and *Dr. Tamal Roy* for their contribution during the time of my research work. I would like to convey my wholehearted gratitude to *Dr. Gautam Sarkar*, who has been a constant source of encouragement and enthusiasm during this period of doctoral research work. I do not have enough words to express my deep and sincere appreciation. I would like to convey my sincere gratitude to *Dr. Ranjit Kumar Barai* for providing me the opportunity to pursue my doctoral research work on such an attractive field of control system engineering. I am very much thankful to him for his wisdom, valuable guidance and encouragement throughout this research work. I would like to convey my heart-felt gratitude to *Dr. Tamal Roy* for his endless co-operation from the initial moment to the final moment of my research work. His valuable comments and passion towards the perfection on every single aspect of research work make the thesis more presentable and perfect.

I wish to thank each individual of control system laboratory, especially, I am indebted to Probir da (Probir Ghosh), without his guidance, it was not possible for me to construct the hardware experimental set-up.

I would like to convey my sincere thanks to Jadavpur University for providing me the RUSA Fellowship during the period, march 2019 to march 2020.

I would like to convey my sincere thanks to all the research scholars of Electrical Engineering department, who are associated with me during this crucial period of my research at the Jadavpur University.

Finally, I would like to convey my deepest gratitude to my precious husband, whom I lost due to COVID -19, without his inspiration I couldn't start, continue, and complete my research work. Today, he is not bodily present with us, but he is the happiest soul looking from the heaven. I would like to express my sincere appreciation to my beloved son, without his enthusiasm and co-operation I couldn't finish my doctoral research work. He always gives me the positive vibes, especially, in this hard time of my life.

At the end, I wish to thank my entire extended family member, whose good wishes always work for me in this struggle period of time and I must thank to almighty God for giving me the strength and patience in this disastrous pandemic situation.

# THESIS SUMMERY

Design of the tracking control system for any dynamic system is indispensable in the area of research and the industrial environment. Controlling the plant is essentially required, so that it can follow the desired trajectory or reference input trajectory. Different feed-forward techniques like model-based feed-forward, enhanced feed-forward, two feed-forward, adaptive feed-forward, feed-forward model inverse control have been applied till date, but among all these techniques, adaptive feed-forward is the most effective control approach. Adaptive feed-forward needs the inverse of the system model, but it is feasible when the system is linear and its inverse model exists.

There are two major classes in the dynamic system in control system engineering, MP and NMP system. RHP zero exhibits non-minimum characteristics in the system leads to internal instability of the system dynamics and obstruct good tracking of desired trajectory. It is one of the difficult tasks to design the tracking control system of the system, which has right hand plane (RHP) zeros in the complex plane. In the frequency domain, RHP zeros produce large phase lag, and in the time domain it exhibits initial undershoot.

Initial undershoot and overshoots are found in the step response of NMP systems, which are unavoidable and obviously undesirable dynamic characteristics as they obstruct the good tracking of NMP systems. Initial undershoot and overshoots are produced due to RHP zeros in the transfer function model of the NMP system. It has been experimented that odd and even number RHP zeros exhibit undershoots and overshoots respectively. So, obviously, an odd number of RHP zeros are responsible for the initial undershoot which needs to be reduced or eliminated for perfect tracking of the NMP system.

Exact set point tracking is required in many of the practical, dynamic systems and it may be possible if the inverse transfer function model of the system is connected in cascade with the original one. But this technical concept is confined to MP systems only, whereas, in the case of an NMP system, it produces unbounded output response. The adaptive inverse model technique is also a very useful tracking control method for dynamic systems, but it is applicable for MP systems only. In the case of an NMP system, it gives an unbounded output response as RHP zero becomes RHP poles. So, an efficient method is essentially needed for achieving the approximate desired trajectory tracking of an unstable inversion-based NMP system. Here, one novel control technique has been proposed, where direct MRAC and arbitrary pole placement method based SFB schemes are considered for feed-forward and

feedback compensation in the 2DOF framework, respectively. The inverse transfer function model of the NMP system is considered as a plant for feed-forward controllers, whereas a non-inverse NMP system has been made for feedback compensation, and both the controllers are decoupled with each other.

The plant for the direct MRAC scheme needs to be inversed, so, the designing of this controller is very difficult for the NMP system, as the inverse transfer function model of NMP becomes unstable due to its RHP zeros.

So, the development of a proper control system for the trajectory tracking control performance of the stable or unstable NMP system with the direct MRAC structure is very difficult, as this control scheme essentially requires inversion of the system model.

As the RHP zeros convert to RHP poles after inversion of the transfer function model of the NMP system model, it obviously produces an unbounded output response. To circumvent this problem regarding the inverse model based tracking approach of the NMP system, a MRAC based 2DOF control technique has been proposed in this research work.

SFB control using appropriate closed loop poles has been considered as one of the best control techniques for an unstable system.

First, we designed the MIT rule based MRAC as a feed-forward compensation of the 2DOF control structure for an inverse model of an NMP system and it has been applied to an unstable NMP system for reference input trajectory tracking. Two major problems are found in this MIT-based MRAC control algorithm. One is, the initial undershoot has not completely been nullified, and the other is, stability proof of the control algorithm could not be performed. To mitigate these disadvantages using the MIT rule MRAC, Lyapunov Design technique has been developed for feed-forward MRAC compensation employing the 2DOF control technique. The same numerical example, which has been taken for the MIT based control scheme, is again considered for the Lyapunov based MRAC control technique, and it has been observed that it produces better results than the previous one.

In this research work, it has been experimented using different types of RMP model, like, un-damped, under-damped, critically damped and over-damped RMP model with the same plant model in this MRAC structure based on proposed control methodology and we observed that the output response of the controlled plant literally follows the trajectory paths and dynamic characteristics of those dissimilar RMP's output trajectories.

Undesirable initial undershoot and overshoots are obvious in the step response of the NMP system and their number depending on the number of zeros placed on the RHP of the complex plane. Here, three types of SISO NMP systems, which have one, two and three RHP zero, respectively, and one TITO NMP system, which has one RHP zero, have been considered for the verification of this MRAC-based proposed control scheme.

We have applied the proposed control methodology to the 3<sup>rd</sup> order and 4<sup>th</sup> order NMP system model. One practical mechanical hardware plant, which consists of mass, spring, damper elements and produces a 4<sup>th</sup> order transfer function model is considered because one part of this mechanical hardware plant exhibits NMP characteristics. The output responses of the above controlled plants have been compared with the PID and SFB and MRAC controlled plants and better results are observed than the other three control schemes. To verify the tracking performance, along with the unit step input, unit ramp and a sine wave signal have also been applied to those closed loop controlled plants. The 4<sup>th</sup> order mechanical NMP system has also been compared with the disturbance rejection controller, which has been presented in other literature and better performance has been observed.

After the satisfactory performance of the proposed MRAC based 2DOF control methodology on the 2<sup>nd</sup> order, 3<sup>rd</sup> order unstable NMP plant and 4<sup>th</sup> order practical mechanical system, a realistic Op-amp based 2<sup>nd</sup> order NMP system has been developed in MATLAB SIMULINK environment, which has one zero at RHP of S plane. Then the inverse model of that NMP system has been constructed. At first, the SFB controller of that NMP plant has been designed, and then the proposed MRAC scheme for the inverse NMP plant has been designed which plays the feed-forward compensation of the 2DOF control structure, whereas the SFB controlled non-inverse NMP plant acts as its feedback counterpart. We have constructed a 1<sup>st</sup> order RMP model using the MATLAB toolbox, for the feed-forward MRAC structure, whose output trajectory should be followed by the closed loop controlled plant. This realistic Op-amp based 2DOF control structure is effectively verified by the software simulation in the MATAB SIMULINK environment.

Next, to validate the proposed control algorithm, an analog simulation of the same Opamp based NMP system with real Op-amp and necessary electronic components in a real environment has been performed.

After examining both the controlled scheme by using software simulation in the MATLAB SIMULINK environment and analogue simulation in a real hardware environment,

it is clearly observed that analogue simulation in a real-time environment yields better results than the software simulation, though the gain adjustment with the electronics components is a really tough task for the researcher. The initial undershoot, which is obvious in the step response of the NMP system, has been reduced in software simulation but completely nullified by hardware simulation.

Finally, to conclude, researchers can take the freedom to claim that the proposed control law is not only able to solve the tracking and stability problem of the NMP system theoretically, but it also exhibits the robust performance of this control approach in a real hardware environment.

				Page No.
Statemen	t of Ori	ginality		iv
Certificat	te from	the Supe	ervisors	V
Acknowledgement				
Thesis Summery				
Table of	content			xi
List of Fi	gure			xvi
List of Ta	able			XX
List of A	cronyms	5		XX
1.	Introd	luction		1-9
	1.1	Non-mi	inimum Phase System	1
	1.2	Motiva	tion	3
	1.3	Probler	n Statement	4
	1.4	Researc	ch Potential	6
	1.5	Thesis	Organization	7
	1.6	Chapter	r Summery	9
2.	Litera	ature Survey		10-14
	2.1	Introdu	ction	10
	2.2	Difficu	Ities in tracking Control of NMP System	10
	2.3	Overvie	ew of Previous and Present Control Technique	12
	2.4	Chapter	r Summery	14
3.	2DOF	Control	Technique	15-19
	3.1	Introdu	ction	15
	3.2	Propose	ed 2DOF Control structure	17
		3.2.1	Feed-forward Control Scheme	18
		3.2.2	Feedback Control Scheme	18
		3.2.3	Justification for Adopting 2DOF Control framework for NMP System	18
		3.2.4	Chapter Summery	19
4.	MRA	C Contro	ol Technique as Feed-forward Compensation	20-22
	4.1	Introdu	ction	20
	4.2	MRAC	Structure	20
		42.1	Uncontrolled Plant	21
		4.2.2	Reference Model Plant	21
		4.2.3	Inner Feedback Control Loop	21

# **CONTENTS**

					Page No.
		4.2.4	Outer Cont	rol Loop with Adjustment mechanism	22
	4.3	Robustr	ness of MRA	C Scheme	22
	4.4	Chapter	Summary		22
5.	SFB	Control a	s Feedback	Compensation	23-27
	5.1	Introdu	ction		23
	5.2	SFB Co	ontrol Schem	e	23
		5.2.1	Controllabi	lity Check	23
		5.2.2.	Choice of a	closed loop poles	24
		5.2.3	Derivation	of State Feedback Gain Matrix	24
	5.3	Chapter	Summary		27
6.	Desig based MIT	n of Adaj Referenc rule base	ptive Two-D ce Input Tra d MRAC	egree-of Freedom Controller for Inversion acking of Non-minimum Phase System using	28-42
	6.1	Introdu	ction		28
	6.2	Problem	n statement		29
	6.3	2DOF c	controller		30
	6.4	Pole pla	acement metl	nod	30
	6.5	MRAC	Control Alg	orithm using MIT rule in 2DOF framework	31
		6.5.1	MIT rule b	based MRAC structure	31
		6.5.2	Control Al	gorithm	32
	6.6	Numeri	cal Example		36
	6.7	Simulat	ion Result		37
	6.8	Result A	Analysis		38
	6.9	Difficul	ties with MI	T Technique	41
	6.10	Chapter	Summery		42
7.	Gene	ralized N	IRAC Empl	oyed in 2DOF Framework	43-49
	7.1	Introduc	ctions		43
	7.2	General Technic	ized Control Jue	Algorithm of MRAC using Lyapunov Design	43
		7.2.1	Flow Cha	rt Diagram	43
			7.2.1.1	Consideration of Plant Model and Reference Model Plant	43
			7.2.1.2	n <sup>th</sup> order Tracking Error	45
			7.2.1.3	Choice of Control Input	45
			7.2.1.4	Derivative of Tracking Error	45
			7.2.1.5	Finding the Parameter Adjustment Rule	46
			7.2.1.6	Choice of Lyapunov Function	47

				Page No.
		7.2.1.7	Derivation of Lyapunov Function	47
		7.2.1.8	Finding Control Parameter	47
		7.2.1.9	Finding Control Input	48
	7.3	Stability Proof		48
	7.4	Barbalat's Lemma		48
	7.5	Chapter Summary		49
8.	Refer Syste Lyap	rence Input Trackin m using Adaptive T unov Design Techni	g of inversion-based Non-Minimum phase wo-Degree-of-Freedom Control using que	50-62
	8.1	Introduction		50
	8.2	Problem Statemen	t	53
	8.3	Two-Degree-of-Fr	eedom Control	53
	8.4	State Feedback Co	ntrol	53
	8.5	MRAC using Lyap	ounov Design Technique	54
	8.6	Numerical Example	le	55
	8.7	Simulation Results	3	57
	8.8	Result Analysis		62
	8.9	Chapter Summery		62
9.	Desig contr struc	n of different refere oller for inversed m ture	ence model based model reference adaptive odel non-minimum phase system in 2DOF	63-72
	9.1	Introduction		63
	9.2	Problem Statemen	t	65
	9.3	MRAC Employing plant	g Lyapunov Stability Theory for second order	65
	9.4	Numerical Examp	le	68
	9.5	Simulation Result		69
	9.6	Result Analysis		71
	9.7	Chapter Summery		72
10	Elimi	ination of Initial uno	lershoot of Non-minimum Phase System	73-83
	10.1	Introduction		73
	10.2	Problem Statemer	nt	76
	10.3	MRAC Scheme		76
	10.4	Numerical Examp	ble	78
		10.4.1 SISO	with one RHP zero	78
		10.4.2 SISO	with two RHP zeros	78
		10.4.3 SISO	with three RHP zeros	78

				Page No.
		10.4.4	Bench marked TITO with one RHP zero	78
	10.5	Simulation	Result	79
	10.6	Result Ana	ılysis	79
	10.7	Chapter Su	Immery	83
11.	Two-I Systen	Degree-of-fre n	edom Control of Non-Minimum Phase Mechanical	84-96
	11.1	Introducti	on	84
	11.2	Problem I	Formulation	87
	11.3	Mechanic	al System Model with NMP Characteristics	87
	11.4	Descriptio	on of Different Controller	88
		11.4.1	SFB Controller	88
		11.4.2	PID Controller	89
		11.4.3	MRAC Compensation employed in 2DOF framework	89
	11.5	Simulatio	n Results	89
	11.6	Result Ar	nalysis	96
	11.7	Chapter S	lummery	96
12.	MRAO 2DOF	C Feed-Forv Framework	vard Control for Non- Minimum Phase System in	97-109
	12.1	Introducti	on	97
	12.2	Problem I	Formulation	99
	12.3	Two-Deg	ree-of-Control Structure	99
	12.4	Different	Controller	99
		12.4.1	MRAC Scheme	99
		12.4.2	SFB Controller	100
		12.4.3	Proposed Control Scheme	100
	12.5	Numerica	l Example	101
	12.6	Simulatio	n Result	101
	12.7	Result Ar	nalysis	108
	12.8	Chapter S	lummery	109
13.	Two-I Non-n	Degree-of-Fro ninimum Pha	eedom based Model Reference Adaptive Control of ase System	110-123
	13.1	Introducti	on	110
	13.2	Problem f	formulation	113
	13.3	Modeling	of Op-amp based NMP System	113
		13.3.1	Mathematical Description of the Model	113
		13.3.2	Block diagram of NMP System Model	114
		13.3.3	Hardware Op-amp based NMP System	114

-Be 1 (01
115
115
116
123
123
25-129
125
128
129

# List of Figures

Figure No	Figure Caption	Page No
Fig. 1.1	Block diagram of series connected inverse and non-inverse transfer function model of MP system	4
Fig. 1.2	Block diagram of 2DOF controlled NMP plant	4
Fig. 1.3	Block diagram of 2DOF controlled NMP plant	5
Fig. 1.4	Block diagram of 2DOF controlled NMP with MRAC as feed-forward and SFB as feedback controller	6
Fig. 3.1	2DOF control structure	17
Fig. 4.1	Block diagram of MRAC structure	20
Fig.5.1	Block diagram of State variable feedback system	23
Fig. 6.1	Block diagram of MIT rule based MRAC structure	31
Fig. 6.2	Model Reference Adaptive Control Architecture using MIT rule	35
Fig. 6.3	Unit step response of uncontrolled NMP system	38
Fig. 6.4	Unit step response of uncontrolled inverse NMP system	38
Fig. 6.5	Unit step response of cascaded non-inverse and inverse NMP system	39
Fig. 6.6	Unit step response of SFB controlled NMP system	39
Fig. 6.7	Unit step response of control input of 2DOF controlled NMP system	40
Fig. 6.8	Unit step response of tracking error of 2DOF controlled NMP system	40
Fig. 6.9	Unit step response of 2DOF controlled inversion based NMP system	41
Fig. 7.1	Flow chart diagram of MRAC Algorithm	44
Fig. 8.1	Unit step response of cascaded inverse and non-inverse MP system	57
Fig. 8.2	Unit step response of cascaded inverse and non-inverse NMP system	57
Fig. 8.3	Unit step response of 2DOF controlled NMP system with 1 <sup>st</sup> order reference model	58
Fig. 8.4	Unit step response of control input for 2DOF controlled NMP system with $1^{st}$ order reference model	58
Fig. 8.5	Unit step response of 2DOF controlled NMP system with 2 <sup>nd</sup> order reference model	59
Fig. 8.6	Unit step response of control input for 2DOF controlled NMP system with the $2^{nd}$ order reference model	59
Fig. 8.7	Unit ramp response of 2DOF controlled NMP system with $2^{nd}$ order reference model	60
Fig. 8.8	Unit ramp response of control input for 2DOF controlled NMP System with the $2^{nd}$ order reference model	60
Fig. 8.9	Unit parabolic response of 2DOF controlled NMP system with 2 <sup>nd</sup> order reference model	61
Fig. 8.10	Unit parabolic response of control input for 2DOF controlled NMP system with the $2^{nd}$ order reference model	61

Figure No	Figure Caption	Page No
Fig. 9.1	Block diagram of MRAC structure	65
Fig. 9.2	Unit step response of uncontrolled inverse model of NMP system	69
Fig. 9.3	Unit step response of MRAC controlled inverse NMP system in 2DOF framework using under-damped (damping ratio=0.1) reference model	69
Fig. 9.4	Unit step response of MRAC controlled inverse NMP system in 2DOF framework using un-damped (damping ratio=0) reference model	70
Fig. 9.5	Unit step response of MRAC controlled inverse NMP system in 2DOF framework using critically damped (damping ratio=1) reference model	70
Fig. 9.6	Unit step response of MRAC controlled inverse NMP system in 2DOF framework using over damped (damping ratio=12) reference model	71
Fig.10.1	Schematic diagram of MRAC structure	76
Fig. 10.2	Unit step response of uncontrolled SISO NMP systems with one RHP zero	79
Fig. 10.3	Unit step response of 2DOF controlled SISO NMP systems with one RHP zero	80
Fig. 10.4	Unit step response of uncontrolled SISO NMP system with two RHP zero	80
Fig. 10.5	Unit step response of 2DOF controlled SISO NMP systems with two RHP zeros	81
Fig. 10.6	Unit step response of uncontrolled SISO NMP system with three RHP zeros	81
Fig. 10.7	Unit step response of 2DOF controlled SISO NMP systems with three RHP zeros	82
Fig. 10.8	Unit step response of uncontrolled TITO NMP system with one RHP zero	82
Fig. 10.9	Unit step response of 2DOF controlled TITO NMP systems with one RHP zero	83
Fig 11.1	Schematic representation of mechanical realization of NMP system	87
Fig. 11.2	Schematic diagram of SFB controller	88
Fig. 11.3	Schematic diagram of PID control scheme	
Fig. 11.4	Unit step response of uncontrolled Mechanical realized NMP system (closure view)	90
Fig. 11.5	Unit step response of uncontrolled inverse mechanical realized NMP system	90
Fig. 11.6	Unit square wave response of cascaded inverse and non-inverse mechanical realized NMP system	91
Fig. 11.7	Unit square wave response of SFB controlled mechanical realized NMP system	91
Fig. 11.8	Unit square wave response of PID controlled mechanical realized NMP system	92
Fig. 11.9	Unit square wave response of a 2DOF controlled mechanical realized NMP system	92
Fig. 11.10	Superimposed unit square wave response of SFB, PID, and 2DOF controlled mechanical realized NMP system	93
Fig.11.11	Superimposed unit step response of SFB, PID, and 2DOF controlled mechanical realized NMP system	93

Figure No	Figure Caption	Page No
Fig. 12.1	Unit step response of uncontrolled 3 <sup>rd</sup> order NMP system	101
Fig. 12.2	Unit step response of cascaded inverse and non-inverse $3^{rd}$ order NMP system	102
Fig. 12.3	Unit step response of SFB controlled 3 <sup>rd</sup> order NMP system	102
Fig. 12.4	Unit step response of MRAC controlled 3 <sup>rd</sup> order NMP system	102
Fig. 12.5	Unit step response of 2DOF controlled 3 <sup>rd</sup> order NMP system	103
Fig. 12.6	Unit step response of control input for 2DOF controlled $3^{\mbox{\scriptsize rd}}$ order NMP system	103
Fig. 12.7	Sine wave response of 2DOF controlled 3rd order NMP system	103
Fig. 12.8	Unit ramp response of 2DOF controlled 3 <sup>rd</sup> order NMP system	104
Fig. 12.9	Superimposed unit step response of SFB, MRAC, and 2DOF controlled 3 <sup>rd</sup> order NMP system	104
Fig. 12.10	Unit step response of uncontrolled 4 <sup>th</sup> order NMP system	104
Fig. 12.11	Unit step response of cascaded inverse and non-inverse $4^{\text{th}}$ order NMP system	105
Fig. 12.12	Unit step response of SFB controlled 4th order NMP system	105
Fig. 12.13	Unit step response of SFB controlled 4 <sup>th</sup> order NMP system (closure view)	105
Fig. 12.14	Unit step response of MRAC controlled 4 <sup>th</sup> order NMP system	106
Fig. 12.15	Unit step response of 2DOF controlled 4 <sup>th</sup> order NMP system	106
Fig. 12.16	Unit step response of control input for 2DOF controlled $4^{\text{th}}$ order NMP system	106
Fig. 12.17	Unit sine wave response of 2DOF controlled NMP system	107
Fig. 12.18	Unit ramp response of 2DOF controlled 4th order NMP system	107
Fig. 12.19	Superimposed unit step response of SFB, MRAC, and 2DOF controlled $4^{\text{th}}$ order NMP system	107
Fig. 13.1	Block diagram representation of NMP system	114
Fig. 13.2	Circuit diagram of Op-amp based NMP system	115
Fig. 13.3	Unit step response of uncontrolled NMP system	117
Fig. 13.4	Unit step response of uncontrolled inverse NMP system	117
Fig. 13.5	Superimposed unit step response of inverse and non-inverse NMP system	118
Fig. 13.6	Unit step response of cascaded inverse and non-inverse NMP system	118
Fig. 13.7	Unit step response of SFB controlled NMP system	118
Fig. 13.8	Unit step response of RMP for MRAC	119
Fig. 13.9	Unit step response 2DOF based MRAC controlled NMP system	119
Fig. 13.10	Square wave response of uncontrolled NMP system	120
Fig. 13.11	Square response of uncontrolled inverse NMP system	121
Fig. 13.12	Square wave response of cascaded inverse and non-inverse NMP system	121

Figure No	Figure Caption	Page No
Fig. 13.13	Superimposed square wave response of inverse and non-inverse NMP system	121
Fig. 13.14	Square wave response of SFB controlled NMP system	122
Fig. 13.15	Square wave response of RMP output for MRAC scheme	122
Fig. 13.16	Square wave response of 2DOF based MRAC controlled NMP system	122
Fig. 13.17	Experimental hardware set-up for analogue simulation of 2DOF based MRAC controlled NMP system	123
Fig. A.1	Realistic Op-amp based Simulink model of NMP system	150
Fig. A.2	Realistic Op-amp based Simulink model of inversed NMP System	150
Fig. A.3	Realistic Op-amp based Simulink model of superimposed inversed NMP and non-inversed NMP system	151
Fig. A.4	Realistic Op-amp based Simulink model of cascaded inversed NMP and non-inversed NMP system	151
Fig. A.5	Realistic Op-amp based Simulink model of reference model plant for MRAC structure	152
Fig. A.6	Realistic Op-amp based Simulink model of SFB controlled Op-amp based NMP system	152
Fig. A.7	Realistic Op-amp based Simulink model of 2DOF controlled Op-amp based NMP system	153
Fig. A.8	Hardware experimental setup for 2DOF controlled NMP system	153

Table No.	Table Caption	Page No.
9.1	Transfer function model of different RMP on the basis of damping ratio	68
9.2	Comparison of characteristics specification of step response of Reference model and plant model	72
11.1	Comparison chart of % Error in RMS value	94
11.2	Comparison chart of controlled system's dynamic characteristics	95
11.3	Comparison of dynamics characteristics between proposed 2DOF controlled plant with disturbance rejection controlled plant	95
12.1	Comparison chart of controlled 3 <sup>rd</sup> order system and 4 <sup>th</sup> order system's dynamic characteristics	109
13.1	Comparison of chart of controlled system dynamics	123
13.2	Comparison of software and hardware simulation	123

# List of Table

# List of Acronyms

NMP	=	Non-minimum Phase System
MP	=	Minimum Phase
2DOF	=	Two-Degree-of -Freedom
SFB	=	State Feedback Control
PID	=	Proportional Integral Derivative
MRAC	=	Model Reference Adaptive Control
RHP	=	Right Hand Plane
RMP	=	Reference Model Plant
SISO	=	Single Input Single Output
TITO	=	Two Input Two Output

# CHAPTER 1 Introduction

#### 1.1 Non-minimum Phase System

In control system engineering, there is considerable research work on control systems found since 1950 and they can be classified in innumerable ways. Depending on the system property, the system can be classified as a minimum phase (MP) or non-minimum phase (NMP) system [1]. The zeros of the system play a major role in the performance of the controlled system, which has been studied from the early decades. An NMP system is that system which has at least one zero placed on the right-hand plane (RHP) of the complex plane. A system with unstable and anti-causal characteristics can also be termed as an NMP system [5].

In MP and NMP systems, both may have the same amplitude, but the NMP system has a larger phase angle than the MP system [3]. As the NMP system has a higher percentage of uncertainty due to the zeros placed at the RHP of the complex plane, it produces a greater level of instability or uncertainty. So, the control system without zero is much easier than the controlling of the system, which consists of RHP zeros [1].

The system representation of any physical model may be done by a transfer function model and a state space model. By observing the pole zero position of the transfer function model in the s plane, the system can be categorized into stable, unstable or minimum phase (MP) and non-minimum phase (NMP)systems. Considering, the two systems can be described by the following equations, respectively [10],

$$G_1(S) = \frac{1 + S\tau}{1 + S\tau_1}$$
[1.1]

and

$$G_2(S) = \frac{1 - S\tau}{1 + S\tau_1}$$
[1.2]

Where, in both the transfer functions, poles are, at  $-\frac{1}{\tau_1}$ , but in the case of zero, it is

$$-\frac{1}{\tau}$$
 for,  $G_1(s)$  and  $\frac{1}{\tau}$  for  $G_2(s)$ .

In the frequency domain, though the amplitude responses of both the above systems are the same, there is a difference in phase angle response. It has been found that the phase contribution of the system, G2(S), is larger than the system, G1(S). The G1(S), G2(S) are named the MP system and the NMP system respectively [2]. There is a limited number of open literature describing the design of controllers for the higher order system model, so, in most cases, the higher order model is approximated to the first order plus a time delay which leads to the NMP system [3]. So, the time delay in the dynamics of the system is another cause of the NMP characteristics of the system. Practical real-time systems like an underwater vehicle system naturally suffer from time delay in its control action, which exhibits NMP characteristics in its dynamics [4].

In these kinds of systems, most of the time, there is an undershoot or over and understood responses have been found in the step response of the system. "Undershoot" is one of the undesirable phenomena which are obvious in the step response of the NMP system. The undershoot is that output response due to the step input signal, which is the first move to the opposite side of the desired output response and then changes to the desired direction and tries to converge to its positive steady state value [5].

Based on the number of zeros and the location of the zeros placed at the RHP of S plane, the system dynamic exhibits the phenomena of undershoot or over and undershoot in the step response of the NMP system.

The step response of a system with an odd number of RHP zero produces the output trajectory, which, initially, goes opposite to the desired steady state output response. Internal instability is inherent in the system dynamics due to these unavoidable undershoots or overshoots in the time response of the NMP system [6].

Unlike the MP system, there are some limitations present in the NMP system, like, good regulation can be obtained from the invertible MP system, whereas the NMP system lacks it [7]. Also, in the closed loop form of the NMP system, it becomes unstable and as a result, root locus based control system design leads to constraints. Zero or zeros placed at RHP reduce the stable region, which is required for the designing of suitable control for NMP systems, but RHP zero makes some of the NMP systems totally unstable and uncontrollable. So, it may be stated that open loop zeros, which are placed at the RHP on the s plane, are responsible for the crucial performance of the closed loop system [7].

2

### 1.2 Motivation

Some peculiar characteristics commonly found in the step response of the NMP system are initial undershoot, overshoot and zero crossing, which are not acceptable as they obstruct perfect tracking of the NMP system. When we want to change the temperature of the water in a hot water system, or shower, we turn the tap in the right direction, but, at first, or, initially, we get cold water in a state of hot water. This phenomenon is very common in our day-to-day life. Another example is, it may be observed frequently, that the tires of a car do not respond according to the movement of the steering of the car. If a car driver wants to move the car in the right direction, and according to that, he turns the steering to that direction, but, the tires first move to the left direction. In the above examples, it has been noticed, that, after some time, the desired performance of the hot water system and car system respond to the command signal.

The same incident happens with the steering of a bicycle or motorcycle [2]. A bicycle or motorcycle is inherently in an unstable state and falls over [8], [9]. In the case of an aircraft system, the nose of the aircraft moves up and the aircraft begins to climb when the pilot pulls back the elevator control stick. As the typical undershoot phenomenon is obvious in the aircraft system, the aircraft first goes down as the pivots of the aircraft body goes down into the center of gravity, after a delay, it moves up, and the passenger sitting in the tail of the aircraft can feel this peculiar phenomenon of NMP characteristics.

In a level control system, the volume of the boiling water may need to be raised, and it has been done by adding the cold water, but in this action, it has been found, that, initially, the water level becomes reduced due to the bubble formation that occurs in the level control system.

These unavoidable phenomena we always experienced in a real practical environment and this inspired us to design the perfect control system required for this type of system, named the NMP system.

When the desired output is just a constant value, it can be treated as a special case of regulating problem [11]. But, in most cases, the design of a tracking control system for any dynamic system is essentially required and the goal of the controlled system is to meet the desired time-varying signal in industrial and research environments. There are so many methods of developing tracking controls found in the literature, but few of them explain perfect set point tracking of controlled systems.

3

Exact set point tracking can be possible by the cascaded connection of an inverse transfer function model with a non-inverse transfer function model, because poles and zeros of both functions are cancelled to each other. But, this technique is only applicable to the MP system.

In the case of an NMP system, it produces an abrupt unbounded output response. This phenomenon also encourages us to build an efficient tracking controller for NMP systems.

## **1.3** Problem Statement

In control system engineering, perfect reference input tracking can be possible by using an inverse transfer function model based system shown in the following diagram-



# Fig .1.1 Block diagram of cascaded inverse and non-inverse system model and outputs for MP systems

But it is restricted to minimum phase (MP) systems only, where all zeros are placed in the left-hand plane (LHP) in the frequency domain; whereas, the inverse model-based control method does not work because the right-hand plane (RHP) zero-converted to RHP poles when the inversion of the transfer function model is done and it leads to instability.



Fig. 1.2 Block diagram representation of the series connected inverse and non-inverse transfer function model of MP system

In Fig. 1.2, exact matching of output with reference input is possible as poles ( $-P_1, -P_2, ..., -P_n$ ) and zeros  $(-Z_1, -Z_2, ..., -Z_m)$  of the polynomials become removed by the technique of pole zero cancellation, but, for the NMP, this method will not work, rather, it will exhibit oscillatory output response.

It is very challenging for NMP systems as the tracking controller has to solve a twofold problem. One is to stabilize the NMP plant, and the other is to track the desired trajectory specified by the set point signal [12].

These difficulties may be overcome by the proposed structure of Fig. 1.3.



Fig. 1.3 Block diagram of 2DOF controlled NMP plant

Here, a feedback controller has been used to stabilize the system, where the feedforward controller is connected in series with a feedback controller to get the required tracking control system.

Adaptive feed-forward compensation is one of the feasible techniques to solve the tracking problem of a dynamic system. But, as it also requires an inverse model of a dynamic system, it is a challenging task to design a proper adaptive inverse feed-forward control technique for the NMP system. In order to circumvent this difficulty in the design of a suitable tracking controller for an NMP system, a 2DOF control structure has been proposed, where the MRAC controller as a feed-forward controller and the state feedback (SFB) compensator as feedback controller and both the controllers are decoupled with each other shown in the Fig. 1.3.



Fig. 1.4 Block diagram of 2DOF controlled NMP with MRAC as feed-forward and SFB as feedback controller

Here, an inversion based model of the NMP system acts as a plant for the feed-forward scheme to get the desired trajectory tracking performance, and a non-inversion based NMP system has been considered for the feedback controller in the 2DOF control structure. Feedback controllers in the 2DOF scheme solve the stabilization problem of the unstable NMP system, which has been frequently observed in the NMP system.

## 1.4 Research Potential

The proposed algorithm has been derived based on the Lyapunov Design technique. As the control law design is based on the Lyapunov Design technique, stability is ensured spontaneously while deriving the parameters of the control law. Moreover, we have employed the error dynamic model for the design of the control law, and it enabled us to directly check the robustness of asymptotic stability as well as perfect tracking, i.e.  $e \rightarrow 0.08t \rightarrow \infty$ . Thus robustness in stability, as well as robustness in tracking, is satisfied. The tracking performance has been verified by Barbalat's Lemma. In this work, we have developed a 2DOF based MRAC for achieving robust tracking of the NMP system. The feed-forward part of the 2DOF control has been derived from the MRAC technique, and a feedback control law has been implemented based on the state feedback control law. The adaptive nature of the feed-forward control law in the model reference framework helped us to achieve the robust tracking performance of the overall system, whereas the feedback control law helps us to stabilize the system. To the best of the knowledge of the authors, the application of 2DOF control employing MRAC in the feed-forward loop and state feedback control in the feedback loop to overcome the ill-effects of NMP systems during tracking control has never been addressed in the literature.

### **1.5** Thesis Organization

Thesis has been organized as follows:

Chapter 1: provides a brief description of the NMP system and its limitations. Problem formulation of this research work has been demonstrated here. Problems that arise in the designing of the tracking control system for exact set point tracking for the NMP system are briefly discussed here. The reasons behind the selection of the NMP system as a plant model for the designing of tracking controller have been demonstrated clearly. In the end, the potentiality of this research work has been stated.

Chapter 2: includes mainly an overview of the literature survey regarding the control problem of stabilization and tracking of NMP systems, and it consists of the introduction, difficulties arise regarding tracking control of NMP systems. It also provides a brief discussion on present and past control techniques of the NMP system.

Chapter 3: presents the application of 2DOF control scheme in different industrial and research areas in control system engineering in its introduction. The block diagram of the proposed 2DOF control scheme has been presented and it demonstrates a brief description of its feed-forward and feedback compensation. At the end, there is a statement of justification for proposing this 2DOF control structure.

Chapter 4: presents the structure of the proposed MRAC control scheme which played the feed-forward control part of the 2DOF structure. It also stated the robustness of the proposed control technique.

Chapter 5: describes the detailed development of an arbitrary pole placement based SFB control algorithm which acts as a feedback counterpart in the 2DOF control structure to stabilize internal instability of the NMP system.

Chapter 6: represent the application of the proposed 2DOF control scheme on a 2<sup>nd</sup> order NMP system. MIT rule based MRAC technique as feed-forward control part of 2DOF has been applied here and SFB compensator as feedback counterpart. Though to some extent, satisfactory results is obtained here, but, MIT rule based MRAC could not prove the closed loop stability and tracking error convergence. Other than that, there are some difficulties arises in this technique, and those are discussed in this chapter.

Chapter 7: provides a demonstration of the generalized control algorithm of the MRAC method, which is based on Lyapunov Design technique. Here, a concise form of the total

algorithm has been presented in the flow chart diagram.

Chapter 8: presents the application of Lyapunov based MRAC in the 2DOF framework on the same numerical example of an NMP system, which has been considered by the MIT rule based MRAC technique (discussed in chapter 6). The problem found by the MIT rule based on MRAC has been easily solved by Lyapunov Design technique based MRAC, which acts as a feed-forward control technique in the 2DOf framework.

Chapter 9: has considered four types of reference model for the MRAC structure to verify the unique properties of the 2DOF based MRAC method based tracking control system. Under-damped, un-damped, critically damped and over-damped transfer function models have been chosen for the reference model plant (RMP) of the MRAC structure. After application of the proposed control technique on the NMP system model, it has been observed, that output responses of the 2DOF controlled system almost track the under damped, un-damped, critically damped, and over-damped trajectory of reference models provided by the MRAC structure.

Chapter10: shows that the number of undershoots and overshoots depends on the number of RHP zeros. The odd number of RHP zeros produces an initial undershoot and an even number produces an initial overshot. The numerical examples of five SISO NMP systems have been taken, and out of them, two one RHP zero, one two RHP zero, and two three RHP zero SISO NMP system models have been considered. One bench marked MIMO NMP system with one RHP zero has also been considered for the implementation of the proposed 2DOF control methodology. All undershoots and overshoots produced by the NMP system have been completely removed with good tracking performance.

Chapter 11: demonstrates the application of a proposed control technique on a practical mechanical realized 4<sup>th</sup> order NMP system. This NMP system has been taken from a referred journal, where it was developed in an experimental laboratory set-up. The square wave response of the proposed 2DOF controlled mechanical realized NMP system has been compared with the PID and SFB control approach.

Chapter 12: consider one 3<sup>rd</sup> NMP system along with the 4<sup>th</sup> order mechanical realized NMP system, which has been presented in the previous chapter. Unit step response of these two systems with the proposed control structure has been compared with the SFB control technique and advanced control technique like MRAC. To verify the tracking performances of the proposed control technique unit ramp and sinusoidal input signal have also been taken as set point of the proposed control scheme.
Chapter 13: validates this research work on an Op-amp based NMP system, which has been developed in a digital as well as analogue simulation environment. At first, the mathematical model of 2<sup>nd</sup> order NMP with one RHP zero was derived using the MATLAB System Identification toolbox, then a realistic Op-amp based NMP system and its 2DOF control structure were developed and experimented in the MATLAB SIMULINK environment. After the satisfactory performance of realistic Op-amp based digital simulation of the proposed control technique, real Op-amp based analogue simulation of hardware experimental set-up has been done and strengthens the demand of the proposed MRAC based 2DOF control technique for NMP systems.

Chapter 14: concludes the dissertation with the concept of an effective control scheme for the solution to the tracking problem of NMP systems, which is hard to control due to its internal instability. Though the cascaded inverse, non-inverse does not work for exact tracking of NMP systems, this inverse and non-inverse NMP model can be used in the proposed 2DOF control structure for perfect tracking of NMP systems.

#### **1.6 Chapter Summery**

In the introductory part of this chapter, it provides a brief description of the NMP system, or system with unstable zero. Then, the motivation behind the designing of the tracking controller for the NMP system has been discussed. The next section represents the problem statement of the proposed research work, and finally, the research potential has been stated.

# CHAPTER 2 Literature Survey

#### 2.1 Introduction

This research work has gone through a wide range of published literature to get the knowledge regarding the NMP system and its different control techniques. We have come to know that the NMP system is one of the interesting types of system in control system engineering which is hard to control. The NMP characteristics found in different practical systems, like hydraulic pumps, a blast furnace, a quadruple-tank system, motion of a conventional aircraft, and transportation lag in the control system, or even if a continuous minimum phase system is converted to an equivalent discrete time system, it is transferred to a NMP system [10]. The mathematical model of Electro-Hydraulic Servo System represents the NMP system model [5]. In the process industry, dry gas humidification is required for the lumped model of membranebased humidifier that uses water to do this task, and it has been observed that the transient behavior of this humidification process exhibits NMP characteristics and the rate of mass vapor becomes lag in control volume [13]. The Bicycle dynamics is also a good example of a nonminimum phase system [9]. Six different NMP transfer functions have been found in quadruple tank systems [2]. We have also found non-minimum phase characteristics in an aircraft system, like six degrees of freedom F-16 aircraft model [14], planar vertical and take-off and landing (PVTOL) aircraft model [29]. A single link manipulator system with a high payload and volume of workspace suffers from the effect of NMP zeros [15]. Bilinear transformation of Boost and buck boost power convertor and DC to DC power convertor suffered from NMP characteristics [16, 17].

#### 2.2 Difficulties in tracking Control of NMP System

In contrast to the MP system, a NMP system has various fundamental limitations, like [10],

- i) Output response of the system initially goes to the opposite of the desired steady state response.
- ii) Problem with the internal stability
- iii) The phase angle contribution is always greater than 90 degrees.
- iv) The speed of the closed loop response is slow.

- v) Disturbance rejection is bounded due to the limitation in control bandwidth. The initial undershoot produced in the step response is a major problem in tracking the performance of the NMP system, but, other than undershoots of NMP zeros, there are more problems observed from the literature given below [18].
- vi) Overshoots frequently occur in the time response of the closed loop system.
- vii) Due to restricted gain margin in the feedback controlled NMP system, destabilization is obvious in the NMP system.
- viii) Robustness of the closed loop system also may be disturbed by the limited gain margin of the closed-loop system.
- ix) The proportional gain of the controller for an NMP system becomes bounded if the dead time is incorporated into the system. In that case, the closed loop system leads to instability.

Other than the above limitations, several cross over frequencies adversely affect the dynamic response of the NMP System [19]. Zero error at sampling time is needed in the process of inverting the control system for iterative learning control design, but this method leads to instability for NMP systems [39].

The exact reference input tracking problem can be solved if the system has RHP zero. The position of the zeros placed at RHP cannot be changed without changing the structure of the system or, in that case, re-formulating of the tracking problem may require [20].

Unstable zero of a closed-loop NMP system decreases bandwidth, dynamic characteristics and robustness against disturbances [21]. The control of practical NMP systems, like the stirred tank reactor, flexible robotic arm, servomechanism, level control of steam in the boiler, the process of increasing the power of tasks are challenging [21]. When a practical higher order time delay is approximated by a pade approximation, unstable zero dynamics is exhibited in the system. [21]. Precise torque and speed of the actuator is required in a hardware-in-loop transmission testing set-up, which mimics the actual engine and vehicle dynamics. This accurate speed or torque can be produced by a feed-forward control scheme, which is based on inversion of the system model, but it does not work for NMP systems. Preview filters can solve the problem of inversion of NMP systems, but it essentially requires the future value of the dynamic response which is not accessible in the hardware-in-loop test [22]. Input output linearization or the steepest decent control method is good for the output tracking of the system. But, for NMP systems, it does not work as this technique requires the system to be in a

minimum phase with respect to a new output [23].

In the case of a discrete time system, if at least one zero of the transfer function model is placed on the outside of the unit circle, the system is called NMP discrete time system [1]. Inverse model based control techniques for discrete time NMP systems also become an unstable tracking control problem [24].

The four-state model of a membrane-based humidifier that applied water to humidify the dry gas exhibits the characteristics of NMP and this undesirable phenomenon is inbred into the system and it falls behind the vapor mass rate in the vapor volume. The coupling effect between the two loop gains, like input to output and disturbance to output, has been affected by the zeros placed at RHP of S plane [13].

### 2.3 Overview of Previous and Present Control Technique

The design of a tracking control system for the NMP system is very difficult as it has to achieve the stabilization of the unstable zero dynamic as well to track the desired output trajectory or command input signal. This two-fold control method is essentially required for the solution of the tracking control problem of the NMP system. But, it has been observed, that, though the output response of the controlled NMP system meets the steady state value of the set point, there is a lower number of methods or techniques have been studied in different referring journal, which can deal with the stability problem of the NMP system, and can produce desired bounded performance [12]. The output tracking of an NMP non-linear system may be achieved by modifying the steepest descent control method, but here, the NMP system must be redefined with respect to a new output in such a way that ultimately it becomes a minimum phase system [23]. In output tracking theory, input output linearization is one of the most available methods [25]. There is a literature which overviews the five fundamental methods for controlling the NMP system. A non-linear system with slight NMP characteristics is converted to an approximate minimum phase system by the approximate feedback linearization technique. This method is first applied on a vertical/short take-off and landing (V/STOL) aircraft. A slightly slight NMP system indicates the RHP zero is placed far away from the imaginary axis of the S plane. The NMP system is converted to a minimum phase system with respect to a new dummy output using the process of output redefinition. It is actually a direct method of feedback linearization when the relative degree of systems is one. It has been applied on to the autopilot design of a tail-controlled skid-to-turn missile system where unstable zero dynamics have been found under model inadequacy. Modification of the output has been done for the approximation

of a non-linear NMP system by a minimum phase system. Here, a new output is constructed with the elimination of any subset of the real zero of the current NMP system. It is basically suitable for the MIMO system only and it has been implemented in the control of flight with fast movement. Some acceptable assumptions have been taken for the stable-inversion of unstable zero dynamics and this inversion process is taken on off-line, and here, the dynamic of reference is integrated in such a way that it can make the input bounded. This technique can be applied on chemical reactors, and as it incorporates major NMP characteristics, previous methodology did not work on it. It was also implemented on planar/vertical take-off and landing (P/VTOL) aircraft for maneuver regulation. The control of an NMP system is difficult when a complete state vector is not available or, rather, few are available. Firstly, output feedback stabilization of a non-linear system has been done with the assumption that no unstable zero is present in the system. Later, a significant output feedback stabilization based controller has been developed and successfully applied on a benchmark inverted pendulum system which exhibits NMP characteristics [26].Desired trajectory tracking by Internal Model Control has been studied, where direct data of the plant model is required, so exact plant model is essentially needed here and fictitious reference iterative tuning (FRIT) method has been used to tune the control parameter with only one shot experimental data [27]. Different constraints have been formed after analyzing the internal stability of the NMP system. In designing a robust disturbance rejection methodology, several constraints with less conservatism are formed after analyzing the internal stability of an unstable NMP system [28]. Under actuated planar, vertical take-off and landing (PVTOL) has been experimented with a prediction-based control approach, which uses the technique of partial feedback linearization and obtained linearized sub-state and internal dynamics [29]. As the stable bandwidth is restricted in NMP system, Active disturbance rejection control (ADRC) is suitable for MP system, ADRC with Linear quadratic regulator (LQR) has been suggested for the system with unstable zero [30]. The MOMI and numerical optimization method based controls have been studied, and it has been observed, that, MOMI technique produces better performance than the numerical optimization method [31]. The Sliding mode model predictive control (SMPC) algorithm actually consists of a sliding mode control (SMC) and a model predictive control (MPC) scheme, and, it has been found, that the problem regarding the MPC controlled NMP system may be solved by the proper tuning of SMPC [32]. To perform the online trajectory tracking of the non-linear NMP system, e.g. A VTOL system, preview based a stable inversion technique, has been implemented where it quantifies the future desired trajectory [33]. Output feedback tracking problem of the NMP system with unknown disturbances and non-linearity may be solved by

an additive-state-decomposition based control technique. It additively decomposes the output feedback tracking problem for systems and the SFB stabilization problem of non-linear systems [34]. Two feed-forward controllers, sharing information with each other, become a perfect tracking control system of a plant, which consists of MP and NMP components [35]. Internal Model Control (IMC) based fractional order controller using constrained minimization optimization technique has been designed for NMP systems, but this technique requires the process model which must be embedded in the controller [36]. It also satisfies the desired phase margin and cross-over frequency in the frequency domain analysis [37].

The n<sup>th</sup> order linear time invariant discrete time causal single input single output NMP system has been considered for output tracking via discrete time variable structure control, where we have observed the tracking control has been done using sliding mode control, but inverse initial response could not be avoided [38]. Control method based on exact feedback linearization with Lyapunov stability theory for a hydraulic turbine unit, which has MIMO nonlinear fifth order NMP characteristics, has been discussed in the literature , where two control input has been developed for the 5<sup>th</sup> order model [40].Fractional filter PID controller combined with smith predictor and inverse compensator has been designed two NMP system with dead time obtained robust closed loop performance [18].Each subsystem of a switched non-linear system represents NMP system, whose uncertain dynamic is controlled by the combined control scheme of exact input output feedback linearization, sliding mode controller and Lyapunov stability theory with the appropriate switching strategy based on infinite diffeomorphism [41]. Performance funnel has been suggested for designing the low complexity controller for a linear NMP system with known relative degree [42]. A new output for implementation of the low complexity funnel controller is formed for a NMP system with a higher relative degree, where unstable zero dynamics has to be eliminated to get the exact reference input trajectory. Then, with funnel control result and a new reference signal, funnel controller has been designed to get better performance [43].

## 2.4 Chapter Summery

The introduction part of this chapter represents the names of those plants which exhibit NMP characteristics in their system dynamics. Then the various difficulties found in the tracking control problem of NMP systems have been discussed. There is an overview of past and present control techniques of systems that has been described later on.

# CHAPTER 3 2DOF Control Technique

#### 3.1 Introduction

The transfer function models of a closed loop control system, which can be controlled separately, is termed as the degree of freedom(DOF) [44]. As, in the industrial field, multi-objective performance is required for the controlled system, the 2DOF control structure has an advantage over 1DOF controller [45].

Though the 1DOF control technique is conventionally used because of its simple and easy realization, as the 2DOF control scheme is very effective for set point tracking as well as for disturbance rejection, it may be stated that the 2DOF method is at an advantageous position than the 1DOF method in control system engineering [46]. It has been observed from the mathematical analysis of the 2DOF controller, the various structures of the 2DOF scheme can be applied to the plant model, which has both transportation delay and disturbance [47]. For reference input trajectory and perturbation recovery, the 2DOF has been applied to the motion of robots as a dynamical movement primitive (DMP) context [48]. Disturbance rejection methodology based on a 2DOF control strategy has been applied on unstable NMP systems, where the 2DOF control structure consists of an inner loop disturbance observer (DOB) and one feedback controller as the outer loop. One 2DOF based disturbance rejection technique to solve the tracking control problem of the system with unstable zeros has been discussed. In that methodology, the inner loop was disturbance observer (DOB) and the outer loop played as feedback controller, whereas DOB has been dealing with external disturbances and internal uncertainties [28]. The combination of model order reduction, approximate model matching concept and optimization techniques in a general 2DOF controller for time delay system formulates a set of non-homogeneous linear equations to find the control parameters of the 2DOF controller by using approximate generalized time moments (AGTM) matching concept [49]. A 2DOF controller has been designed to study how the closed loop RHP zeros affect the closed loop poles in the strictly proper NMP plant [50]. An magnetic levitation system has been taken as a plant model to verify the 2DOF control strategy, where command input tracking and rejection of load disturbance have been performed in such a way that these control task can be tuned separately [51]. Serious wavering movement commonly occurs in steering mechanism of automated guided vehicle (AGV), when the vehicle speed rises above 5m/s, and it hampers the perfect set-point tracking of the vehicle, the 2DOF strategy not only overcomes this tracking problem, it also solves the load disturbance rejection problem [52].2DOF PID controller has been implemented on second order like processes with a proper tuning based on MATLAB optimization toolboxes. The kick characteristics incorporated with the PID compensation have been completely removed by this technique and its control performance compared with I-PD and PI-PD control techniques [53].

For set point tracking and load disturbance rejection problem problems, an event based 2DOF PID controller has been designed for a first order with dead time model [54]. Advantages of 2DOF PID over its classical form have been verified by a fixed set point weighting PID controller [55]. The one-step adjustment method of the 2DOF controller of a closed loop system can be possible by using the least squares method, where it is applied to feedback and feedforward controllers simultaneously [56]. To get the desired transportability matching under model uncertainty and flurry disturbances, Reference Feed-forward type 2DOF control system has been developed for an experimental aircraft of multipurpose aviation laboratory-a  $(MuPAL-\alpha)$  [57]. It has been shown that the step response without overshoot and servo response with no oscillation could be achieved by the proper tuning of PI and PID controller in 2DOF control structure for the integrating plant and the first order plant with time delay [58].PI controller based 2DOF control scheme for first order plus dead time system model obtain robustness to model uncertainty, rejection of disturbance and fine command input response using butterworth rules and genetic algorithm optimization [59]. It has been observed that 2DOF PID controller is better than the traditional PID controller for a 6DOF rigid robotic manipulator in real time, which is of multi input multi output (MIMO), highly non-linear complex structure [60]. The 2DOF based integral plus derivative and derivative (IDD), proportional, integral and derivative (PID) and proportional plus integral (PI) control with multi-objective optimization technique, like the thermal and wind system of an automatic generation control (AGC) of a power plant have been compensated by flower pollination algorithm [61].

The control aspects like set point tracking and disturbance rejection of process industry are hard to fulfill the desired performance by using one degree of freedom control as they are basically contradicting to each other. So, here 2DOF control technology can solve the problem and it has been checked by the 2DOF PID controlled temperature control laboratory set up [62]. A direct synthesis method based on lag-lead compensator with simple PID controller in 2DOF control strategy for a system which has one unstable pole or two unstable poles or an integrator or a zero reduce the overshoot and settling time of closed loop system [63]. By using

the method of model order reduction, optimization and approximate model matching technique like AGTM/AGMP, a single low order 2DOF controller can obtain the desired tracking and disturbance rejection performance simultaneously of a coupled tank process [64].

The real-time pressure process is quite hard to control due to its immense sensitivity and non-linearity. Moreover, its performance degrades a lot when set point changes and high external disturbance occurs. Here, both parallel and series configuration of 2DOF fractional order PID controller have been applied [65]. The control problem of unstable NMP systems has been solved by disturbance rejection methodology by using an inner loop disturbance observer (DOB) of 2DOF control scheme, where the outer loop acts as a feedback controller [28]. Non-oscillatory and under-damped response can be obtained by a closed loop model reference optimization method applied to a 2DOF proportional integral controller for an integrating controlled process [66]. After knowing the desired output signal, by fractional input output technique, the needed set point signal has been achieved and based on this synthesized command signal, set point filter is computed and applied into 2DOF fractional controller and it can able to achieve the required tracking performance and it may not be dependent on feedback part of the controller [67].2DOF controller also can be applied on single chip microcontroller (MCU), where SCADA/HMI system monitors the controlled system, and the serial interface exists between industrial personal computer and MCU [68].

#### **3.2 Proposed 2DOF control structure**



Fig. 3.1 Two degrees of freedom control structure [71]

A two-degree-of-freedom (2DOF) control structure, which is a combination of feedforward and feedback control, significantly improves the tracking performance of the control system [9]. Two-degree-of-freedom theory consists of feed-forward and feedback control structure. It has two transfer functions which can be designed independently and hence it is called a two-degree-of-freedom system [24]. Feedback Control is used for stabilization of unstable systems, whereas feed-forward controls improve the tracking performance of the system. The feedback and feed-forward compensation in the 2DOF control system may be connected in different ways. The control input of the feedback controller is actually the output of the feed-forward control scheme, which has been shown in Fig. 3.1. To stabilize the unbounded system, a feedback controller is applied here, whereas the feed-forward control scheme is implemented in a 2DOF structure to obtain the desired tracking performance.

#### 3.2.1 Feed-forward Control Scheme

Controlling a plant with different set point signal feed-forward control is essentially required, but it must include an inverse plant model that leads to instability to the NMP system dynamics [70]. It has been observed that the circular contour tracking system of a NMP discrete system model has produced better results with a feed-forward controller than a tracking system without feed-forward control [71]. Model predictive control (MPC) based feed-forward model inverse control has been used in the discrete time model of the LTI diesel generator [72].

Here, MRAC scheme has been considered for the feed-forward control method of the proposed 2DOF structure. This algorithm performs, the direct derivation of the required control parameters, and the system parameter identification, simultaneously by adaptive mechanism. The gain of the feed-forward controller has been adjusted by trial and error method.

#### 3.2.2 Feedback Control Scheme

To solve the tracking problem as well as stabilization problem of a system, the controller scheme mainly consists of two parts, those are, feedback controller and the feed-forward controller. The feedback controller has been applied commonly for the regulating purpose, and it has been found in many literatures [11]. Arbitrary pole placement method based SFB controller acted as feedback counter part of the feed-forward compensator in the proposed 2DOF framework.

Pole-placement feedback controller is successfully experimented on the mathematical model of a plant, whose system representation has been derived by using the MATLAB system identification toolbox with open loop experimental input output data [69].

#### 3.2.3 Justification for adopting 2DOF Control framework for NMP System

2DOF control technique is very much effective for solving the multi-tasking control problem in control system engineering. As NMP system is inherently unstable due to its RHP zero, first it needs to be stabilized. Then, to solve the tracking problem, the design of suitable tracking controller is required. This two-fold control task only can be performed by the appropriate control methodology in 2DOF framework.

## **3.3** Chapter Summery

The overview of the different application of 2DOF control technique has been described by the introduction part of this chapter. Then, the suggested 2DOF control structure and its feed-forward and feedback control part have been presented. Finally, justification behind adopting the 2DOF control structure for the solution of tracking control problem of NMP system has been stated.

## **CHAPTER 4**

# **MRAC** Control Technique as Feed-forward Compensation

#### 4.1 Introduction

The Model Reference Adaptive Control (MRAC) was mainly proposed to solve the control problem in which the characteristics specifications are provided in terms of a reference model plant (RMP). The model reference adaptive control (MRAC) knows many industrial applications, and especially in the field of aeronautics. The MRAC is an adaptive control strategy that creates a control law, subject to an adaptation gain, which causes the system's plant to continuously track a reference model until a zero tracking error is achieved.

The control system is a device that compensates for the dynamics of any other plant or process. Adaptive control is one of the commonly used control methodologies to develop advanced control systems for better performance and accuracy. MRAC is a direct adaptive scheme with an adjustment mechanism, which adjusts the control parameters.



## 4.2 MRAC structure



Model reference adaptive control consists of four parts shown in four blocks in Fig. 4.1; those are the plant, the reference model, the adjustment mechanism and the controller. An error signal is produced by comparing the output of the reference model and the controlled plant's output. By using MIT rule [92] or Liapunov's stability theory [91]. Adjustment mechanism block calculates the appropriate control parameters and the controller with the help of these parameters produces output which is actually the control input of the plant. Controller designers selected the reference model as per the design requirement.

The error dynamics is produced by comparing the dynamic response of a plant model and model reference plant and is used to produce the required adaptive algorithm by parameter adjusting mechanism. To get the desired trajectory given by the reference model plant error dynamics asymptotically converges to zero. Among the two loops of the MRAC structure, the inner loop consists of plant and controller, whereas the outer loop adjusts the parameters to minimize the error between reference model and plant model to acquire the pre-specified trajectory of reference model. Four components of this control scheme are required; those are reference model, plant, controller and parameter adjustment mechanism. Adjustment mechanisms may be developed by using the MIT rule, theory of augmented error, and Lyapunov theory. Two approaches to the MRAC design scheme have been done for antenna azimuth position control system e.g. gradient and Lyapunov rule. Though the design of the gradient approach is easier than the Lyapunov approach, its stability may hamper the increase in high adaptive gain. Lyapunov approach overcomes the staggering system response in closed loop system stability [112]. So, we have proposed the Lyapunov Design technique based on MRAC for the proposed 2DOF controller.

#### 4.2.1 Uncontrolled Plant

The plant which is to be controlled has been placed in the block 'Plant'. In this proposed control technology, the MRAC scheme has been used as a feed-forward control technique and in feed-forward compensation inversion of plant model is essentially required, in case of the NMP system, inversion of the system leads to system instability, even if the non-inverse NMP system is stable one.

## 4.2.2 Reference Model Plant (RMP)

The reference plant model is an essential part of the MRAC structure. It is selected according to the desired characteristics of the plant output. First, the control designer has to find the desired characteristics specifications of the plant output and construct the reference plant model according to those desired dynamic characteristics. Then deriving the error dynamics by comparing the plant model and the reference plant model output, obtain the required control parameters by using proposed control algorithm.

## 4.2.3 Inner Feedback Control Loop

It's an ordinary feedback control loop in the MRAC structure which actually stables the plant and helps to find the required control input to get the desired trajectory of the closed loop controlled system.

## 4.2.4 Outer Control Loop with Adjustment Mechanism

The adjustment mechanism block is an important part of the MRAC structure as this block is actually here to find out the essential control parameters for the closed loop control system. The tuning of the control parameters has also been done by this block.

## 4.3 Robustness of MRAC Scheme

MRAC schemes are known as an effective method to deal with the system's non-linearity. High adaptive gain is usually needed to achieve fast adaptation. Different control logic may be implemented for the development of control algorithm for MRAC compensation. Among them, Lyapunov Design technique is very effective as it not only stabilizes the system but it also ensures the stability of the closed loop controlled system. The robustness of this control structure has also been proved by its tracking performance. By using Barbalat's Lemma, the 2<sup>nd</sup> derivative of the Lyapunov function establishes the robustness of the tracking performance.

## 4.4 Chapter Summery

Model Reference Adaptive Control is an effective method for command input tracking of a system. The advantage of the MRAC scheme is that the RMP, which is an essential part of MRAC structure, plays the main role in determining the transient dynamic characteristics of a controlled plant's output response. A brief description of MRAC structure has been provided in this chapter and robustness of the MRAC technique has also been stated here.

# **CHAPTER 5**

# SFB Control as Feedback Compensation

### 5.1 Introduction

The arbitrary pole placement method is a well-known and popular technique, which is very useful for designing modern state feedback control systems. To compensate for the adverse effect of the unstable poles or zeroes, arbitrary closed loop poles are placed at the left-hand side (LHP) of the S plane. Then the state feedback gain matrix is derived using different techniques.

## 5.2 SFB Control Scheme



#### Fig. 5.1 Block diagram of state variable feedback system [92]

Considering a closed loop control system where all the state variables are measurable and are available for feedback. In this case, all closed loop poles can be located at any desired location by means of an appropriate state feedback gain matrix. This process of designing a closed loop control system is known as Pole Placement technique.

#### 5.2.1 Controllability Check

For the application of SFB control techniques to any dynamic system, all the state of the system must be completely state controllable, and before checking the controllability, it must be ensured that that system matrix must be non-singular.

#### Chapter 5

## 5.2.2 Choice of closed loop poles

There is an option for the control designer to choose the closed loop poled for the required transient performance of the dynamic system using MATLAB toolbox.

## 5.2.2 Derivation SFB Gain Matrix

Let us consider the state equation of n<sup>th</sup> order system is

$$\dot{X}(t) = AX(t) + Bu(t)$$
(5.1)  
where,  

$$X(t) \text{ is } n \times 1 \text{ state vector}$$

$$u(t) \text{ is } n \times 1 \text{ input vector}$$

$$A \text{ is } n \times n \text{ system matrix}$$

$$B \text{ is } 1 \times m \text{ input matrix}$$
Let, the control input is  

$$u(t) = -KX(t)$$
(5.2)  
Where, K is the state feedback gain matrix  
and the closed loop poles are located at  

$$s = \mu_1, \mu_2, \dots, \mu_n$$
From equation (5.1) and (5.2), we get,  

$$\dot{X}(t) = (A - BK)X(t)$$
(5.3)  

$$X(t) = e^{(A - BK)}X(0)$$
(5.4)

If the matrix K is chosen appropriately, the matrix (A - BK)

can be made asymptotically stable matrix for all  $X(0) \neq 0$ .

The eigen values of the matrix (A - BK) is called regulator poles.

Let,

$$\overline{A} = A - BK \tag{5.5}$$

$$SI - A + BK = SI - \overline{A} = (S - \mu_1)(S - \mu_2)....(S - \mu_n) =$$
  

$$S^n + \alpha_1 S^{n-1} + \alpha_2 S^{n-2} + .... + \alpha_{n-1} S + \alpha_n = 0$$
(5.6)

As per Caley-Hamilton theorem, every matrix satisfies its own characteristics,

$$\phi(\overline{A}) = \overline{A}^n + \alpha_1 \overline{A}^{n-1} + \alpha_2 \overline{A}^{n-2} + \dots + \alpha_{n-1} \overline{A} + \alpha_n I = 0$$
(5.7)

Let,

$$n=3$$
 (5.8)

Then,

$$\phi(\overline{A}) = \overline{A}^3 + \alpha_1 \overline{A}^2 + \alpha_2 \overline{A} + \alpha_3 I = 0$$
(5.9)

Now,

$$\overline{A}^{2} = (A - BK)^{2} = A^{2} - 2ABK + B^{2}K^{2}$$
  
=  $A^{2} - ABK - ABK + B^{2}K^{2}$   
=  $A^{2} - ABK - (A - BK)BK$  (5.10)

Or,

$$\overline{A}^2 = A^2 - ABK - BK\overline{A} \tag{5.11}$$

$$\overline{A}^{3} = (A - BK)^{3} = \overline{A}(A - BK)^{2} = (A - BK)(A^{2} - ABK - BK\overline{A})$$
(5.12)

Or.

$$\overline{A}^{3} = A^{3} - A^{2}BK - ABK\overline{A} - BK\overline{A}^{2}$$
(5.13)

$$\phi(\overline{A}) = \alpha_3 I + \alpha_2 \overline{A} + \alpha_1 \overline{A}^2 + \overline{A}^3$$
  
=  $\alpha_3 I + \alpha_2 (A - BK) + \alpha_1 (A - BK)^2 + (A - BK)^3$  (5.14)

Or,

$$\phi(A) = \alpha_3 I + \alpha_2 (A - BK) + \alpha_1 (A^2 - ABK - B^2 K^2)$$
  
+  $(A^3 - A^2 BK - ABK\overline{A} - BK\overline{A}^2)$  (5.15)

Or,

$$\phi(\overline{A}) = \alpha_3 I + \alpha_2 A + \alpha_1 A^2 + A^3 - \alpha_2 B K - \alpha_1 A B K - \alpha_1 A B K - \alpha_1 A B K \overline{A} - A^2 B K - A B K \overline{A} - B K \overline{A}^2$$
(5.16)

We know that,

$$\alpha_{3}I + \alpha_{2}\overline{A} + \alpha_{1}\overline{A}^{2} + \overline{A}^{3} = \phi(\overline{A}) = 0$$
(5.17)  
Also, we have,  

$$\alpha_{3}I + \alpha_{2}A + \alpha_{1}A^{2} + A^{3} = \phi(A) \neq 0$$
(5.18)  
So,  

$$\phi(\overline{A}) = \phi(A) - \alpha_{2}BK - \alpha_{1}BKA - \alpha_{2}BK - ABK\overline{A} - BK\overline{A}^{2}$$
(5.19)  

$$\phi(\overline{A}) = \phi(A) - \alpha_{2}BK - \alpha_{1}BK\overline{A} - BK\overline{A}^{2}$$
(5.20)  

$$\phi(\overline{A}) = \phi(A) - \alpha_{2}BK - \alpha_{1}BK\overline{A} - BK\overline{A}^{2}$$
(5.20)  
As  

$$\phi(A) \neq 0$$
(5.21)

$$\phi(A) = B(\alpha_2 K + \alpha_1 K A + K A)$$

$$-AB(\alpha_1 K + K \overline{A}) - A^2 B K$$
(5.22)

$$\phi(A) = \begin{bmatrix} B : AB : A^2B \end{bmatrix} \begin{bmatrix} \alpha_2 K + \alpha_1 K \overline{A} + K \overline{A}^2 \\ \alpha_1 K + K \overline{A} \\ K \end{bmatrix}$$
(5.23)

Since the system is completely state controllable, the inverse of the controllability matrix exists, Pre-multiplying the inverse of the controllability matrix in both sides, we get,

Multiplying both sides of the equation no (5.23) by

[0 0 1], we get,

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B : AB : A^2B \end{bmatrix}^{-1} \phi(A) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_2 K + \alpha_1 K\overline{A} + K\overline{A}^2 \\ \alpha_1 K + K\overline{A} \\ K \end{bmatrix} = K$$
(5.24)

For any arbitrary positive integer n, we have,

$$K = [0 \dots 1] \left[ B : AB : A^2B \dots A^{n-1}B \right]^{-1} \phi(A)$$
(5.25)

This is the Ackermann's formula for the determination of state feedback gain matrix of  $n^{th}$  order state model.

Here, we have assumed that all states are available for state feedback control. When the plant's own states are not available, the observer states can be used to provide feedback control. The concept of observability is very important because, in practice, the difficulty encountered with state feedback control is that for direct measurement, with the result that it becomes necessary to estimate the un-measureable state variable in order to construct the control signal [104]. A state of observer is a system that models a real system in order to provide an estimate of the internal state of the system [191].

## 5.3 Chapter Summery

This chapter presents a brief description of arbitrary pole placement based SFB controller. The necessary condition has been stated here. The derivation of the state feedback gain matrix (using Ackermann's formula) of the n<sup>th</sup> order state model of the system has been demonstrated.

## **CHAPTER 6**

# Design of Adaptive Two-Degree-of-Freedom Controller for inversion based Reference Input Tracking of NMP system using MIT rule based MRAC

#### 6.1 Introduction

The set point tracking of dynamic systems is an essential control task in control system engineering and it finds its extensive applications in the industrial and research areas. A reference input trajectory needs to be predefined, so that the output trajectory of a system is directed to follow the reference input trajectory [86].

By using feed-forward control of an inversion based system, exact set point tracking can be achieved, but it is confined to the minimum phase system only as this inversion system approach leads to the unbounded response of the NMP system. Standard inversion technique also does not work for the NMP system [74].

Researchers tried to find their best to solve the tracking problem associated with NMP systems. Preview-based output tracking of systems has been done by utilizing the inverse input, which has been computed by the finite-previewed (in time) future desired trajectory. If the previewed time extended to infinity, then only exact output tracking is possible here. But, some physical constraints always reduce the preview time [75].

The predictive control approach basically follows two methodologies, like the classical predictive approach, which is non-optimal, and the other is the modern optimization-based approach, which is generalized predictive control. Both are model based and their controller produces the required control input by using the predicted future changes in the output response. The well-known Smith Predictor and IMC control structure are an example of classical predictive control [76].

The control input, required to solve the tracking control problem, can be developed by the direct relationship between the reference input signal and control input signal of the linear stable NMP system, and which can be expressed as the linear transformation of their Fourier co-efficient [77].

The adaptive control scheme is a very effective technique for solving the trajectory tracking problem of MP and NMP systems. Depending on Fourier approximation theory and two adaptive observers, an output feedback based adaptive learning controller has been designed for the local solution of the output tracking problem of an uncertain linear system using periodic reference input signals with known periods, but some assumptions for systems are required here [78].

In the history of adaptive control techniques, several methods of adaptation are present as the real application of this method is interesting. The researcher nurtured a different scheme of adaptive control method. Among them, Model Reference Adaptive control, Adaptive pole placement (self-tuning regulator), Adaptive Sliding Mode Control are very popular. The extremum seeking is occasionally treated as a method of adaptive control, and has been extensively applied in the industrial environment [79].

The application of the Direct MRAC algorithm on SISO, a non-linear, continuous and possibly NMP system has been studied, where surrogated tracking is used to construct the needed control input. This adaptive technique must have the information of the first non-zero Markov parameter and then, zeros at the RHP of S plane. Preliminary stability analysis of MRAC using this surrogated tracking error method has been performed in this literature [80].

But the mentioned plants, which are being used for the applications of the 2DOF control method, do not have non-minimum phase characteristics. The indirect Adaptive Pole placement control scheme is generally applied to NMP systems. But the main shortcoming found with this adaptive scheme is that the estimated plant on which the computation of the control parameter has been performed may possibly lose its stability [85].

The adaptive control scheme exhibits non-linearity in the dynamic characteristics of the NMP system [93]. So, it is hard to develop the adaptive algorithm for the NMP system.

The purpose of this research work is to develop a MIT rule based MRAC control algorithm as feed-forward compensation in the 2DOF framework, where an arbitrary pole placement method based on SFB control technique has been used as a feedback counter part of the suggestive control technique to solve the tracking problem of the NMP system.

### 6.2 Problem Statement

Any inversed model based system can produce exact reference input tracking, but it is only applicable for minimum phase systems, whereas, in the case of a non-minimum phase (NMP)

system, the concept of inversion technique gives unstable output response. To overcome this problem, a two-degrees-of-freedom (2DOF) control technique has been considered for the inversion based reference input tracking control problem of the NMP system, where the combination of the Model Reference Adaptive Control structure (MRAC) using the MIT rule and the state feedback control methodology in 2DOF framework has been taken as total compensation technique of NMP System.

### **2DOF controller**

The 2DOF internal model control structure has been designed for non-square processes, where input/output disturbances, such as step disturbances, dynamic disturbance rejection are provided by this controller [82]. Preview information of the desired trajectory employed in the designing of the feed-forward control scheme for NMP systems and it has been analyzed theoretically and its performance compared with the preview-based stable inversion and optimum zero phase error tracking controller [86]. Two different NMP models have been developed using input output experimental data from a linear servo system using MATLAB system identification toolbox, and they are controlled by a feed-forward zero phase error tracking controller [87].

If the model is error free and there is no external disturbance affecting the model, a controller architecture is developed, which consists of two feed-forward control components sharing information in between them that can employ the perfect reference tracking for both minimum phase and non-minimum phase systems with time delays [88].

The advantage of the 2DOF control structure efficiently performs the disturbance free set point tracking even if for an uncertain process. Here, the load disturbance rejection controller and set point tracking controller have been designed individually [89]. We have studied these different successful applications of the 2DOF control strategy and are very much inspired to design the proposed control structure.

#### 6.3 Pole Placement Method

In the linear time invariant (LTI) system, 'Pole placement' is a conventional design methodology. In this method, closed-loop stable poles are considered in a suitable region of the complex plane to nullify the adverse effect of right hand plane (RHP) pole/zeros [90]. Here, we have used a state feedback method to get the feedback compensation in the 2DOF control structure.

The stability and transient characteristics are determined by the eigen values of a closed loop system matrix A - BK (shown in Fig.5.1). If the matrix K is chosen properly, the matrix A - BK can be made an asymptotically stable matrix [91].

## 6.4 MRAC Control Algorithm using MIT rule in 2DOF Framework

## 6.4.1 MIT rule based MRAC Structure



Fig. 6.1 Block diagram of MIT rule based MRAC structure

Fig. 6.1.describe the block diagram of the MIT rule based MRAC structure

Plant Transfer Function is  $G_{p}(S)$ .

Reference Model Transfer function is  $G_m(S)$ .

Reference Input is  $U_c$ .

Control Input to the Plant is U.

Plant Output is *Y* 

Reference Model Plant Output  $y_m$ 

Error between Plant Output and Reference Model Output is e

Required control parameter for the control input  $\theta$ .

Tuning gain of the control parameter is  $\gamma$ 

#### 6.4.2 Control Algorithm

We can get tracking error like the following equation,

$$e = y - y_m \,. \tag{6.1}$$

The output y of the plant is compared to a desired response  $y_m$  from a reference model. The controller parameters are updated based on this error.

After comparing, y and  $y_m$ , error is developed and parameters of the controller is obtained.

From this error, a cost function  $J(\theta)$  is formed, where  $\theta$  is the parameter that will be adapted inside the controller.

A typical cost function

$$J(\theta) = \frac{1}{2}e^{2}(\theta) \tag{6.2}$$

Here, :

$$\frac{d\theta}{dt} = -\gamma' \frac{\partial J}{\partial \theta} = -\gamma' e \frac{\partial e}{\partial \theta}$$
(6.3)

The  $\frac{\partial e}{\partial \theta}$  is known as sensitivity derivative.

MIT rule is defined as the calculation of the rate of change of control parameters and cost function and it is very much responsive to the modulus of the signal. The  $\gamma$  has been used to tune the control parameter, and usually it should be kept very low. Different parametric values of the control input affect the inputs and outputs of the controlled system. In this control algorithm, sensitivity will be multiplied by the error.

The block diagram of the MIT rule based MRAC control algorithm has been shown in the Fig. 6.1.The reference model plant has been considered for having the desired value of the gain, and it has been used three times in the MATLAB SIMULINK model of control structure.

The gain in the feed-forward part of the controlled part must be matched with the reference model plant. Here, the unknown constant  ${}^{'}K_{p}{}^{'}$  and  ${}^{'}K_{m}{}^{'}$  are the gain of MIT rule based MRAC structure.

$$\frac{Y(S)}{U_c(S)} = K_m G(S) \tag{6.4}$$

The same cost function is chosen as (6.2)

$$J(\theta) = \frac{1}{2}e^{2}(\theta) \tag{6.5}$$

And same derivative of cost function is chosen as (6.3)

$$\frac{d\theta}{dt} = -\gamma' \frac{\partial J}{\partial \theta} = -\gamma' e \frac{\partial e}{\partial \theta}$$
(6.6)

$$y_m = G_m U_c = K_m G U_c \tag{6.7}$$

$$e = y - y_m = K_p G U - K_m G U_c = K_p G \theta U_c - K_m G U_c$$
(6.8)

The parameter  $\theta$  of the above expression is to be updated.

The sensitivity derivative is calculated and restated in terms of the model output.

$$e = y - y_m = K_p G \theta U_c - K_m G U_c \tag{6.9}$$

$$\frac{\partial e}{\partial \theta} = K_P G U_C = \frac{K_P}{K_m} y_m \tag{6.10}$$

$$y_m = G_m U_c = K_m G U_c \tag{6.11}$$

$$GU_C = \frac{1}{K_m} y_m \tag{6.12}$$

Now the MIT rule to give an expression for updating  $\theta$ ,

 $K_{p}$  and  $K_{m}$  are used to construct the  $\gamma$ .

From equation (6.9) and (6.10), we get,

$$\frac{d\theta}{dt} = -\gamma' e \frac{\partial e}{\partial \theta} = -\gamma' \frac{K}{K_0} y_m e = -\gamma y_m e$$
(6.13)

By integrating  $\frac{d\theta}{dt}$  of equation (6.13) with respect to t, we get,

$$\theta = \frac{\gamma}{s} y_m e \tag{6.14}$$

$$U = \theta_1 U_c - \theta_2 y \tag{6.15}$$

$$e = y - y_m = G_p U - G_m U_c \tag{6.16}$$

$$y = G_p U = G_p [\theta_1 U_c - \theta_2 y]$$
(6.17)

$$[1+G_p\theta_2]y = G_p\theta_1U_c \tag{6.18}$$

$$e = y - y_m = \frac{G_p \theta_1 U_c}{1 + G_p \theta_2 U_c} - G_m U_c$$
(6.19)

$$\frac{\partial e}{\partial \theta_1} = \frac{G_p U_c}{1 + G_p \theta_2 U_c} \tag{6.20}$$

$$\frac{\partial e}{\partial \theta_2} = \frac{G_p^2 \theta_1 U_c}{\left(1 + G_p \theta_2 U_c\right)^2} \tag{6.21}$$

MRAC architecture which is used for feed-forward control is shown in Fig. 6.2

$$U = \theta_1 U_c - \theta_2 y \tag{6.22}$$

$$e = y - y_m = G_p U - G_m U_c$$
 (6.23)

Where, Transfer function of the plant is  $G_p$ , the transfer function of the RMP is  $G_m$ , and is reference model type T.F. is  $G_m$ .

$$y = G_p U = G_p [\theta_1 U_c - \theta_2 y] = G_p \theta_1 U_c - G_p \theta_2 y$$
(6.24)

Or,

$$[1+G_p\theta_2]y = G_p\theta_1 U_c \tag{6.25}$$

or, 
$$y = \frac{G_p \theta_1 U_c}{1 + G_p \theta_2}$$
 (6.26)

Chapter 6 Design of Adaptive Two-Degree-of-Freedom Controller for inversion based Reference Input Tracking of NMP system using MIT rule based MRAC



Fig 6.2. Model Reference Adaptive Control architecture using MIT rule

The error has been written again in terms of adaptive terms  $\theta_1$  and  $\theta_2$ .

$$e = y - y_{\rm m} = \frac{G_p \theta_1 U_c}{1 + G_p \theta_2 U_c} - G_m U_c$$
(6.27)

Now,

Derivative of sensitivity error is,

$$\frac{\partial e}{\partial \theta_1} = \frac{G_p U_c}{1 + G_p \theta_2 U_c} \tag{6.28}$$

$$\frac{\partial e}{\partial \theta_2} = \frac{G_p^2 \theta_1 U_c}{\left(1 + G_p \theta_2 U_c\right)^2} \tag{6.29}$$

or, 
$$\frac{\partial e}{\partial \theta_2} = \frac{G_p}{1 + G_p \theta_2 U_c} y$$
 (6.30)

$$\frac{\partial e}{\partial \theta_1} = G_m(S)U_C \tag{6.31}$$

$$\frac{\partial e}{\partial \theta_2} = G_m(S) y \tag{6.32}$$

Now from the equation (6.13), we get,

$$\frac{d\theta_1}{dt} = -\gamma \frac{\partial e}{\partial \theta_1} e \tag{6.33}$$

$$\frac{d\theta_2}{dt} = -\gamma \frac{\partial e}{\partial \theta_2} e \tag{6.34}$$

The equations (6.33) and (6.34) have been integrated, and then, we get the updated parameters,  $\theta_1$  and  $\theta_2$ .

#### 6.5 Numerical Example

The following system representation has been considered for the experimentation of the MIT rule based MRAC as a Feed-forward control scheme in the 2DOF framework.

The state space representation of system is given below,

The state equation is given by,

$$\dot{X} = AX + Bu \tag{6.35}$$

Output equation is given by,

$$y = CX + Du \tag{6.36}$$

Where,

System Matrix,

$$A = \begin{bmatrix} -5 & -4 \\ 1 & 0 \end{bmatrix},\tag{6.37}$$

Input Matrix,

$$B = \begin{bmatrix} 1\\0 \end{bmatrix} \tag{6.38}$$

**Output Matrix** 

$$C = [-4 \ -10] \tag{6.39}$$

Transmission Matrix,

$$D = [1]$$
 (6.40)

Plant Transfer Function:

$$G_p(S) = \frac{S^2 + b_{1n}S + b_{0n}}{S^2 + a_{1n}S + a_{0n}}$$
(6.41)

	Input Tracking of NMP system using MIT rule based MI
Where	
$b_{1n} = 5$	(6.42)
$b_{0n} = 4$	(6.43)
and	
$a_{1n} = 1$	(6.44)
$a_{0n} = -6$	(6.45)
Reference Model,	
$G_m(S) = \frac{b_m}{S + a_m}$	(6.46)
Where,	
$b_m = 3$	(6.47)
$a_m = 3$	(6.48)
Defense Medalli	

Reference Model like system,

$$G_m(S) = \frac{a_m}{S + a_m} \tag{6.49}$$

Reference model plant has been used for sensitivity derivatives.

The above numerical example for plant model and reference model plant has been constructed using the MATLAB toolbox.

#### 6.6 Simulation Result

Unit step input signal has been used here for all the software simulation in the MATLAB SIMULINK environment.



Fig. 6.3 Unit step response of uncontrolled NMP system





#### 6.7 Result Analysis

The objective of this work is to design a 2DOF controller to stabilize the unbounded output response of the NMP system as well as to follow the command input trajectory which is given as an input signal to the 2DOF controlled system. Fig. 6.3 shows the output response of an uncontrolled NMP system which is unstable in nature and, as a result, it could not track the desired output trajectory, which was placed as a reference input signal. The inverse model of the NMP system obviously produces an unstable response as RHP zero is transformed into a RHP pole, and that is shown in Fig. 6.4. Though the SFB controller is able to stabilize the

plant, a large initial undershoot is found in its step response, it's not at all acceptable for the tracking control system shown in Fig. 6.6. When the inverse and non-inverse NMP systems are connected in a cascade, unlike the minimum phase system, the output becomes unbounded as shown in Fig. 6.5.



Fig .6.5 Unit step response of cascaded non-inverse and inverse NMP system



Fig. 6.6 Unit step response of SFB controlled NMP system


Fig. 6.7 Unit step response of control input for 2DOF controlled NMP system



Fig. 6.8 Unit step response of tracking error of 2DOF controlled NMP system

After application of the MIT rule based direct MRAC scheme as feed-forward and SFB control method as feedback in 2DOF framework, the proposed closed loop 2DOF controlled system follows the reference input trajectory shown in Fig. 6.9. The steady state error of the SFB controlled system, found in Fig. 6.6, become nullified here, but initial undershoot has not been completely removed by the proposed control structure. Control input and tracking error of 2DOF controlled system have been shown in the Fig. 6.7 and Fig. 6.8 respectively.

#### Chapter 6 Design of Adaptive Two-Degree-of-Freedom Controller for inversion based Reference Input Tracking of NMP system using MIT rule based MRAC



Fig. 6.9 Unit step response of 2DOF controlled inversion based NMP system

# 6.8 Difficulties with the MIT Technique

Main drawbacks of this technique are,

- The proof of stability of the MRAC controlled closed loop system cannot be possible, which is the most essential part of the designing of any closed-loop controlled system.
- 2) Barbalat's Lemma cannot be applicable to prove the tracking error tends to zero.
- Two extra reference model type transfer functions are needed in the SIMULINK model of the closed loop controlled system.
- The initial undershoot may not be completely removed by this MIT rule-based MRAC technique.
- 5) The Adaptive Control scheme, which is used for the NMP system, exhibits nonlinearity in the transient response of the NMP system. The direct MRAC, which is based on the MIT rule, is unable to solve the nonlinear system's stability problem [93].

Chapter 6 Design of Adaptive Two-Degree-of-Freedom Controller for inversion based Reference Input Tracking of NMP system using MIT rule based MRAC

#### 6.9 Chapter Summery

To achieve the set point tracking of the NMP system, a 2DOF control structure, based on an inversion model for unstable NMP systems, has been proposed here. This 2DOF control methodology consists of feed-forward and feedback compensation, where, the MIT rule-based MRAC structure connected as a feed-forward control part and its feedback counterpart has been successfully implemented by the SFB controller, which is based on an arbitrary pole placement technique. The design of this two-fold control structure has been done independently and they are not directly linked with each other. The tracking problem has been solved by the MIT rule-based MRAC scheme as feed-forward compensation, where an inversion model of the NMP system has been used. The large steady state error found in the SFB controlled NMP system has been eliminated, but the initial undershoot is not completely removed.

#### **CHAPTER 7**

# **Generalized MRAC Employed in 2DOF Framework**

#### 7.1 Introduction

Direct MRAC is a particular class of adaptive control system, where RMP plays an important role in the control structure, which is used to present the desired transient response of the plant to be controlled. The block diagram of MRAC has been provided by Fig. 4.1. The MRAC structure consists of four parts shown in four blocks, e.g. reference model plant, the plant to be controlled, the adjustment mechanism and the controller. An error signal is derived by comparing the reference model plant's output and the controlled plant's output. This section describes the MRAC control algorithm.

# 7.2 Generalized Control Algorithm using Lyapunov Design Technique

Tracking error e will be produced due to the difference of output signals y from plant and  $y_m$  from the reference model plant. Error dynamics will converge to zero by the technique of adaptation mechanism [92].

# 7.2.1 Flow Chart Diagram

The MRAC technique has been employed as feed-forward control technique in 2DOF framework. Lyapunov Design technique based MRAC algorithm has been represented in a concise form of the flow chart diagram in Fig. 7.1 provided by the next section.

# 7.2.1.1 Plant Model and Reference Model Plant

Considering the differential equations of the  $n^{th}$  order plant and reference model plant are represented by equation (7.1) and (7.2)

$$\frac{d^{n} y}{dt^{n}} = -a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} - a_{n-2} \frac{d^{n-2} y}{dt^{n-2}} - \dots - a_{1} \frac{dy}{dt} - ay$$

$$+b_{n-1} \frac{d^{n-1} u}{dt^{n-1}} + b_{n-2} \frac{d^{n-2} u}{dt^{n-2}} + \dots + b_{1} \frac{du}{dt} + bu$$
(7.1)





$$\frac{d^{n} y_{m}}{dt^{n}} = -a_{m(n-1)} \frac{d^{n-1} y_{m}}{dt^{n-1}} - a_{m(n-2)} \frac{d^{n-2} y_{m}}{dt^{n-2}} - \dots$$

$$-a_{m1} \frac{dy_{m}}{dt} - a_{m} + b_{m} r$$
(7.2)

Here,

y is the output of the plant

 $y_m$  is the output of the reference model plant

*u* is control input

r is reference input

 $[a_{n-1}, a_{n-2}, \dots, a; b_{n-1}, b_{n-2}, \dots, b_1, b]$  are the parameters of the plant

 $[a_{m(n-1)}, a_{m(n-2)}, ..., a_{m1}, a_m; b_m]$  are the parameters of the reference model plant

# 7.2.1.2 n<sup>th</sup> order Tracking Error

*e* is the tracking error, which is produced due to the difference between the plant output and RMP output

$$e = y - y_m \tag{7.3}$$

By subtracting equation (7.2) from equation (7.1), we get the  $n^{th}$  order error dynamics,

$$\frac{d^{n}e}{dt^{n}} = \frac{d^{n}y}{dt^{n}} - \frac{d^{n}y_{m}}{dt^{n}} = -a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} - \dots + b_{n-1}\frac{d^{n-1}u}{dt^{n-1}} + \dots$$

$$-(-a_{m(n-1)}\frac{d^{n-1}y_{m}}{dt^{n-1}} - a_{m(n-2)}\frac{d^{n-2}y_{m}}{dt^{n-2}} - \dots + b_{m}r$$
(7.4)

#### 7.2.1.3 Choice of Control Input

Consider the control input is,

$$u = \theta_1 r - \theta_2 y \tag{7.5}$$

Where,

 $\theta_1$  and  $\theta_2$  are control parameters.

The objective of this control algorithm is to find out the control parameters, which are an essential part of the control input u.

#### 7.2.1.4 Derivation of error dynamics

Replacing *u* from equation (7.5) to equation (7.4), we get the  $n^{th}$  order error dynamics model,

$$\frac{d^{n}e}{dt^{n}} = -a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} - a_{n-2}\frac{d^{n-2}y}{dt^{n-2}} - \dots + b_{n-1}\frac{d^{n-1}(\theta_{1}r - \theta_{2}y)}{dt^{n-1}} + \dots - (-a_{m(n-1)}y_{m}^{n-1} - \dots + b_{m}r)$$
(7.6)

By adding and deducting

$$a_{m(n-1)} \frac{d^{n-1}y}{dt^{n-1}}, a_{m(n-2)} \frac{d^{n-2}y}{dt^{n-2}}, \dots, a_{m1} \frac{dy}{dt}, a_m y$$

with equation (7.6), we get,

$$\frac{d^{n}e}{dt^{n}} = \left(-a_{m(n-1)}\frac{d^{n-1}y}{dt^{n-1}} + a_{m(n-1)}\frac{d^{n-1}y_{m}}{dt^{n-1}}\right) + \left(-a_{m(n-2)}\frac{d^{n-2}y}{dt^{n-2}} + a_{m(n-2)}\frac{d^{n-2}y_{m}}{dt^{n}}\right) + \left(-a_{m(n-2)}\frac{d^{n-1}y}{dt^{n-1}} - a_{n-1}\frac{d^{n-1}y}{dt^{n-1}}\right) + \left(-a_{m(n-2)}\frac{d^{n-2}y}{dt^{n-2}} - a_{n-2}\frac{d^{n-2}y}{dt^{n-2}}\right) + \left(a_{m(n-2)}\frac{d^{n-2}y}{dt^{n-2}} - a_{n-2}\frac{d^{n-2}y}{dt^{n-2}}\right) + \left(-a_{m(n-1)}\frac{d^{n-1}e}{dt^{n-1}} - a_{m(n-2)}\frac{d^{n-2}e}{dt^{n-2}} - \dots\right) + \left(a_{m(n-1)}\frac{d^{n-1}e}{dt^{n-1}} - a_{m(n-2)}\frac{d^{n-2}e}{dt^{n-2}} - \dots\right) + \left(a_{n-1} - a_{m(n-1)}\frac{d^{n-1}y}{dt^{n-1}} - \left(a_{(n-2)} - a_{m(n-2)}\right)\frac{d^{n-2}y}{dt^{n-2}} - \dots\right) + \left(a_{n-1} - a_{m(n-1)}\frac{d^{n-1}y}{dt^{n-1}} - \left(a_{(n-2)} - a_{m(n-2)}\right)\frac{d^{n-2}y}{dt^{n-2}} - \dots\right) + \left(a_{n-1} - a_{m(n-1)}\frac{d^{n-1}y}{dt^{n-1}} + \dots - b_{m}r\right) + \left(a_{n-1} - a_{m(n-1)}\frac{d^{n-1}y}{dt^{n-1}} + \dots - a_{m}r\right) + \left(a_{n-1} - a_{m(n-1)}\frac{d^{n-1}y}{dt^{n-1}} + \dots - a_{m}r\right) + \left(a_{n-1} - a_{m}\frac{d^{n-1}y}{dt^{n-1}} + \dots - a_{m}r\right)$$

By integrating equation (7.8), (n-1) times with respect to time and considering the initial value of all control variables, we get,

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - a_m)r$$
(7.9)

# 7.2.1.5 Finding the Parameter Adjustment Rule

From the error dynamic equation (7.9), it is observed that tracking error converges to zero if the parameters are adjusted like equation (7.10) and (7.11),

$$(b\theta_2 + a - a_m)y = 0 \tag{7.10}$$

$$(b\theta_1 - a_m)r = 0 \tag{7.11}$$

As  $y \neq 0$  and  $r \neq 0$ , we get,

$$\theta_1 = \frac{a_m}{b} \tag{7.12}$$

$$\theta_2 = \frac{a - a_m}{b} \tag{7.13}$$

Here,  $\theta_1$  and  $\theta_2$  are the required control parameters which consist of the parameters of plant and reference model plant.

# 7.2.1.6 Choice of Lyapunov Function

Considering the following quadratic equation as a Lyapunov function,

$$v(e,\theta_1,\theta_2) = \frac{1}{2} \left( e^2 + \frac{1}{b\lambda} (b\theta_2 + a - a_m)^2 + \frac{1}{b\lambda} (b\theta_1 - b_m)^2 \right)$$
(7.14)

Where,  $b\lambda > 0$ .

Here, the Lyapunov function is the function of error *e* and required control parameters  $\theta_1$  and  $\theta_2$ .

 $\lambda$  is considered for tuning of the required control parameters.

#### 7.2.1.7 Derivation of Lyapunov function

First order derivative of the equation (7.14), gives

$$\frac{dv}{dt} = e\frac{de}{dt} + \frac{1}{\lambda}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\lambda}(b\theta_1 - b_m)\frac{d\theta_1}{dt}$$
(7.15)

Putting the value of  $\frac{de}{dt}$  from equation (7.9) into (7.15),

we get,

$$\frac{dv}{dt} = -a_m e^2 + \frac{1}{\lambda} (b\theta_2 + a - a_m) (\frac{d\theta_2}{dt} - y\lambda e) + \frac{1}{\lambda} (b\theta_1 - b_m) (\frac{d\theta_1}{dt} + \lambda r e)$$
(7.16)

# 7.2.1.8 Finding Control Parameter

From equation (7.16), it is clear that,  $\frac{dv}{dt}$  will be negative if the extra terms of equation

(7.16) are equal to zero.

If the derivative of Lyapunov function is negative definite, it indicates that

v(t) > v(0) and it ensures that,  $e, \theta_1, \theta_2$  must be bounded.

If we put the extra term equal to zero, we also get the control parameters from the

equation (7.17) and equation (7.18).

$$\frac{d\theta_2}{dt} - y\lambda e = 0 \tag{7.17}$$

$$\frac{d\theta_1}{dt} + \lambda re = 0 \tag{7.18}$$

From equation (7.17) and (7.18), we get the required control parameters.

$$\theta_1 = -\frac{1}{S} yre \tag{7.19}$$

$$\theta_2 = \frac{1}{S} y \lambda e \tag{7.20}$$

From equations (7.19) and (7.20), it has been found that control parameters can easily avoid the plant parameters and reference model plant parameters by using this Lyapunov Design technique. Control parameters are only consisting of the reference input signal, plant output and tracking error. Tuning gains are provided to tune the controlled plant output.

#### 7.2.1.9 Finding Control Input

Now, if we put the value of  $\theta_1$  and  $\theta_2$  from equations (7.19) and (7.20) into equation (7.5), we can get the required control input for the closed-loop MRAC controlled plant.

$$u = -\frac{1}{S} yr^2 e - \frac{1}{S} y^2 \lambda e \tag{7.21}$$

By observing the equation (7.21), it has been observed that, control input obtained by using this technique, is independent of plant parameter and reference model plant parameters.

# 7.3 Stability Proof

From the equation (7.16), it has been observed that the stability of a closed loop controlled system will be automatically satisfied if the parameter adjustment rule is achieved.

#### 7.4 Barbalat's Lemma

If a scalar function V(x,t) satisfies the following conditions [92]

- 1. V(x,t) is lower bounded.
- 2. V(x,t) is negative semi-definite.
- 3.  $\dot{V}(x,t)$  is uniformly continuous in time.

Then,  $V(x,t) \rightarrow 0$  as  $t \rightarrow \infty$ 

If we put the extra term of equation (7.9) equal to zero, we will get equation (7.22),

$$\frac{dv}{dt} = -a_m e^2 \tag{7.22}$$

The second derivative of equation (7.22) is required to ensure the zero tracking error

$$\frac{d^2 v}{dt^2} = -2a_m e \frac{de}{dt}$$
(7.23)

Since all the parameters are bounded and  $y = e + y_m$  is bounded, and,  $\frac{d^2v}{dt^2}$  is also bounded, which implies that  $\frac{dv}{dt}$  is uniformly continuous. Now using Barbalat's Lemma, we conclude that tracking error converges to zero.

#### 7.5 Chapter Summery

The proposed algorithm has been derived based on Lyapunov Design technique. As the control law design is based on Lyapunov Design technique, stability is ensured spontaneously while deriving the parameters of the control law. Moreover, we have employed the error dynamic model for the design of the control law and it enabled us to directly check the robustness of asymptotic stability as well as perfect tracking, i.e.  $e \rightarrow 0$  as  $t \rightarrow \infty$ . Thus, robustness in stability as well as robustness in tracking is satisfied. The tracking performance has been verified by the Barbalat's Lemma.

# **CHAPTER 8**

# Reference Input tracking of inversion-based Non-Minimum Phase System using Adaptive Two-Degree-of-Freedom Control

#### 8.1 Introduction

Tracking control is one of the essential requirements for the control system. To solve the tracking problem, model based feed-forward, enhanced feed-forward, two feed-forward, adaptive feed-forward, and feed-forward model inverse control have been studied in different literature.

In the early days, the regular change of the set point or command input signal is not necessary in control system engineering. So, there is no requirement of optimal tuning for the disturbed output response and, as a result, for a conventional PID controller, it is sufficient to solve the control problem. But in these modern days, frequent set point changes occur and, consequently, a more powerful control system is essentially required. Here, the 2DOF PID controller can overcome these difficulties by replacing the conventional classical PID Controller [35].

A real time XY table can be realized by two mathematical models using the MATLAB Identification toolbox. Out of the two models, one is the MP model and another is the NMP model, and these two models are controlled by adaptive feed-forward zero phase error tracking control (ZPETC) with the adaptive gain adjustment by recursive least square [RLS] algorithm [94].

An unstable and linear time-invariant model of a diesel generator exhibits NMP characteristics when it is converted to a discrete time model. At first, the model predictive controller (MPC) has been applied to the diesel generator system model, and then the transfer function of the closed loop system model has been derived. After that, the feed-forward controller has been developed using the inverse model of the derived transfer function [95]. Out of these control schemes, the inversion based adaptive feed-forward technique is very popular for tracking control systems.

But inversion based techniques are restricted to the MP system only. In the case of an NMP system, it shows an abrupt output response due to unstable zero, which is obvious in it.

Adaptive feed-forward zero phase error tracking control and feed-forward model inverse control using a predictive model applied in diesel generators are proposed for discrete time systems. For a continuous time NMP system, it is difficult to design a tracking control system that asymptotically tracks an arbitrary reference desired trajectory [96].

The 2DOF control approach has been demonstrated on a bioreactor, which exhibits maximum NMP characteristics. The gain scheduling plant has been used here to stabilize the plant. The nominal state vector and current nominal control signal are scheduled to construct the parameters of the controller. Here, based on the nominal trajectory, the linearization of the plant is done because the linear parameter varying model is required for the designing of this controller [97].

A feed-forward control scheme has been applied to one discrete time system model, which exhibits NMP characteristics due to small or reduced sampling time. This controller used a closed loop inverse model and its internal stability could not be assured. Zero phase error tracking controller (ZPETC) overcomes this problem of stability by cancelling all poles and zeros after approximating the NMP system model using the MATLAB system identification toolbox [98].

Combination of feed-forward and feedback control in a two-degree-of-freedom framework has been effectively applied in several control systems, like hydraulically actuated hexapod robot, motion system, piezoelectric actuators, pumped storage power plant, unstable electro-hydraulic actuator with NMP characteristics [81].

An efficient electro-hydraulic servo system is used as the actuator of walking robots, because large power is required for walking on rough surfaces. Due to changes in the hydraulic properties, a high degree of uncertainty and non-linearity, time delay in the oil flow and deadzone in the electromagnetic valve have occurred, and as a consequence, locomotion control of the actuator for the legged robot becomes challenging. To mitigate this control problem, onestep ahead, a fuzzy pre-filter as feed-forward compensation and a PI-based fuzzy controller as its feedback counterpart in the 2DOF structure has been suggested here [99].

Advancements in micro/nano technology lead to the controlling of piezoelectric elements, as these elements are the key devices for the actuation of micro/nano positioning stages. Inherent hysteresis and creep nonlinearity are found in piezoelectric materials, which may reduce efficiency, even if the stability of the whole system may be affected. The combination of feed-forward and feedback may be the solution to this problem in its application

on the electro-hydraulic actuator (EHA) as this actuator suffers from time-varying parameters with non-linearity and uncertainty [100] [102].

To get better performance of DSMC, applied on EHA, a point to point trajectory concept has been implemented in the system model. The DSMC in 2DOF control method provides robustness against uncertainty and disturbances during the time of position tracking at different points of trajectory [103].

The static feed-forward part of a 2DOF control method is usually used in pumped storage power plants. Due to the NMP nature of the storage plant, at the time of set point changes in generator output, the controlled pumped storage plant leads to pressure oscillation. To avoid these negative phenomena, a flatness based feed-forward is developed and implemented in the already existing 2DOF control structure [104].

The adaptive feed-forward scheme is efficiently used in tracking the control system structure of any dynamic system, but if it is used in the linear NMP system, it introduces non-linearity in its dynamic characteristics [93]. One of the suitable methods to solve the stability problem of non-linear systems is the Lyapunov stability theory method [105]. The Lyapunov stability theory and input output system variables of the system are used to develop the MRAC type adaptive control structure [106]. The Lyapunov Design technique based on MRAC has been applied to the simplified mathematical model of permanent magnet linear motor drives, and here, it has been observed, that the dynamic and steady state performance of the Lyapunov based model reference adaptive System based controller provides better performance than conventional PID controller for the permanent magnet linear motor [107].

Again, problems regarding the stabilization and tracking of non-linear systems like the Vander Pol Oscillator may be solved by Lyapunov stability theory discussed in a master's thesis [108].

There is a comparative analysis performed between MRAC using Lyapunov stability theory and a fuzzy learning control for the balancing of the benchmarked system model of an inverted pendulum and both control techniques have been designed on the basis of the reference model [109].

Comparative analysis of MIT rule based MRAC and Lyapunov stability theory based MRAC has been performed on the first order slow process, which is a part of the level process control in the MATLAB simulation environment and proves the superiority of the Lyapunov Design technique based on MRAC [110].

In previous research work, the MIT rule based MRAC method was implemented on a  $2^{nd}$  order linear NMP system model using step input signal [111]. One major problem was found that an undesirable initial undershoot has not completely been removed and robustness with respect to stability and tracking performance cannot be proved.

Here, we have considered the Lyapunov based Direct Model Reference Adaptive feedforward control for tracking the reference input signal for an inversion-based second order NMP System. State feedback control using arbitrary pole placement acts as a feedback part of a two-degree-of-freedom structure.

# 8.2 Problem Statement

Direct Model Reference Adaptive Control (MRAC) of an inverse transfer function model of the NMP system has to be proposed here as a feed-forward compensator in the 2DOF framework to track the reference input trajectory, and a SFB control scheme has been considered here for feedback compensation. Initially, we had designed the MIT rule-based MRAC as feed-forward compensation in the 2DOF structure, but we had faced some difficulties with this control technique discussed in the chapter 6.

# 8.3 Two-degree-of-Freedom Control

In the 2DOF structure, the inverse transfer model-based NMP system has been considered for feed-forward control and a non-inverse transfer function model-based NMP system is considered for feedback control. Here, feed-forward and feedback controllers are decoupled with each other.

# 8.4 State Feedback Control

The state feedback control using an arbitrary pole placement method has been used as a feedback counterpart for the non-inverse NMP employed in the 2DOF control structure. Here, the transient characteristics as well as stability of the system are determined by the eigen values of the system matrix (refer to Fig. 5.1) If the matrix is chosen accurately, the matrix can be made an asymptotically stabilized matrix [90].

#### 8.5 MRAC Using Lyapunov Design Technique

The generalized Lyapunov Design technique of the MRAC scheme has been provided in chapter 7. Here we are presenting a Lyapunov based MRAC technique for an inverse-based NMP system.

The basic structure of the MRAC scheme is shown in Fig. 4.1. It has four parts: the reference model, the plant, the controller and the adjustment mechanism [112]. The MRAC structure and its description are provided in chapter 4.

The reference model is chosen to get the desired trajectory. The output response of the reference model must be tracked by the output response of the closed loop plant using the adjustment mechanism. MRAC is divided into direct and indirect methods. In direct MRAC, the parameter vector is updated directly by adaptive law [113].

Let the plant Transfer function [114]

$$G(S) = \frac{b_m S^m + b_{m-1} S^{m-1} + \dots + b_0}{a_n S^n + a_{n-1} S^{n-1} + \dots - a_0} = \frac{Y(S)}{U(S)}, \ m \le n$$
(8.1)

$$a_{n}S^{n}Y(S) + a_{n-1}S^{n-1}Y(S) + \dots - a_{0}Y(S) = b_{m}S^{m}U(S) + b_{m-1}S^{m-1}U(S) + \dots + b_{0}U(S)$$

$$(8.2)$$

$$\frac{1}{a_n} \left[ b_m u^{(m)} + b_{m-1} u^{(m-1)} + \dots + b_0 u - a_{n-1} y^{(n-1)} - \dots + a_0 y \right] = y^n$$
(8.3)

and reference model transfer function

$$G_m(S) = \frac{c_1}{S^2 + c_2 S + c_3} = \frac{Y_m(S)}{R(S)}$$
(8.4)

$$S^{2}Y_{m}(S) + c_{2}SY_{m}(S) + c_{3}Y_{m}(S) = c_{1}R(S)$$
(8.5)

By using inverse Laplace transformation, we get

$$y_m^{(2)} = c_1 r - c_2 y^{(1)} - c_3 y \tag{8.6}$$

Assuming plant output matches with the output of the reference model.

$$c_{1}r - c_{2}y^{(1)} - c_{3}y = \frac{1}{a_{n}} \left[ b_{m}u^{(m)} + b_{m-1}u^{(m-1)} + \dots + b_{0}u - a_{n-1}y^{(n-1)} - \dots + a_{0}y \right]$$
(8.7)

Initial values of the control parameters,

$$\theta_1 = \frac{c_1 a_n}{b_0} \tag{8.8}$$

$$\theta_2 = \frac{a_0 - c_1 a_n}{b_0}$$
(8.9)

Error,

$$e = y - y_m \tag{8.10}$$

Defining the Lyapunov Function,

$$V(e,\theta_1,\theta_2) = \frac{1}{2} \left[ e^2 + \frac{1}{b_0 \gamma} (b_0 \theta_2 + c_1 a_n - a_0)^2 + \frac{1}{b_0 \gamma} (b_0 \theta_1 - c_1 a_n)^2 \right]$$
(8.11)

Derivative of Lyapunov function,

$$\frac{dV}{dt} = -b_0 e^2 + \frac{1}{\gamma} (b_0 \theta_2 + c_1 a_n - a_0) (\frac{d\theta_2}{dt} - \gamma ye) + \frac{1}{\gamma} (b_0 \theta_1 - c_1 a_n) (\frac{d\theta_1}{dt} + \gamma re)$$
(8.12)

To get  $\frac{dV}{dt}$  negative semi definite,

$$\frac{d\theta_2}{dt} = \gamma ye \tag{8.13}$$

$$\frac{d\theta_1}{dt} = -\gamma re \tag{8.14}$$

From equation (8.13) and (8.14), we can get,

$$\theta_1 = -\frac{\gamma}{S} re \,, \tag{8.15}$$

$$\theta_2 = \frac{\gamma}{S} ye \tag{8.16}$$

Where,  $\theta_1$  and  $\theta_2$  are control parameters.

# 8.6 Numerical Example

Here we have examined the proposed Lyapunov based 2DOF control methodology by using the  $2^{nd}$  order NMP system whose state model has been represented by the following equation (same as Chapter 6)

	Adaptive Two-Degree-of-Freedom Co		
$\dot{X} = AX + Bu$ $Y = CX + Du$	(8.17)		
Where,			
$A = \begin{bmatrix} -5 & -4 \\ 1 & 0 \end{bmatrix}$	(8.18)		
$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$	(8.19)		
$C = \begin{bmatrix} -4 & -10 \end{bmatrix}$	(8.20)		

$$D = \begin{bmatrix} 1 \end{bmatrix} \tag{8.21}$$

We have chosen the following state space model of the 2<sup>nd</sup> order reference model plant

$$A_m = \begin{bmatrix} -5 & -16\\ 1 & 0 \end{bmatrix}$$
(8.22)

$$B_m = \begin{bmatrix} 1\\ 0 \end{bmatrix} \tag{8.23}$$

$$C_m = \begin{bmatrix} 0 \ 16 \end{bmatrix} \tag{8.24}$$

$$D_m = \begin{bmatrix} 0 \end{bmatrix} \tag{8.25}$$

We have also chosen 1<sup>st</sup> order reference model whose transfer function is in the following equation,

$$G_m(s) = \frac{1}{S+1}$$
(8.26)

is,

# 8.7 Simulation Result

Simulation results shows that the combination of Lyapunov theory based MRAC and SFB controller can effectively solve the tracking problem of inversion based NMP system. The unit step, parabolic and ramp signals are chosen as reference input trajectories for this controlled system.



Fig. 8.1 Unit step response of cascaded inverse and non-inverse MP system



Fig. 8.2 Unit step response of cascaded inverse and non-inverse NMP system



Fig. 8.3 Unit step response of 2DOF controlled NMP system with 1<sup>st</sup> order reference model



Fig. 8.4 Unit step response of control input for 2DOF controlled NMP system with 1<sup>st</sup> order reference model



Fig. 8.5 Unit step response of 2DOF controlled NMP system with 2<sup>nd</sup> order reference model



Fig. 8.6 Unit step response of control input for 2DOF controlled NMP system with the 2<sup>nd</sup> order reference model



Fig. 8.7 Unit ramp response of 2DOF controlled NMP system with 2<sup>nd</sup> order reference model



Fig. 8.8 Unit ramp response of control input for 2DOF controlled NMP system with the 2<sup>nd</sup> order reference model



Fig. 8.9 Unit parabolic response of 2DOF controlled NMP system with 2<sup>nd</sup> order reference model



Fig. 8.10 Unit parabolic response of control input for 2DOF controlled NMP system with the 2<sup>nd</sup> order reference model

# 8.8 Result Analysis

In the MATLAB SIMULINK environment, software experimentation has been performed on the same numerical example of an NMP system as stated in chapter 6, where we have found that an uncontrolled plant's output responses are shown in Fig. 6.3 (chapter 6) and unbounded output response of the inverse NMP system shown in Fig. 6.4 (chapter 6). When the inverse and non-inverse transfer function model of the NMP system are connected in series, it cannot cancel the poles and zeros like the minimum phase system in Fig. 8.1, and rather it produces an unbounded oscillatory output response shown in Fig. 8.2.

The Lyapunov based MRAC scheme as feed-forward of 2DOF control technique not only stabilizes the uncontrolled NMP system, its initial undershoot has been completely removed as shown in Fig. 8.3 and Fig. 8.5 with 1<sup>st</sup> order and 2<sup>nd</sup> order reference models respectively. Both the 2DOF controlled output response (shown in Fig. 8.3 and Fig. 8.5) effectively track the reference input trajectory. To prove the efficiency of the proposed tracking control system, unit ramp and parabolic command signals also have been applied, shown in Fig. 8.7 and Fig. 8.9 respectively, strengthening the concept of this proposed control methodology. The unit step control inputs with 1st order and 2<sup>nd</sup> order reference models are shown in Fig. 8.4 and Fig. 8.6 respectively. Fig. 8.8 and Fig. 8.10 show the ramp and parabolic control input respectively.

# 8.9 Chapter Summery

This approach uses a feed-forward direct MRAC of the inverse model plant using Lyapunov Design technique for desired trajectory tracking and state feedback control for stabilization of the NMP system. The arbitrary pole placement method has been used in feedback control to get the bounded time response of the NMP system. The proposed controller shows that it can provide impressive tracking performance for the step, ramp and parabolic reference input trajectories. Initial undershoot is an undesirable characteristic evolved from step response of the NMP system, and also can also be neutralized by the proposed control methodology.

# **CHAPTER 9**

# Design of different reference model based model reference adaptive controller for inversed model non-minimum phase system in 2DOF structure

#### 9.1 Introduction

Design of the tracking control system for minimum or non-minimum phase system is one of the indispensable as well as challenging tasks in the area of control system engineering. Different control techniques have been studied so far, and among them chattering free convergence has been developed by robust sliding mode learning control. A recursive learning mechanism based on sliding mode control has been designed to experiment on a class of non-linear NMP system, where the internal dynamics are stabilized completely [115]. To minimize the tracking errors of a system with NMP properties, a linear combination of user-defined basis functions with unknown coefficients the forwarded to filter using the dynamics of NMP system. Here, the frequently used non uniform rational B-spline (NURBS) curve has been analyzed to derive the set of basis functions [116]. State-based uncertainty has been found in some classes of uncertain nonlinear NMP systems. To get the output feedback tracking of those systems, the uncertainty is divided into two decomposed subsystems through an adaptive state decomposition process. The non-linear small gain theorem has been utilized for the development and stability analysis of the controlled system [117].

The output trajectories of the controlled plant cannot be shaped by the above discussed tracking control system. The control designer has no option to choose the shape of the trajectory path as the requirement of the control objective.

But, MRAC methodology can give this opportunity to select the dynamics of a reference model plant (RMP) as per the control designer's or researcher's choice. The output response of the plant always tracks the output trajectory of the RMP in this control scheme. Moreover, the MRAC scheme has been effectively implemented in different areas in control system engineering.

An air vehicle with a dual multi-degree-freedom manipulator uses Lyapunov-based MRAC to solve the control problem when both the manipulator movement and interaction with the target objects are performed by the air vehicle. Oscillation, produced from the unstable PID

#### Chapter 9 Design of different reference model based model reference adaptive controller for inversed model non-minimum phase system in 2DOF structure

control loop has been stabilized by this effective control method [118]. Simulation results of Lyapunov stability theory based by MRAC on the simplified mathematical model of permanent magnet linear motor shows that a dynamic performance and steady precision of the controlled system model is better than the conventional PID control of the said system model [119]. The vibration suppression of a smart piezoelectric mechanical structure has been controlled by nonlinear MRAC algorithm based on the finite element model structure. Robustness on stability of this control method is done by adding the integral term to the control algorithm [120]. The MRAC method based on reference model and adaptation has been implemented on a 1cm x 0.5 cm length Ionic polymer metal composite (IMPC) strip based on a Nafion 117 Na + membrane in the Lab view platform [121]. The mechanical and electrical equations have been used to develop the system model of a mechatronics elevator system, which is driven by a permanent magnet synchronous motor. The self-learning particle swarm optimization method identified the experimental parameters of the elevator. This system model has been considered for the application of the Lyapunov-based MRAC scheme to achieve the trajectory of the reference model's trajectory of the scheme [122]. A twin rotor system with multiple inputs and multiple outputs is a very complex non-linear system, as it decomposes into two subsystems, and the cross coupling between them exhibits disturbances to each other make the system very hard to control. In this twin motor system, the desired and perfect dynamic performance specified by the reference model has been achieved by a hyper-stability system based on adaptive control, and it has been applied to both the subsystems separately [123].

Both the MIT rule based and Lyapunov stability theory based MRAC are simultaneously implemented on a standard level process control system, where the system is a first order slow process on the MATLAB SIMULINK platform [109].

In the biological environment, the application of MRAC is praiseworthy. In this application, a master-slave structure of a soft tissue cutting process is modeled, and in this process, the D.C. motor-driven slave has to accurately track the movement of the master; though there are different disturbing forces affecting their movement, but this critical tracking problem can be solved by the dual functionality of the Lyapunov's 2<sup>nd</sup> technique based MRAC [124]. MRAC is also applied in the simulation of glucose regulation in blood [125]. Its efficiency has also been checked by comparing it with other control actions. It has been seen that the output of the MRAC compensated plant is more accurate than the fuzzy Logic controller and the PI controller when it is applied to the level control of a coupled tank system

[126].

Though the observed research work highlighted the accuracy of the MRAC scheme, they haven't shown any comparison between the reference model plant's output and the output of the plant to be controlled. This untouched part of the MRAC scheme will be presented in this research work.

The objective of this simulation is to verify whether the proposed MRAC controlled 2DOF control of the NMP system literally follows the output trajectory of a reference model, whose dynamic characteristics are specified by the control designer.

# 9.2 Problem Statement

It is well known, that the output trajectory of a MRAC controlled plant always tracks the reference model output trajectories. In this work, it has to be verified that the proposed MRAC controlled inversion based NMP system in the 2DOF framework actually tracks the RMP's output trajectory or not.

# 9.3 MRAC Employing Lyapunov Stability Theory for second order Plant

Tracking error is simply difference between the plant output and the reference model outputs refer to Fig. 9.1



# Fig. 9.1 Block diagram of MRAC structure

Considering 2<sup>nd</sup> order Reference Model:

$$\frac{d^2 y_m}{dt^2} = -a_m \frac{dy_m}{dt} + b_m r \tag{9.1}$$

And a 2<sup>nd</sup> order Plant Model:

$$\frac{d^2 y}{dt^2} = -a\frac{dy}{dt} + bu \tag{9.2}$$

Where,  $y_m$  is reference model output, y is plant output, r is reference input signal Let,

Where  $\theta_1$  and  $\theta_2$  are the control parameters

Control input: 
$$u = \theta_1 r - \theta_2 \frac{dy}{dt}$$
 (9.3)

Replacing u in equation(9.2) from equation (9.3), we get,

$$\frac{d^2 y}{dt^2} = -a\frac{dy}{dt} + b(\theta_1 r - \theta_2 \frac{dy}{dt})$$
(9.4)

Tracking error:

$$e = y - y_m \tag{9.5}$$

By subtracting the equation (9.1) from equation (9.2), we get,

$$\frac{d^2 e}{dt^2} = \frac{d^2 y}{dt^2} - \frac{d^2 y_m}{dt^2} = -a \frac{dy}{dt} + bu - (-a_m \frac{dy_m}{dt} + b_m r)$$
(9.6)

By adding and deducting  $a_m \frac{dy}{dt}$  with equation (9.6), we get,

$$\frac{d^2e}{dt^2} = -a\frac{dy}{dt} + a_m\frac{dy}{dt} + b\theta_1 r - b\theta_2\frac{dy}{dt} - a_m\frac{dy}{dt} + a_m\frac{dy_m}{dt} - b_m r$$
(9.7)

$$\frac{d^2e}{dt^2} = -a_m \left(\frac{dy}{dt} - \frac{dy_m}{dt}\right) - \left(b\theta_2 + a - a_m\right)\frac{dy}{dt} + \left(b\theta_1 - b_m\right)r\tag{9.8}$$

Or,

$$\frac{d^2e}{dt^2} = -a_m \frac{de}{dt} - (b\theta_2 + a - a_m)\frac{dy}{dt} + (b\theta_1 - b_m)r$$
(9.9)

Assuming the initial values of the control parameter  $\theta_1$  and  $\theta_2$ ,

and integrating equation (9.9) with respect to time 't',

We get,

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)r$$
(9.10)

From the error dynamics it is observed that the tracking error will go to zero, if

$$b\theta_2 = a_m - a \tag{9.11}$$

$$\theta_2 = \frac{a_m - a}{b} \tag{9.12}$$

and

$$b\theta_1 = b_m \tag{9.13}$$

$$\theta_1 = \frac{b_m}{b} \tag{9.14}$$

In this way, the control parameters have been developed by adjustment mechanism. Now, considering the Lyapunov function for the error dynamics,

$$v(e,\theta_1,\theta_2) = \frac{1}{2} \left( e^2 + \frac{1}{b\gamma} (b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma} (b\theta_1 - b_m)^2 \right)$$
(9.15)

where  $b\gamma > 0$ .

$$V(e,\theta_1,\theta_2) = \frac{1}{2}e^2 + \frac{(b\theta_2 + a - a_m)^2}{2b\gamma} + \frac{(b\theta_1 - b_m)^2}{2b\gamma}$$
(9.16)

This function becomes zero when e is zero and the controller parameters are equal to the correct values. For a valid Lyapunov function ,time derivative of lyapunov function must be negative. The derivative is given by,

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma} \left( b\theta_2 + a - a_m \right) \frac{d\theta_2}{dt} + \frac{1}{\gamma} \left( b\theta_1 - b_m \right) \frac{d\theta_1}{dt}$$
(9.17)

Substituting the value of  $\frac{de}{dt}$  from equation (9.10) into equation (9.17), we get,

$$\frac{dV}{dt} = e[-a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)r] + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt}$$
(9.18)

$$\frac{dV}{dt} = -a_m e^2 - (b\theta_2 + a - a_m)ye + (b\theta_1 - b_m)re + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt}$$
(9.19)

#### Chapter 9 Design of different reference model based model reference adaptive controller for inversed model non-minimum phase system in 2DOF structure

$$\frac{dV}{dt} = -a_m e^2 + \frac{1}{\gamma} (b\theta_2 + a - a_m) (\frac{d\theta_2}{dt} - \gamma ye) + \frac{1}{\gamma} (b\theta_1 - b_m) (\frac{d\theta_1}{dt} + \gamma re)$$
(9.20)

If the parameters are updated as follows

$$\frac{d\theta_1}{dt} = -\gamma re \tag{9.21}$$

$$\frac{d\theta_2}{dt} = \gamma \, ye \tag{9.22}$$

After integrating the equation (9.21) and (9.22), we get,

$$\theta_1 = -\frac{\gamma}{S}re \tag{9.23}$$

$$\theta_2 = \frac{\gamma}{S} ye \tag{9.24}$$

 $\theta_1$  and  $\theta_2$  are the control parameters with adjustable gain  $\gamma$ .

# 9.4 Numerical Example

The transfer function of the NMP system has been provided by the following numerical value:

$$G(s) = \frac{S^2 + S - 6}{S^2 + 5S + 4} \tag{9.25}$$

Reference Models used in this control stucture are of different damping nature and demonstrated in the table below.

# Table 9.1 Transfer function of different RMP on the basis on damping ratio

Transfer Function	Nature of Damping	
$\frac{25}{S^2 + S + 25}$	Underdamped	
$\frac{25}{\overline{S^2 + 10S + 25}}$	Critically damped	
$\frac{25}{S^2 + 120S + 25}$	Over damped	
$\frac{5}{S^2+5}$	Undamped	

# 9.5 Simulation Result

The inversion based NMP system has been considered as an unstable system for MRAC structure. The RMP with four types of damping nature has been incorporated with the proposed 2DOF based MRAC structure and step input as a set point has been applied here. The following plots demonstrate the output responses of the plant before and after the application of the proposed 2DOF control scheme.



Fig. 9.2 Unit step response of uncontrolled inverse model of NMP system



Fig. 9.3 Unit step response of MRAC controlled inverse NMP system in 2DOF framework using un-damped (damping ratio=0.1) reference model





Fig 9.4 Unit step response of MRAC controlled inverse NMP system in 2DOF framework using un-damped (damping ratio=0) reference model



Fig. 9.5 Unit step response of MRAC controlled inverse NMP system in 2DOF framework using critical damped (damping ratio=1) reference model



Fig. 9.6 Unit step response of MRAC controlled inverse NMP system in 2DOF framework using over-damped (damping ratio=12) reference model

# 9.6 Result Analysis

The previous example has been taken here to verify whether the proposed MRAC truly follows the desired trajectory provided by the reference model or not. On the basis of the damping ratio, four types of reference model have been considered here, which are incorporated into the MRAC structure. These are un-damped (damping ratio=0), under-damped (damping ratio=0.1), critical-damped (damping ratio=1), and over-damped (damping ratio=12).

The inverse model of the NMP system produces unbounded, and, which is obviously an unstable output response, as RHP zero is transferred into the RHP pole, shown in Fig. 9.2. It has been observed that all the output responses or trajectories of the proposed controlled NMP plant follow their respective reference model's output trajectories or, clearly say, dynamic characteristics of the reference model. Fig. 9.3, Fig. 9.4, Fig. 9.5, and Fig. 9.6 show that controlled plant output literally tracks the output trajectory of models with the characteristics of under-damped, un-damped, critically damped, and over-damped respectively. All the simulations have been done in the MATLAB SIMULINK environment, and it has been observed that all output responses almost track the reference model output trajectory.

# Table.9.2 Comparison of characteristics specification of step response of reference model and plant model

Characteristics Specification	Reference Model	Plant Model
Rise time (Secs)	0.33	0.32 seconds
Peak time (Secs)	0.63	0.59
Settling time (Secs)	1.9	1.8
Steady state error	Nil	Nil

# 9.7 Chapter Summery

This work demonstrates that the plant, which has been used as a reference model for the MRAC scheme for the 2DOF control structure, has an important role in tracking the control system. It provides a lot of advantage to the designers as they can get the chance to modify the plant output as per their needs. The RMP with four types of damping ratio has been employed in the MRAC structure to verify the role of the reference model. The inverse based NMP system model has been taken from chapter 6 and the step input signal has been considered as a set point. Lyapunov Design technique has been applied to obtain the control parameters of the MRAC algorithm. It has been shown that output trajectories of the controlled plant always follow the path obtained by the RMP of this MRAC structure.
# **CHAPTER 10**

# Elimination of Initial undershoot of Non-minimum Phase System

### 10.1 Introduction

In control systems, extensive research work on the system property has been done for many decades and the linear time invariant (LTI) system can be classified in various ways on the basis of the system property. If at least one of the zeros of the transfer function model of any LTI system is placed at the RHP of the S plane, then it is named as an NMP system and the peculiar characteristic of an NMP system is that its initial response goes opposite to it's steady state response [18]. It is basically the step response of the system, which, at first, moves inverse to the desired output trajectory provided by the set point, and after some time it follows the desired trajectory of a linear time invariant NMP system and, obviously, it is produced due to the zero, which is placed at RHP of the S plane. These NMP zeros cannot be cancelled by unstable poles as it hampers the stability of the system [127]. Presence of RHP zeros not only produces initial undershoots, some more unavoidable time domain characteristics, like zero crossing, overshoot also exhibits in its step response, and comparison of initial undershoots in different NMP system has been mathematically analyzed by using some theorems [128].

It has been studied that, complex conjugate RHP zeros has no contribution in the production of initial undershoot or, we can say the complex conjugate RHP zero has no influence on the characteristics of initial undershoot [129]. The time delay system also exhibits NMP characteristics in its dynamics after the pade approximation of its delay component [28]. Though the same magnitude is produced by the MP and NMP systems in frequency response analysis, the NMP system exhibits large phase lag, which is not treated as a good property of the system [5]. As input and input variables of the system being coupled to each other and represented as zeros of transfer function of the system, the response to an input signal is characteristics like undershoot, undershoot and overshoot are obvious in the step response of the NMP system. These phenomena in time response obstruct good tracking control of the NMP system. The NMP system is difficult to handle as closed poles move towards the RHP zero as the gain increases.

Researchers have nurtured different technologies to mitigate these unacceptable characteristics of the NMP system. Conventional control methods are not sufficient to control the deep-rooted characteristics of the undershoot of the NMP system [131].

There are different control methods that have been observed for the solution to the control problem of NMP systems. The active disturbance rejection control (ADRC) method is very effective for minimum phase systems. As there is limitation in the stable bandwidth of the NMP system, ADRC cannot be able to stabilize the plant by determining the observer poles and the controller poles. So, to overcome this, the linear quadratic regulator (LQR) method using an extended system of ADRC has been used for the disturbance removal and stabilization of the NMP system [30]. The fractional order PID controller with a simple optimization method has been applied on a multi variable non-minimum phase quadruple-tank system, but initial undershoot and overshoot have not completely been removed [2]. A fractional order cancellation of zeros has been introduced in the literature, though the cancellation of zero by unstable is not possible for NMP as it leads towards instability [127]. Internal instability is the main cause for inaccurate tracking of NMP systems. It is a difficult task for linear control techniques to counteract these undershoots characteristics of the NMP system. The genetic algorithm based fuzzy PID controller with its two-stage control structure is able to cancel the effect of unstable zero and help to provide good transient response of NMP system [131].

Another unavoidable phenomenon, like initial overshoot occurs due to the RHP zeros, which are even in number, and reference input tracking without overshooting is a challenging task in many applications.NMP systems under noise and disturbance conditions are controlled by magnitude-optimum multiple integration and numerical optimization based on PID control. Here, mathematical analysis of NMP systems with time delay has been done using numerical optimization techniques [132].An experimental set-up of a quadruple-tank system has been considered as an NMP system with an undershoot and an overshoot, where six single variable transfer function models are present, and a fuzzy PID controller with simple tuning method is applied here. It has been observed that, though the initial undershoot has not completely been removed by the above-said control technique, it is better than conventional PID control [2]. Different optimization methods, like, imperialist competitive algorithm, particle swarm optimization, genetic algorithm, artificial bee colony, and cuckoo search techniques have been employed to develop suitable autopilot for missiles with NMP model, which incorporates the speed, undershoot, overshoot, and steady-state error and compare these performance on unit step set point and, here innovative cost function has developed to deal with these problems

[133].Particle swarm optimization-based two-stage PI/PD controller has been explored on three NMP systems to reduce the peak overshoot, maximum overshoot, settling time and rise time in the unit step response of the NMP system in the MATLAB SIMULINK environment[134]. An unstable NMP system may be decomposed into two sub system, one is stable system model with NMP properties and other is unstable MP subsystem. The root locus design technique may be used here to shift the RHP zero to the left-hand side of the S plane to convert the NMP system model to an MP one. Now, the feed-forward control logic may be implemented on the resultant system. Control techniques like adaptive PID and model reference control are not capable of meeting the desired transient response and steady state error of the unstable NMP system, but, the described method can overcome this problem, and the above said control algorithm can be easily applied on it [135]. Initial undershoot of TITO NMP system has been reduced by simulation based predictive controller, where the controller incorporated the extended setting of the constraints, and it is actually the modified version of the predictive control for SISO system with NMP behavior. The transfer matrix from the mathematical model of the TITO NMP system has been used for the experimentation of initial undershoots control by a predictive control scheme [136].

Inspired by predictive control of the TITO NMP system, here one benchmarked TITO NMP system is considered to verify this proposed 2DOF based MRAC control strategy. The main objective of this work is to eliminate the undesirable initial undershoot, which is obvious as well as responsible for difficulties arising in the NMP system. Along with this initial undershoot, initial overshoot, and more undershoot overshoot are to be removed using the proposed MRAC scheme.

The main contributions of this research works are:

- 1. Recent literature on behavioral control problems of the NMP system has been extensively studied.
- 2. Analyzing the effect of odd and even number the RHP zeros of the NMP system.
- 3. Implementing the proposed control technique to remove the initial undershoot and overshoot to get good tracking performance of the NMP system.
- 4. Validation has been done on the different SISO and one TITO NMP systems.

### **10.2 Problem Statement**

On the basis of the position of the zeros in the S plane, the dynamic system is classified into two categories, like the MP and the NMP system. They may be of the same amplitude, but in anti-phase to each other. Odd and even number of RHP zero exhibits initial undershoot and overshoot in its step response, respectively, which lead to internal instability and consequently restrain the smooth trajectory tracking of reference input. To overcome these obstacles of tracking problems on NMP systems, a MRAC control approach has been considered here to nullify the initial undershoot as well as an initial overshoot phenomenon of systems.

# 10.3 MRAC Scheme



Fig. 10.1 Schematic diagram of MRAC structure[112]

MRAC is an advanced control scheme in which desired performance can be achieved by utilizing a minimum phase plant that acts as the referenced model in its control structure [79].

The error dynamics is produced by comparing the dynamic response of the plant model and the reference model plant and it is used to produce the required adaptive algorithm by a parameter adjusting mechanism. To get the desired trajectory given by the reference model, dynamics asymptotically converges to zero. Among the two loops of the MRAC structure, the inner loop consists of plant and controller, whereas the outer loop adjusts the parameters to minimize the error between the reference model and plant model to acquire the pre-specified trajectory of the reference model. The four components of this control scheme are required; those are reference model, plant, controller and parameter adjustment mechanism. Adjustment mechanisms may be developed by using the MIT rule, theory of augmented error, and Lyapunov theory [137].Tracking error, e will be produced due to the difference between the output signal of a plant and the reference model. Error dynamics will converge to zero by the technique of adaptation mechanism [92].

Let, the 2<sup>nd</sup> order reference model is,

$$y_m^{(2)} = -a_m y_m^{(1)} + b_m r$$
(10.1)

The 2<sup>nd</sup> order plant model is

$$y^{(2)} = -ay^{(1)} + bu \tag{10.2}$$

In equation (10.1) and (10.2), u is the control input and r is the reference input. Let,

$$u = \theta_1 r - \theta_2 u \tag{10.3}$$

Tracking error,

$$e = y - y_m \tag{10.4}$$

By subtracting (10.1) from (10.2), we get,

$$e^{(2)} = y^{(2)} - y_m^{(2)}$$
(10.5)

$$e^{(2)} = -ay^{(1)} + bu - (-a_m y_m^{(1)} + b_m r)$$
(10.6)

Replacing u in equation (10.6), we get,

$$e^{(2)} = -ay^{(1)} + b(\theta_1 r - \theta_2 y) - (-a_m y_m^{(1)} + b_m r)$$
(10.7)

By adding and deducting  $a_m y^{(1)}$  in equation (10.7), we get,

$$e^{(2)} = -a_m(y^{(1)} - y_m^{(1)}) - (b\theta_2 + a_1 - a_m)y^{(1)} + (b\theta_1 - b_m r)$$
(10.8)

Considering the control parameter's initial value  $\theta_1$  and  $\theta_2$ , and integrating with respect to t, we get,

$$e^{(1)} = -a_m e - (b\theta_2 + a_1 - a_m)y^{(1)} + (b\theta_1 - b_m r)$$
(10.9)

By observing the equation (10.9),

We find, that, tracking will tends to zero for,

$b\theta_2 = a_m - a$	(10.10)
$b\theta_1 = b_m$	(10.11)

In this way the parameter adjustment rule is achieved.

### **10.4** Numerical Examples

In this section we have demonstrated five numbers of SISO NMP systems and one TITO NMP system transfer function model, which have been taken to verify the MRAC structure for the elimination of initial undershoot and overshoot. Initial undershoot produced by the following SISO transfer function models [138].

### 10.4.1 SISO NMP with one RHP zero

$$G_1(S) = \frac{-S^2 + 7S + 8}{S^2 + 5S + 6} \tag{10.12}$$

$$G_2(S) = \frac{-6S+6}{S^2+5S+6} \tag{10.13}$$

### 10.4.2 SISO NMP with two RHP zeros

Initial overshoot and undershoot produced by the following SISO transfer

function model [138],

$$G_3(S) = \frac{24S^2 - 60S + 24}{S^3 + 11S^2 + 34S + 24}$$
(10.14)

### 10.4.3 SISO with three RHP zeros

Initial undershoot, overshoot and then undershoot produced by

the following SISO transfer function models [138],

$$G_5(S) = \frac{-15S^3 + 13.5S^2 - 138.6S + 67.8}{S^4 + 17S^3 + 101S^2 + 247S + 210}$$
(10.15)

$$G_4(S) = \frac{-2S^3 + 16S^2 - 50S + 52}{S^4 + 16S^3 + 86S^2 + 176S + 105}$$
(10.16)

# 10.4.4 Benchmarked TITO with one RHP zero

Initial undershoot produced by the following TITO transfer Matrix model [139]

$$G_{6}(S) = \begin{bmatrix} \frac{-0.495S+1}{0.08118S^{2}+0.659S+1} & \frac{-0.495S+1}{0.08118S^{2}+0.659S+1} \\ \frac{-1.98S+1}{0.3247S^{2}+2.144S+1} & \frac{-0.495S+1}{0.08118S^{2}+0.659S+1} \end{bmatrix}$$
(10.17)

### **10.5** Simulation Results



Fig. 10.2 Unit step response of uncontrolled SISO NMP systems with one RHP zero

In this experiment, five SISO NMP system models and one TITO system model have been considered for the elimination of their unavoidable dynamic characteristics using the proposed 2DOF based MRAC technique.

In the MATLAB SIMULINK environment, the NMP systems with different numbers of zeros have been considered for the verification of the proposed control scheme. The NMP system exhibits undershoot and overshoot in its step response and it depends on the number of RHP zeros. The odd number of RHP zeros exhibits an initial undershoot and an even number of RHP zeros produce an initial overshoot.

### **10.6 Results Analysis**

First, two NMP SISO systems have been considered, and it has been observed, as they contain one RHP zero, the unit step response from them produces one undershoot shown in Fig. 10.2, and one of the output responses exhibits the steady state error . It is observed in Fig. 10.3, undershoot and steady state error are completely removed by using the proposed MRAC based 2DOF control scheme.



Fig .10.3 Unit step response of 2DOF controlled SISO NMP systems with one RHP zero



Fig 10.4. Unit step response of uncontrolled SISO NMP systems with two RHP zeros

One SISO NMP system with two RHP zeros has been considered next, and it produced initial overshoot first and then undershoot in its step response shown in Fig. 10.4, but proposed controller eliminate these undesirable phenomena shown in Fig. 10.5.



Fig. 10.5 Unit step response of 2DOF controlled SISO NMP system with two RHP zeros



Fig. 10.6 Unit step response of uncontrolled SISO NMP systems with three RHP zeros

The unit step response of two SISO NMP systems with three numbers of RHP zeros exhibits initial undershoot, overshoot, then again undershoot in its step response shown in Fig. 10.6. After application of the proposed control technique on those NMP systems, all undershoots and overshoots are totally removed as shown in Fig. 10.7.



Fig. 10.7 Unit step response of 2DOF controlled SISO NMP systems with three RHP zeros



Fig. 10.8 Unit step response of uncontrolled TITO NMP system with one RHP zero

In the end, one benchmark TITO NMP system has been experimented in a software simulation environment. Both the outputs of this NMP system exhibit large initial undershoot and steady state error in its step response shown in Fig. 10.8. In Fig. 10.9, it has been noticed that the proposed control method reduces these undesirable characteristics without changing the system parameters, whereas, in the original literature, it was done by changing the value of the system's own parameter [139].

All the output responses of controlled NMP systems track the reference input trajectory perfectly. It has been verified that the magnitude of the undershoot inevitably tends to infinity as the settling time tends to zero [140]. Consequently, it has also been observed that settling time becomes larger with the elimination of the initial undershoot and overshoot for all the controlled output responses of an NMP system.



Fig .10.9. Unit step response of 2DOF controlled TITO NMP system with one RHP zero

### **10.7** Chapter Summery

Initial undershoot is an unavoidable and undesirable common phenomenon in the step response of a non-minimum phase (NMP) system and it creates difficulties in tracking control of the NMP system. Odd and even number of right half plane (RHP) zeros of the transfer function model of the NMP system produce initial undershoot and overshoot respectively, and it creates obstacles in the tracking control of the NMP system. To overcome these difficulties, Lyapunov Design technique based Model Reference Adaptive Control (MRAC) has been suggested here and the novelty of this technique is that it not only nullifies undershoots and overshoots, it also tracks the reference input trajectory simultaneously. Different SISO NMP systems have been considered for the verification of suggested control methodology and one benchmark two inputs two outputs (TITO) NMP system has been considered to strengthen its claim, on elimination of an unacceptable initial undershoot of the NMP system in the MATLAB SIMULINK environment.

# **CHAPTER 11**

# Two-Degree-of-freedom Control of Non-minimum Phase Mechanical System

### 11.1 Introduction

Design of a tracking control system for any dynamic system is essentially required for industry and research areas, but it becomes a very challenging task for a non-minimum phase (NMP) system due to its right hand plane zero (RHP) in the s plane. In the frequency domain, it has been found that though it generates the same magnitude as an MP system, it gives rise to large phase shifts, which are always greater than 90 degrees. This large phase shift is the main cause of the internal instability of the NMP system. Initial undershoot is an undesirable phenomenon that occurs in the step response of the NMP system, which is the inverse of the steady state output response and obviously obstructs the perfect tracking of the NMP system. So, it is very much needed to develop suitable tracking controllers for a practical real time dynamic system which has NMP characteristics. These undesirable phenomena are obvious in many industrial processes, like flexible link manipulators, aircraft, steam generators, electronic circuits etc. whose dynamic response exhibits NMP characteristics, and it is obviously a demanding task for the control engineer to tackle this type of system [141-146].

During the past decade, several control techniques have been developed for the NMP system. The combination of the cancellation method of NMP zeros and the classical PID control method has been studied for flexible arm robots. Here, at first, the fractional order or partial cancellation of NMP zeros has been done by the pre-compensator to minimize undershoots of the NMP system and form an undershoot less system. Then the conventional controller is developed to implement the cascaded combination of these with undershoot and without undershoot NMP system [147].

The quantitative feedback theory (QFT) design techniques can be applied to the linear SISO NMP system with unstructured uncertainty, even if its mathematical model is unknown. The new nominal plant required for this method may be reformulated to get better stability performance [148]. The control input of a discrete time SISO LTI NMP system can be expressed as a linear combination of the basis function using the filter basis function (FBF) method. Here, co-efficient of the basis function using the dynamics of the NMP system helps to minimize the tracking error of a given desired trajectory [149]. Any suitable set of basis

functions can be freely selected by the control designer. The commonly used uniform rational B-spline (NURBS) curve can be utilized to derive the needed basis function and the proposed approach has been compared with the zero phase error tracking control system [151]. Linearization procedure of the NMP system may produce RHP zero, leading to NMP characteristics of the system. The procedure of removing the RHP zero has been applied for the approximate tracking control of the NMP system and applied in flight control application [150].Sliding mode learning control with a recursive learning mechanism has been applied to a non-linear NMP system, where, sliding variable of the controlled system reach and remain on the sliding surface to make complete stabilization of internal dynamics [152].To control a SISO, input-affine nonlinear NMP system, a new input-output linearization technique with the redefinition of the output of the system has been considered. The LQR method utilized for the development of the redefined output, which guaranteed the stabilization of zero dynamics [153].

Simulation has been done on the series connected inverse and non-inverse transfer function model of the NMP system, and it has been observed, that instead of getting exact reference input trajectory tracking, it produces an unstable unbounded output response [110, 154].

It is well understood, that this phenomenon will not work for the minimum phase systems. This unusual nature of RHP zero dynamics inspired the researcher to design an appropriate control system for the NMP system and the thinking process led to the 2DOF control structure.

To achieve the precise control performance of dynamic systems, the 2DOF strategy is obviously a better design technique in control system engineering. The number of variables in the control problem of the system can be regulated or manipulated in a process that is named as 2DOF, or, in other words, the number of closed loop transfer functions which have to be controlled individually is called 2DOF. Some research on 2DOF PID control structures have been surveyed by the tutorial of Araki and Taguchi [155]. Regarding the 2DOF control concept, we have gone through the different literatures, and it has been observed that this control structure has become a reliable tool for researchers. The 2DOF control technique with feed-forward and feedback combination may be an effective control tool for the real time XY table application, whose mathematical model consists of both MP and NMP characteristics [93]. The algebraic method based on 2DOF control training, where the ball has to track the square wave

85

with a certain design specification [156]. Industrial processes with model uncertainty are effectively tuned with a 2DOF based model predictive control scheme [157]. The model predictive control (MPC) based 2DOF controller has been applied for the automatic tuning of the control parameters of the SISO industrial process, which consists of model uncertainties, and this 2DOF structure can control the oscillation, worst-case overshoot and it can also minimize the settling time in the process output [157].

Among all the adaptive controllers, MRAC is very interesting as it is a direct technique to force the uncertain system to obtain the desired performance [158]. We have gone through the different applications of techniques. Here is an advantage for control designers or researchers that they are able to select the shape of the transient part of the output trajectory, and it is only possible by RMP, which is essentially required for the MRAC structure [159]. The problem of crawl phenomena in a very low speed servo system, like permanent magnet synchronous motor [PMSM] has been solved by the MRAC technique, and, the simulation result shows that the MRAC scheme can also remove the environmental disturbances and improve the steady state performance of the controlled mathematical model [160]. Both MIT and Lyapunov theory based MRAC technique has been used to derive the iterative learning control of the chemical reactor system, which consists of uncertain parameters, input disturbances and resetting error of input-output of the system [161]. It has been studied, that the stability of the controlled system is completely ensured by the Lyapunov Design technique [141]. In order to improve the stability of articulated heavy vehicle (AHV), the MRAC algorithm has been implemented on active trailer steering (ATS) under the given condition, that the parameters and operating condition of the vehicle remain constant [162]. Matched and unmatched uncertainties of the non-linear system can be estimated by an observer model and removed by the hybrid direct-indirect MRAC technique [158]. The adaptive law of Lyapunov based MRAC has been formulated in terms of tracking and prediction error and applied on a fractional order non-linear system to generate smooth system output [163].

The feedback counterpart of the 2DOF control strategy may stabilize the unstable system dynamics and be able to remove the disturbance occurring in the system. SFB controllers can play the role of feedback compensation for the 2DOF-based control structure. The proportional plus integral (PI), proportional plus integral plus derivative (PID), phase lead or phase lag controller may not be able to control all the poles of the higher order system independently.

To circumvent these difficulties, state feedback control with an arbitrary pole placement approach can solve this problem independently under the certain condition, that the system must be completely state controllable [164]. Transient dynamics of the piezo-actuated bimorph atomic force microscopy (AFM) probe is controlled by a state feedback controller, where the quality factor and the resonance frequency of the probe have been adjusted simultaneously [165]. An optimal state feedback design has been considered for the classical unstable plant like the Inverted Pendulum by calculating the appropriate state feedback gains using a flower pollination algorithm (FPA) [166].

# **11.2 Problem Formulation**

Exact reference input tracking is possible by series connected inverse and non-inverse transfer function model of the system, but it is true for the minimum phase system. For the NMP system, it exhibits unbounded output response, so, it is very difficult to design the proper inversion based tracking controller for the perfect trajectory tracking of the NMP system.

## 11.3 Mechanical System Model with NMP Characteristics

A mechanical experimental setup developed by C. T. Freeman, P. L. Lewin and E. Rogers [167], where the NMP part has been placed in the upper left portion of the test bed and two more mass-spring-damper components have been linked with the NMP system.



# Fig. 11.1 Schematic representation of mechanical realization of NMP System [167]

Electricals analogous to the mechanical system model, shown in Fig. 11.1 show NMP dynamic characteristics. This NMP system includes inertia, a damper, a torsional spring, a timing belt, pulleys, and gears. In the schematic diagram [167], inertia is represented by J,  $J_g$ , G represents gear and B, K represents damping friction co-efficient and spring constant

respectively. The spring mass damper systems have been placed before the NMP component to increase the relative degree of the total system. The hardware experimental setup has been constructed by the following transfer function model.

$$G(s) = \frac{123.853 * 10^4 (3.5 - S)}{(S^2 + 6.5S + 42.25)(S + 45(S + 190))}$$
(11.1)

### **11.4 Description of Different Controllers**

Along with the MRAC compensation employed in the 2DOF structure, SFB and PID Controllers also have been considered here. The PID controller is a very popular conventional classical control technique, and most of the process industry has adopted this control technique as the most suitable one though it is applied to SISO systems. But, to solve the control problem of MIMO systems, the SFB control technique is essentially required, state space representation of the mathematical model is needed here. The brief description of the SFB and the PID control scheme has been provided by the following two sub sections.

### 11.4.1 SFB Compensation



Fig. 11.2 Schematic Diagram of SFB Controller [90]

In pole placement technique, all the closed loop poles are placed at desired locations. It may be assumed that the entire state variable is measurable and is available for feedback. A necessary and sufficient condition for this arbitrary pole placement technique is that the system must be state controllable. If the condition is satisfied, the control signal will be. It means that the control signal is determined by the immediate state of the system. Such a scheme is called the state feedback control scheme and the matrix is called the state feedback gain matrix. The eigen values of the closed loop system may be placed at any desired location through this state

feedback gain matrix.

# 11.4.2 PID Compensation

A PID controller is a conventional controller which includes three different control approaches; proportional action P, an Integral action I and derivative action D in Fig. 11.3. All the three control modes require three independent operational amplifiers to be adjusted independently and it requires electronic circuits. The mathematical model of PID control is given by the following:

$$u(t) = k_{P}e(t) + k_{i} \int_{0}^{t} e(\tau)d\tau + k_{D} \frac{de}{dt}$$
(11.2)

u(t) is an input, e(t) is an error signal, and  $k_p$ ,  $k_i$ ,  $k_d$  are the proportional, integral and derivative adjustable constant respectively.

The transfer function of PID Controller is



Fig. 11.3 Schematic diagram of PID control scheme

# 11.4.3 MRAC Compensation employed in 2DOF framework

The MRAC Control Algorithm employed in the 2DOF framework has been demonstrated in chapter 7 by the equations no (7.1) to (7.21).

# **11.5** Simulation Results

An NMP system with a mass spring damper has been considered here, which is actually part of a mechanical experimental set-up. A unit step signal is applied to the NMP system, which is not controlled.

The step input signal with unit amplitude has been applied on the uncontrolled NMP system. The NMP system, and its inverse model, have been connected in a cascade first for the pole zero cancellation.

The SFB, PID, and 2DOF controlled mechanically realized NMP system has been simulated with square wave signal in the MATLAB SIMULINK environment.



Fig. 11.4 Unit step response of uncontrolled mechanical realized NMP system (closure view)



Fig. 11.5 Unit step response of uncontrolled inverse mechanical realized NMP system



Fig. 11.6 Unit square wave response of cascaded inverse and non-inverse mechanical realized NMP system



Fig. 11.7 Unit square wave response of SFB controlled mechanical realized NMP system



Fig. 11.8 Unit square wave response of PID controlled mechanical realized NMP system



Fig. 11.9 Unit square wave response of 2DOF controlled mechanical realized NMP system



Fig. 11.10 Superimposed unit square wave response of SFB, PID, and 2DOF controlled mechanical realized NMP system



Fig. 11.11 Superimposed unit step response of SFB, PID, and 2DOF controlled mechanical realized NMP system

# **11.6** Results Analysis

Simulation of the entire controlled, uncontrolled system model has been performed in the MATLAB SIMULINK environment. Fig. 11.4 shows the output response of an uncontrolled mechanical released NMP system with a close view, where a deep undershoot is found. The inverse model of that NMP system is obviously an oscillatory, unbounded system and Fig. 11.5 shows its unbounded output response. Unlike the minimum phase system, an NMP system connected in cascaded with its inverse model produces an unbounded output response shown in Fig. 11.6. The Fig. 11.7, Fig. 11.8 and Fig. 11.9 represent the square wave response of SFB controlled, PID controlled and 2DOF controlled NMP systems respectively. It is observed that a 2DOF controlled NMP system produces far more acceptable results than PID controlled and SFB controlled NMP systems when a square wave signal has been applied as a reference input signal shown in superimposed response in Fig. 11.10. Unit step response of state feedback, PID and 2DOF controlled NMP systems have been superimposed as shown on Fig. 11.11. It has been found that the output trajectory of a 2DOF controlled NMP plant is very near to the reference input trajectory and also, the initial undershoot becomes nullified by this method. Percentage RMS errors from the square wave responses of the 2DOF controlled NMP system with the existing compensation technique have been listed below.

# Table 11.1: Comparison of percentage error in RMS value

Type of Controller	State Feedback	PID	2DOF	
% Error in RMS 46%		7.8%	0.79%	

<b>Table 11.2:</b>	: Comparison	chart of	controlled	system's	s dynamic	characteristics
				~		

Name of the Controller	Rise time	Steady state error	Initial undershoot
SFB	Infinite	0.4	1.27
PID	6.1	1.2	0.47
2DOF	2.8	Nil	0.03

The proposed 2DOF control technique for this 4<sup>th</sup> order mechanical NMP system has been compared to the disturbance rejection controller with and without disturbance observer (DOB), as referred to the literature [6].

 Table 11.3 Comparison of dynamic characteristics between proposed controlled with disturbance rejection controlled system.

Name of the Controller	Initial Undershoot	Number of Undershoot	Number of Overshoot	Steady State Error
Disturbance Rejection without DOB	0.35	One	Nil	1.3
Disturbance Rejection with DOB	0.4	Three	Two	Nil
Proposed 2DOF	0.03	One	Nil	Nil

By observing the above comparison table, it is clear, that the proposed 2DOF control technique exhibits far better performance than the disturbance rejection controller.

### **11.7** Chapter Summery

This work presents a methodology for designing a MRAC scheme in the 2DOF scheme for a practical mechanical system, which has unstable zero characteristics. The compensators like feedback and feed-forward are the major part of the proposed control, which are decoupled in nature. As the inversed NMP systems are not able to exhibit stable output, the inverse model NMP system is controlled by the MRAC scheme as a feed-forward controller and the non-inverse NMP system has been compensated by SFB in the proposed control method. The proposed 2DOF methodology has established the capability of the 2DOF control methodology. Initial undershoot; the most unwanted output of the plant was totally removed. PID control and SFB control techniques have been compared and the result shows that the proposed 2DOF controller can produce better performance than PID and SFB control schemes. Finally, the output response of the proposed control structure has been compared with the output response of the disturbance rejection controller with and without DOB for the same mechanical realized NMP system and achieved satisfaction regarding the proposed control scheme.

# **CHAPTER 12**

# Model Reference Adaptive Feed-Forward Control for Non-Minimum Phase System in Two-Degree-of-Freedom Framework

### 12.1 Introduction

In the field of control system engineering, solving the tracking problem of the control system is essentially a demanding task, but, when the system exhibits NMP characteristics, it becomes harder to control. A system having at least one RHP zero on the S plane is named as a NMP system. Its large phase contribution in its frequency domain analysis indicates the internal instability in its system dynamics. Due to this, RHP zero, an unavoidable 'initial undershoot' occurs in the output response, which is actually the inverse of the desired steady state response, when the set point is step signal, and it obviously obstructs good tracking of the NMP system. Exact reference input trajectory tracking can be easily achieved if inverse and non-inverse transfer function models are connected in series as the numerator and denominator polynomial, which consists of poles and zeros, cancel each other. But, this method can only be applied to MP systems. In the case of an NMP system, this technique produces a peculiar or unusual output response, like an unbounded or oscillatory output trajectory in place of an exact set point trajectory.

So, it has been observed, that designing an appropriate tracking controller for the NMP system is a really difficult and challenging task, and this unavoidable effect of RHP zeroes inspired many researchers to design an effective tracking control system for the NMP system. We have gone through much of the research work on tracking control systems.

The DOF is actually the number of closed loop transfer function models of a system, which are to be compensated independently [155]. It has been observed by different literature surveys on 2DOF control strategy that the concept of a 2DOF technique has become a more and more reliable control tool for experimental work. The ball can accurately track the square wave signal with some given specifications in the ball and beam system by using the 2DOF control structure [156]. The SISO mathematical model of industrial process with model uncertainty can be automatically tuned by model predictive control (MPC) in 2DOF structure [157]. The MP and NMP mathematical models are derived from a true plant of the XY table using input-output experimental data with different sampling times via MATLAB System Identification toolbox, and these system models were successfully experimented by adaptive

### Chapter 12 Model Reference Adaptive Feed-Forward Control for Non-Minimum Phase System in Two-Degree-of-Freedom Framework

feed-forward zero phase error tracking control (ZPETC) in 2DOF structure [93]. The adaptive control techniques on MP and NMP, both system models, inspired us to think about the design of a combined feed-forward and feedback control in a 2DOF framework to solve the tracking problem of NMP systems.

The servo system, like permanent magnet synchronous motor (PMSM), when it is operated at a very low speed, crawls phenomena occur and crawls are presented by hyperstability theory. The MRAC technique may not only solve these crawling phenomena, it also removes environmental disturbance, swells the clarity of the system and improves the steady state performance [160]. A chemical reactor with uncertain parameters, input disturbance, and initial output resetting error is considered as the plant model for implementation of iterative learning control using the MRAC method, which is based on both MIT rule and the Lyapunov stability theory, but, it has been observed that the closed loop stability of the system is only guaranteed by the Lyapunov theory based control technique [161].To improve the lateral stability of articulated heavy vehicles (AHVs), MRAC method has been applied on its active trailer steering under the assumption that vehicle model parameter and operating condition are constant [162]. The MRAC algorithm has been applied on a fractional order non-linear system to get smooth system output and here, the indirect Lyapunov method analyzed the closed loop stability and tracking convergence of the system [163].

Control of a higher order system is difficult by phase lead or phase lag controller, proportional plus integral plus derivative (PID), proportional plus integral (PI), as all the poles of the system cannot be placed in the right position independently from said controller above. To overcome this control problem of the higher order system, arbitrary pole placement, based on SFB compensation, under the condition, that the system must be completely state controllable [164]. The quality factor and resonance frequency of a piezo-actuated bimorphatomic force microscopy (AFM) probe have been adjusted simultaneously by using a state feedback controller, where the dynamic response of the probe is first observed, then the time constant of the probe becomes less by minimizing its quality factor or increasing its resonance frequency [165]. The flower pollination algorithm (FPA) has been used for deriving the state feedback gain and successfully applied on a benchmarked unstable system like an inverted pendulum [FPA].

# **12.2 Problem Formulation**

Perfect set point tracking is required in most of the industrial application in the control system engineering. It may be possible in a very easy way of cancelling the pole and zeros of the transfer function model. If the inverse and non-inverse transfer function models are connected in series, their numerator and denominator polynomial cancel each other and obviously produce reference input as output response. It has been observed in the previous work, that in case of NMP system this concept does not work. As this method exhibits an unbounded output response for NMP system, a 2DOF control strategy has been contemplated here, which consists of direct MRAC based feed-forward and SFB compensation based feedback control scheme.

# 12.3 2DOF Control Structure

The structure of the 2DOF control scheme has been provided by Fig. 1.4 of chapter 1 for the tracking control system of the NMP system. The control scheme consists of feed-forward and feedback controllers, which are not coupled to each other. As the feed-forward controller needs an inverse model of the system, here an inverse transfer function model of the NMP system has been taken as plant for the feed-forward scheme and the MRAC algorithm have been implemented in this feed-forward compensation to deal with the tracking problem of NMP system. The stabilization of the unstable dynamics of the NMP system has been done by the arbitrary pole placement based SFB control scheme.

# 12.4 Different Controllers

To prove the efficiency of the proposed 2DOF control strategy, two other control schemes have been considered here. One is the direct MRAC controller and another is arbitrary pole placement based SFB controller. The elaborate description has been provided by the chapter 4 and chapter 5 respectively.

# 12.4.1 MRAC Scheme

The schematic diagram of MRAC structure has been provided by the figure 5. MRAC structure is consists of four blocks and two loops. The one block contains plant, which is to be controlled by its control input and controlled output is placed in the middle of the structure. The reference model is the most important block of the MRAC structure, which is essentially required to shape the desired trajectory. Adjustment mechanism block produced required control parameter with the help of plant output, reference input tracking error, which is based on Lyapunov Design Technique. Control input, which is the output of the controller block,

contains control parameters produced from the adjustment mechanism, reference input, and plant output. By adjusting the adaptive gain, which are associated with the control input, desired trajectory can be achieved by the MRAC structure.

# 12.4.2 SFB Controller

In this modern year, SFB control application for controlling the linear and nonlinear system is very much effective control technique. It is a linear combination of the state variable of the state model.

The state feedback control law is a linear combination of the state variable of state model of any system.

Refer to Fig. 5.1, we get,

Uncontrolled state model has the following representation,

State equation is,

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$
(12.1)

Output equation is,

$$y(t) = Cx(t) + Du(t)$$
 (12.2)

Considering the control input,

$$u(t) = -Kx(t) \tag{12.3}$$

Where,

K is state feedback gain matrix derived by the Ackerman's formula.

Putting the value of the control input into equation (12.1),

the closed loop state equation becomes,

$$x(t) = (A - BK)x(t) + Cx(t)$$
 (12.4)

# 12.4.3 Proposed MRAC Control Scheme employed in 2DOF framework

The direct MRAC feed-forward based 2DOF controller represented by the Fig. 1.4, where the pole placement method based SFB controller has been placed as its feedback counterpart. The

generalized control algorithms of SFB and MRAC have been provided by the chapter 5 and chapter 7.

### 12.5 Numerical Example

In order to examine the proposed control methodology, two numerical examples have been chosen here.

Example 1. 
$$G(S) = \frac{(S - 0.526)(S^2 + 1.526S + 3.803)}{(S + 1.185)(S^2 - 0.185S + 4.219)}$$
 (12.5)

Example 2. 
$$G(S) = \frac{123.853 \times 10^4 (3.5 - S)}{(S^2 + 6.5S + 42.25)(S + 45)(S + 190)}$$
 (12.6)

The 4<sup>th</sup> order transfer function model is one part of a real mechanical system, which consists of mass, spring damper element and which produce NMP characteristics as it has RHP zero placed at 3.5[167].

### 12.6 Simulation Result

All the simulation experiments on 3<sup>rd</sup> order and the 4<sup>th</sup> order NMP system have been performed in MATLAB SIMULINK environment. Unit step, ramp and sinusoidal input signals have been used to verify the proposed 2DOF control structure. To compare the effectiveness of the proposed control method, SFB and MRAC scheme also have been implemented on the same 3<sup>rd</sup> and 4<sup>th</sup> order NMP system.



Fig. 12.1 Unit step response of uncontrolled 3<sup>rd</sup> order NMP system



Fig. 12.2 Unit step response of cascaded inverse and non-inverse 3rd order NMP system



Fig. 12.3 Unit step response of SFB controlled 3<sup>rd</sup> order NMP system



Fig. 12.4 Unit step response of MRAC controlled 3<sup>rd</sup> order NMP system



Fig. 12.5. Unit step response of 2DOF controlled 3<sup>rd</sup> order NMP system



Fig. 12.6 Unit step response of control input for 2DOF controlled 3<sup>rd</sup> order NMP system



Fig. 12.7. Sine wave response of 2DOF controlled 3<sup>rd</sup> order NMP system



Fig. 12.8 Unit ramp response of 2DOF controlled 3<sup>rd</sup> order NMP system



Fig. 12.9 Superimposed unit step response of SFB, MRAC, and 2DOF controlled 3<sup>rd</sup> order NMP system



Fig. 12.10 Unit step response of uncontrolled 4<sup>th</sup> order NMP system

Chapter 12 Model Reference Adaptive Feed-Forward Control for Non-Minimum Phase System in Two-Degree-of-Freedom Framework



Fig.12.11 Unit Step response of cascaded inverse and non-inverse 4<sup>th</sup> order NMP system



Fig. 12.12 Unit step response of SFB controlled 4<sup>th</sup> order NMP system



Fig. 12.13 Unit step response of SFB controlled 4<sup>th</sup> order NMP system (closure view)



Fig. 12.14 Unit step response of MRAC controlled 4<sup>th</sup> order NMP system



Fig. 12.15 Unit step response 2DOF controlled 4<sup>th</sup> order NMP system



Fig. 12.16 Unit step response of control input for 2DOF controlled 4<sup>th</sup> order NMP system
Chapter 12 Model Reference Adaptive Feed-Forward Control for Non-Minimum Phase System in Two-Degree-of-Freedom Framework



Fig. 12.17 Unit Sine wave response f 2DOF controlled 4th order NMP system



Fig. 12.18 Unit ramp response of 2DOF controlled 4th order NMP System



Fig. 12.19 Superimposed unit step response of SFB, MRAC, and 2DOF controlled 4<sup>th</sup> order NMP system

#### 12.7 Results Analysis

Unit step response of the uncontrolled 3<sup>rd</sup> order system produced unbounded oscillatory response shown in Fig. 12.1. Large initial undershoot and steady state error is found in the unit step response of 4<sup>th</sup> order mechanical realized NMP system. At first SFB controller has been applied on both the system, the 3<sup>rd</sup> order NMP system becomes stable, but large steady state error and initial undershoot exhibits in the output response shown in Fig. 12.3. Steady state error of SFB controlled 4<sup>th</sup> order NMP system become reduced, but still there exists large initial undershoot shown in Fig. 12.12 and Fig. 12.13 (closer view). After this experiment, MRAC control scheme applied on both the NMP system, MRAC only could not able to stabilize the 3<sup>rd</sup> order NMP system shown in Fig. 12.4 and output trajectory of MRAC controlled 4<sup>th</sup> order NMP system becomes very sluggish in nature shown in Fig. 12.14.

Though the numerator and denominator polynomial of cascaded inverse and noninverse transfer function model cancelled each other and able to produce exact reference input trajectories, but the NMP system could not produce an exact set point trajectory like MP system, it exhibits unbounded and oscillatory output response. It has been verified by the unit step response of cascaded inverse and non-inverse connected 3<sup>rd</sup> order and the 4<sup>th</sup> order NMP system shown in the Fig. 12.2 and Fig. 12.11 respectively.

So, to overcome this tracking control problem, the proposed 2DOF control technique have been applied on the 3<sup>rd</sup> order and the 4<sup>th</sup> order NMP systems and obtained the satisfactory step response shown in Fig. 12.5 and Fig. 12.15. Unit step response of the control input of the 2DOF controller for both the NMP systems is provided by Fig. 12.6 and Fig. 12.16.

The tracking performance of the proposed control technique has been verified by the sinusoidal and ramp input signals, which have been applied to the 3<sup>rd</sup> order NMP system, and obtained the output responses shown in Fig. 12.7 and Fig. 12.8 respectively.

The sinusoidal and ramp response of the 4<sup>th</sup> order system has been observed in Fig. 12.17 and Fig. 12.18 and it must say that both the systems acquired good tracking performance by using the proposed control scheme.

By observing the superimposed unit step response of SFB, MRAC and 2DOF controlled for both the 3<sup>rd</sup> order and 4<sup>th</sup> order systems, shown in Fig. 12.9 and Fig. 12.19, it has been clear that the proposed control technique exhibits far better result compared to the SFB and MRAC methods like modern and advanced control technique. The tracking performance

of the proposed control technique has also been validated by sinusoidal and ramp input signals. A comparison chart for Controlled 3rd and 4<sup>th</sup> order system's dynamics has been shown below:

Name of the Controller	Order of the System	Initial Undershoot	Steady state error	Rise time (seconds)
SFB	Third	2	1.3	Infinity
	Fourth	1.27	0.4	Infinity
MRAC	Third	Nil	Infinity	Infinity
	Fourth	Nil	0.1	152.5
2DOF	Third	Nil	Nil	3
	Fourth	0.03	Nil	2.8

 Table. 12.1 Comparison chart of 3<sup>rd</sup> order and 4<sup>th</sup> order system

# 12.6 Chapter Summery

This work demonstrates a scheme for developing MRAC as a feed-forward control part in the 2DOF structure to obtain the trajectory tracking control problem of the NMP plant model. The 3<sup>rd</sup> and 4<sup>th</sup> order NMP systems have been considered here, and the 2DOF control methodology obtained satisfactory results.

# **CHAPTER 13**

# Two-Degree-of-Freedom based Model Reference Adaptive Control of Non-minimum Phase System

#### 13.1 Introduction

In the research and industrial domain of control system engineering, the solution of tracking problems of dynamic systems is one of the crucial control tasks. As perfect set point tracking is essentially required in the real, practical industrial area, researchers are very motivated to design the appropriate tracking control system for the system model. There is different literature which has been found regarding the different tracking control techniques. Among them, the cascaded control technique proves its ability to solve the tracking problem. The trajectory tracking and stabilization problem of the ball and plate system has been successfully resolved by the integer order proportional derivative (IOPD) and fractional order proportional derivative (FOPD), and they are tuned to track circular, square, cardioids and star shaped trajectories by three different optimization algorithms, like, Bat Algorithm, Gravitational Search Algorithm, and Firefly Algorithm [168]. The SISO fuzzy feed-forward and fuzzy PID as a feedback controller in combined control mode with electromagnetic field optimization algorithm has been applied on PEM Fuel Cell Air Feed System, and this control scheme tracks the command input with less overshoot, undershoot and settling time compared to other control schemes [169]. A cascaded control scheme, consisting of input-output feedback linearization and gradient descent control techniques has been suggested for a non-linear NMP system, where compensation of non-linearity has been performed by an inner loop input-output feedback linearization method and a gradient descent technique placed in the outer loop to solve the internal stability problem of the closed loop system [170].But the design of the tracking control system for the NMP system is a challenging task as its right-hand plane (RHP) zeros in its system dynamics provoke a poor tracking performance [171,172]. The inversion based tracking control system is very effective for good tracking performance, but this method cannot be directly applied to the NMP system as it leads to instability in the system dynamics [173]. The output tracking problem of the causal NMP MIMO system has been converted to a state tracking problem and then the sliding mode control method is applied to approximate output tracking of that system model [171]. The set point tracking performance of the time delay NMP system has become improved by generalized predictor-based control, where output response is obtained without time delay [172]. The asymptotic output tracking of an arbitrary reference input signal has been achieved by the reduced order sliding mode controller, when it is applied to the NMP system [12].

The adaptive feed-forward control method is the most efficacious for the tracking control system [86], [35]. MIT and Lyapunov rule-based adaptive controllers have been experimenting with a disturbed thermodynamics system and obtaining an accurate and robust performance [174]. The fixed structure feed-forward controller with inverse dynamics of the system model has been designed with the linear combination of command input trajectory and its time derivatives with an appropriate weighting factor, and applied to NMP plant [70]. Inverse dynamic feed-forward torque cancelled the pole and zero of the NMP system models and developed the asymptotic tracking compensation for the flexible manipulator [144].

The inversion of the system transfer function model is essentially required, and it is obvious, that the inversion-based feed-forward control technique leads to instability in the system dynamics of the Non-minimum element and system [173], [22].To overcome these difficulties regarding trajectory tracking found in the dynamic response of the NMP system, two control methods, feed-forward and feedback controller, which are decoupled with each other in a 2DOF control structure, has been suggested here. Various 2DOF control structures have been considered for the system model with transport delay and load disturbance [47].

Different kinds of literature have been studied to verify the 2DOF control strategy. To achieve the desired output of a plant, without its mathematical model, optimal feed-forward control parameters can be derived from a 2DOF control structure [176]. An LTI uncertain system with fuzzy co-efficient is known as a fuzzy parametric uncertain system and has been considered for experimentation in 2DOF control structure to obtain the robust stability of the closed loop system [177]. In the robot learning and control domain, robot motion is represented in dynamical movement primitives (DMP) framework and it has been controlled by 2DOF control technique to get reference trajectory tracking and perturbation recovery [48].

For designing the perfect tracking control system, the MRAC method may be a good choice as feed-forward compensation in the 2DOF framework. There is different literature describing the potency of MRAC techniques. The MRAC technique may be a good choice as feed-forward compensation in the 2DOF structure. The effectiveness of MRAC has been studied in different literature, like; good set-point tracking achieved in the cylindrical tank system, magnetic levitation system, and disturbed thermodynamic system [178-179 and 174].

In the liquid level control in the process industry, the cylindrical tank system has been taken as a plant, and here the MRAC technique produces better transient performance than classical control [178]. As MRAC introduces non-linearity in system dynamics, the Lyapunov stability theory will be appropriate for designing the control parameters in the model reference adaptive scheme.

One unique property of the MRAC approach is that, by using the inverse plant model, its control parameters are directly adapted or modified to drive the plant output to track the reference model output [181]. The selection of reference models is an important factor in designing the control system, and it gives a lot of option to the designer as they can modify the plant output as per their choice [182, 183]. As MRAC introduces non-linearity in system dynamics, the Lyapunov stability theory will be appropriate for designing the control parameters in the model reference adaptive scheme.

State feedback control has been considered as the feedback counterpart of the 2DOF control structure as feedback control is inevitable for the standard MRAC approach as a strictly positive real (SPR) reference model, and the NMP plants become mismatched with each other and produce an unbounded output response [184].

In an engineering application, it is desirable to examine the dynamic behavior of a complex system utilizing analogue simulation [185]. Analog simulation may accomplish reduced frequency, and the basic purpose of the analogue simulation is to obtain accurate performance data from the operating model that provides the real characteristics [186]. After going through the benchmark Op-amp realization of different NMP systems [187], it is assured that the Op-amp based design of a closed-loop controlled NMP system is also possible.

The purpose of this work is to propose a 2DOF based MRAC scheme that serves as an adaptive feed-forward, inverse control technique that can find the solution to the tracking problem of the NMP system, and be connected in series with a state feedback-controlled non-inverse model of the NMP system to get stable steady-state performance. The feed-forward and feedback control schemes are decoupled with each other in the 2DOF framework. To verify the proposed control structure of the NMP system, one Op-amp based NMP model has been developed in a digital and analog environment. The proposed 2DOF control scheme has been implemented in the MATLAB SIMULINK environment and then on a real Op-amp based hardware model in the analog environment.

#### **13.2** Problem Formulation

In control system engineering the design of tracking control is an essential part of the research and industrial environment. Perfect set point trajectory tracking can be achieved if the transfer function model of any dynamic system is connected in series with its inverse transfer function model. But this technique is restricted to a MP system. In the case of the NMP system, inversion of its transfer function model leads to instability as the right-hand plane (RHP) zero converted to RHP poles.

Adaptive feed-forward compensation is one of the feasible techniques to solve the tracking problem of a dynamic system. But, as it also requires an inverse model of a dynamic system, it is a challenging task to design a proper adaptive inverse feed-forward control technique for the NMP system. In order to circumvent this difficulty in the design of a suitable tracking controller for an NMP system, a 2DOF control structure has been proposed, where MRAC controller as a feed-forward controller and state feedback (SFB) compensation as a feedback controller and both the controllers are decoupled with each other.

#### 13.3 Modeling of Op-amp based NMP System

The transfer function of any system model can be realized using electronic components like an Op-Amp, resistor and capacitor [186]. Here, at first the 2<sup>nd</sup> order NMP system with one RHP zero has been developed using MATLAB System Identification toolbox, then the realistic Opamp based NMP system model with all electronic virtual components needed for the NMP system model, has been constructed in the MATLAB SIMULINK environment. After getting a satisfactory system response with 1 volt input supply as a unit step signal using MATLAB SIMULINK toolbox, the 2<sup>nd</sup> order NMP system model has been constructed with the real Op-Amp (LM741), resistor and capacitor on the electronics board using laboratory hardware experimental set-up.

## 13.3.1 Mathematical description of the Plant Model

The differential equation is the most indistinguishable tool for the development of the mathematical model of many control systems [188].

First, one single input, single output 2<sup>nd</sup> order differential equation is formulated in such a manner that it consists of two poles and two zeros. Out of them, at least one zero is placed at the RHP to create the NMP characteristics in the system.

Considering the differential equation, which will be used to represent a 2<sup>nd</sup> order non-

minimum phase system is  $\frac{d^2y}{dt^2} + a\frac{dy}{dt} = \frac{d^2u}{dt^2} + \frac{du}{dt} - bu$  where *u* is input to the system and *y* 

is the output of the system. Here, poles are at 0 and –a position and zeros are at  $\frac{-1\pm\sqrt{1+4b}}{2}$ 

. With the help of the state model and transfer function model, the NMP system has been constructed, which will be taken as a plant to examine the proposed tracking control methodology.

#### 13.3.2 Block Diagram of NMP System Model



Fig. 13.1 Block diagram representation of NMP system

The transfer function of the NMP system has been developed from the block diagram shown in Fig. 13.1. Input and output voltages have been represented by  $V_{in}(S)$  and  $V_{out}(S)$  respectively. *K* is the gain of the NMP system, which will be used to find out the zero placed at RHP of the complex plane.

From the transfer function of this 2<sup>nd</sup> order NMP system, it will be clear that it has two poles and two zeros.

Poles are placed on the left-hand plane (LHP) and their values are, 0 and -P.

Out of two zeros, one is placed at the LHP of s plane with the value -z, and the other is placed at RHP of s plane with the value  $k_1 \left( = \frac{1}{k} \right)$ . This zero is responsible for developing the NMP characteristics in the system.

#### 13.3.3 Hardware Op-amp based NMP System

In this section, the circuit diagram of the NMP system for hardware simulation has been shown.



Fig. 13.2 Circuit Diagram of Op-amp based NMP system

In the above circuit diagram, four operational amplifiers have been used to construct the NMP system. The first Op-amp is used for pole-zero construction and the second one acts as an inverting amplifier. The third and fourth Op-amp is being used for integrator and adder respectively. Resistances and capacitances are the main components of the NMP circuit. Here, the LM741 operational amplifier has been used for the construction of the NMP system.

#### 13.4 Proposed Control System

The MRAC based 2DOF control structure has been suggested here, where MRAC. In the 2DOF framework, feed-forward compensation is needed for the solution of tracking problem of NMP system and it required inverse model of NMP system. The Lyapunov Design technique based on MRAC has been placed as the feed-forward control part of the 2DOF structure. To get stable and steady state performance, feedback compensation is very useful and, so SFB controller using arbitrary pole placement technique has been taken as the feedback counterpart of the 2DOF control scheme. A detailed control algorithm has been provided in chapter 5 and chapter 7.

#### 13.5 Selection of reference model using SISO tool

The selection of a plant for the reference model of the MRAC structure is an important task, as the output response of the closed-loop plant essentially follows the dynamic response of the reference model. In MRAC, the choice of the reference model is made by the control engineers, based on their experience [190]. Construction of an Op-amp based first-order system in a real-time environment is easier than any higher-order system. So, here, one first-order system has been selected for the reference model in the MRAC structure. Initially, the chosen reference

model using the SISO tool of the MATLAB control system toolbox was developed using the trial and error method and then implemented in the MRAC scheme.

#### 13.6 Experimental Result & Analysis

In this section, the effectiveness of the proposed control law has been verified by software simulation using MATLAB toolbox as well as an analogue simulation in a real hardware environment. At first, software simulation was done on a realistic Op-amp based plant considering all virtual electronic components required for the NMP system and its control components in the 2DOF structure. Afterward, the performance of the proposed control law has been validated in a real-time environment using analogue simulation to verify its effectiveness during practical implementation.

Inspired by software simulated Op-amp realization of the NMP system [187], a new 2<sup>nd</sup> order Op-amp-based non-minimum phase, as well as its inverse model, has been developed using MATLAB toolbox and on electronics hardware board respectively.

Initially, the proposed control structure was verified with the Op-amp based realistic control structure using MATLAB SIMULINK toolbox. Later, it has been implemented on electronic boards, and analog simulation has been performed by a square wave signal generated from the function generator. All the output responses have been observed by Cathode Ray Oscilloscope (CRO).

In software simulation, the unit step response of NMP, inverse NMP, and the superimposed inverse, non-inverse NMP model have been shown in Fig. 13.3, Fig. 13.4, and Fig. 13.5 respectively. It has been checked that, unlike the minimum phase system, the cascaded inverse and non-inverse transfer function model of that NMP system produces unstable output response shown in Fig. 13.6. though the feedback control is necessary for the standard MRAC technique, SFB controlled NMP system contributes large initial undershoot and steady-state error shown in Fig. 13.7. The reference model is an essential part of robust MRAC, and here we have selected one first-order system as the reference plant model for MRAC and checked its response in Fig. 13.8. Finally, a robust 2DOF based MRAC controlled Op-amp based NMP system's output has been obtained and shown in Fig. 13.9. All the above responses have been checked using one volt supply as a step input signal in the realistic Op-amp environment of the MATLAB SIMULINK toolbox.



Fig. 13.3 Unit step response uncontrolled NMP system

We have implemented the experimental setup employing the analog simulation technique. The analog simulation-based plant model helped us to mimic a real plant on the laboratory scale. Analog simulation helps us to achieve fast response times, like a real plant, compared to larger response times in digital simulation. However, while validating the control law designs, we have simulated the entire closed-loop system on the digital platform (MATLAB SIMULINK) for faster prototyping.



Fig. 13.4 Unit step response uncontrolled inverse NMP system



Fig. 13.5 Superimposed unit step response of inverse and Non-inverse NMP system



Fig. 13.6 Unit step response cascaded inverse and non-inverse uncontrolled NMP system



Fig. 13.7 Unit step response of SFB controlled NMP system



Fig. 13.8 Unit step response of RMP for MRAC



Fig. 13.9 Unit step response robust 2DOF based MRAC controlled NMP system

For analog simulation, the real Op-amp-based 2<sup>nd</sup> order system with its real electronics components has been developed on an electronics board, and which has one RHP zero, responsible for the NMP characteristics of the plant. As the inverse transfer function model is essentially required for feed-forward control in the 2DOF framework, here we have developed the 2<sup>nd</sup> order NMP plant with zero relative degrees. The square wave signal as a step signal has been applied as a reference input to verify the analogue simulation of this real electronic NMP system. Like software simulation, the output response of NMP, Inverse NMP, cascaded, and superimposed inverse, non-inverse NMP system models have been verified, shown in Fig. 13.10, Fig. 13.11, Fig. 13.12, and Fig. 13.13 respectively. Steady-state errors and initial undershoot also have been observed in Fig. 13.14 after applying SFB control on the NMP system.

To shape the desired trajectory as per the designer's choice, a reference model is an essential part of the MRAC scheme [182, 183]. The output response of the developed 1<sup>st</sup> order reference model system has been shown in Fig. 13.15. Finally, the Op-amp based feed-forward and feedback combination of the MRAC and SFB control structure in the 2DOF framework has been applied on the Op-amp based system model, and the square wave response of this proposed controlled system structure found satisfactory results shown in Fig. 13.16, where, controlled plant model's output trajectory able to track the reference input trajectory with the help of RMP. The total hardware experimental set up has been demonstrated in Fig. 13.17.

The tracking performances of the 2DOF controlled NMP system have been validated both in software and the hardware simulation environment shown in Fig. 13.9 and Fig. 13.16. The close observations reveal the fact that the analog simulation in a real-time environment ensures better performance than the software simulation in MATLAB's realistic environment. The initial undershoot remains in the digital simulation, whereas it has been completely nullified in analog simulation in real hardware experimental set up. The incorporation of real electronics components may help this proposed control law to pursue better desired performance than virtual components in the MATLAB environment.

Analog simulation of the NMP system has not been able to remove the unavoidable initial undershoots completely by manipulating its system parameters [139]. But, here, the initial undershoot of the NMP system has been completely removed by the analog simulation of this proposed 2DOF control technique in the real hardware environment.



Fig. 13.10 Square wave response of uncontrolled NMP system



Fig. 13.11 Square wave response of uncontrolled inverse NMP system



Fig.13.12 Square wave response of cascaded inverse and non-inverse NMP system



Fig. 13.13 Superimposed square wave response of inverse and non-Inverse NMP system



Fig. 13.14 Square wave response of SFB controlled NMP system



Fig. 13.15 Square wave response of reference model output for MRAC scheme



Fig. 13.16 Square wave response of 2DOF MRAC controlled NMP system



# Fig. 13.17 Experimental hardware set-up for analogue simulation of 2DOF controlled NMP system

## 13.7 Comparison between Software and Hardware

Name of the System	Steady State Error		Initial Undershoot	
	Software	Hardware	Software	Hardware
RMP	Nil	Nil	Nil	Nil
SFB Controlled NMP	0.5	0.8	1.9	0.5
2DOF Controlled NMP	0.005	Nil	0.05	Nil

#### Table 13.1 Comparison chart of controlled system dynamics

 Table 13.2: Comparison of software and hardware simulation

Type of Simulation	Type of response
Software	Slow
Hardware	Fast

## **13.8** Chapter Summery

2DOF based MRAC as a feed-forward control technique has been proposed to find more reliable solutions to the tracking control problems of the NMP system. Unlike the MP system, inversion-based feed-forward tracking control is difficult to construct for the NMP system as its inverse transfer function model is unstable. To circumvent this problem of set-point tracking of the NMP system, the model reference adaptive inverse control as feed-forward and the state feedback compensation as a feedback control approach in the 2DOF framework have been applied to finding out more reliable solutions to the two important control problems of the NMP system, namely, tracking of the reference input trajectory and stabilization of the output response of NMP system.

The performance of the suggested control scheme has been studied in both software simulation using the MATLAB toolbox and analogue simulation in a real hardware environment, and it is clearly observed that analogue simulation in a real-time environment yields better results, though the gain adjustment with the electronics components is really a tough task for the researcher. The initial undershoot, which is obvious in step response of the NMP system, has been reduced in software simulation but completely nullified by hardware simulation.

# CHAPTER 14 Conclusion

#### 14.1 Concluding remarks

Perfect set point tracking is essentially required in the research and industrial area of control system engineering. It can be possible by the cascaded connection of the inverse and non-inverse transfer function model of the system, where the numerator and denominator polynomials are cancelled or, specifically, it may be said poles and zeros of the system cancelled with each other. As a result, exact reference input tracking can be achieved. But this method is only applicable for MP systems, whereas the NMP system exhibits unbounded and oscillatory output response using this method.

In this research work, 2DOF based MRAC as a feed-forward control technique has been proposed to find more reliable solutions to the tracking control problems of the NMP system. Unlike the MP system, inversion-based feed-forward tracking control is also difficult to construct for the NMP system as its inverse transfer function model is unstable. To circumvent this problem of set-point tracking of the NMP system, the model reference adaptive inverse control as feed-forward and the state feedback compensation as a feedback control approach in the 2DOF framework have been applied to finding out more reliable solutions to the two important control problems of the NMP system, namely, the tracking of reference input trajectory and stabilization of the output response of NMP system. In this proposed method, feed-forward and feedback control are decoupled with each other in the 2DOF framework.

At first, MIT rule-based MRAC as feed-forward control and SFB controller as feedback compensation has been developed for the 2DOF structure, and applied on a 2nd order NMP system with one RHP zero. Though the set point tracking is achieved by this method, the initial undershoot, which is not acceptable for good tracking, could not completely be removed by this MIT rule-based method. There are some major difficulties that arise with this method, like closed loop stability and tracking error convergence that could not be proved by this MIT rulebased MRAC technique.

To overcome the shortcomings regarding the MIT technique, the generalized Lyapunov Design technique based on MRAC has been designed as a feed-forward control part of the proposed 2DOF control scheme, and it has been applied to the same numerical example of the NMP system, which has been used in MIT rule-based control technique. It has been observed that the initial undershoot has been completely removed with the perfect set point tracking. Along with the step input signal, the ramp and parabolic signal also have been applied for the verification of tracking performance. Closed loop stability has been automatically proved at the time of derivation of control parameters, and the proof of tracking error convergence has been established by Barbalat's Lemma.

The RMP is an important part of the MRAC structure as the output response of a closed loop plant follows the output trajectory of the reference model plant. So, designers can freely choose the reference model as per their design requirements. On the basis of the damping ratio, four types of the RMP with under damped, un-damped, critically damped, over-damped dynamic characteristics have been selected for the verification of this unique property of the MRAC, which has been used as a feed-forward control scheme in 2DOF control framework.

This work demonstrates that the plant, which has been used as a reference model for the MRAC scheme for the 2DOF control structure, has an important role in the tracking problem of the control system. It gives a lot of advantage to the researcher or designer, as they can get the opportunity for the modification of the plant's output response as per their needs. The RMP with various damping ratios has been selected to check the role of the RMP employed in the MRAC structure. The inversion based NMP system model has been selected from chapter 6 and the step input signal has been considered as command input. Lyapunov stability theory has been applied to obtain the control parameters of the MRAC algorithm. It has been observed, that the output response of the controlled plant always tracks the path produced by the RMP of this MRAC structure.

Initial undershoot is obvious in the step response of systems, which is the most undesirable characteristic of the output response, as it obstructs perfect reference input tracking of NMP systems. The number of undershoots and overshoots in the step response of the NMP system depend on the number of odd and even numbers of RHP zeros respectively. For the undershoot and overshoot control by the proposed control methodology, five SISO NMP systems have been considered, out of which, two SISO NMP systems with one RHP zero, one SISO NMP system with two RHP zeros, and two SISO NMP systems with three RHP zeros respectively. One benchmarked TITO NMP system with one RHP zero has also been considered for the verification of the proposed control method. The entire initial undershoots and overshoots of uncontrolled NMP systems have been completely nullified and the controlled NMP systems achieved perfect trajectory tracking of the set point. It has been studied, that undershoot control of the benchmarked TITO NMP system in its original literature has been performed by manipulating the system parameters, whereas, the proposed control structure demands its ability by avoiding the option of manipulating the system parameters of the plant and it itself is sufficient to eliminate the undesirable initial undershoots of the TITO NMP system.

After the experimentation on undershoot, overshoot control of NMP systems, 2DOF control methodology has been implemented on a practical, laboratory experimental set-up based on a 4<sup>th</sup> order mechanical realized NMP system from a referred journal. Here, the square wave signal, as a set point, has been used for the verification of the 2DOF control scheme, and finally, it assured that the proposed control performance is better than PID and SFB control techniques.

In the next work, the 3<sup>rd</sup> order NMP system has been considered along with the previous 4<sup>th</sup> order system. The totally unbounded, oscillatory response of the 3<sup>rd</sup> order NMP system has been stabilized by the proposed control technique. Here, the unit step response of 2DOF controlled 3<sup>rd</sup> order and 4<sup>th</sup> order NMP system has been obtained and compared their satisfactory performance with the SFB and advanced MRAC control scheme. The effective tracking performance of this 3<sup>rd</sup> and the 4<sup>th</sup> order NMP has also been verified by the unit ramp and sinusoidal input signals.

Finally, to strengthen the concept of this proposed 2DOF control structure, an Op-amp based NMP system model has been developed in digital and analog simulation environments. At first, the mathematical model of a 2<sup>nd</sup> order NMP system with zero relative degree and one RHP zero has been derived using MATLAB System Identification toolbox, then a realistic Op-amp based NMP system with virtual electronic components and its 2DOF control structure have been developed and experimented in the MATLAB SIMULINK environment. After the satisfactory performance of realistic Op-amp based software simulation of the 2DOF controlled Op-amp based NMP system, a real Op-amp based NMP system with its real electronics components on an electronic board has been constructed, and analogue simulation of this hardware experimental set-up has been performed satisfactorily and strengthen the demand of the proposed MRAC based 2DOF control technique for NMP system.

After the detailed study of the suggested 2DOF control scheme's performance in both software simulation using MATLAB toolbox and analogue simulation in a real hardware environment, it is clearly observed that analogue simulation in a real-time environment yields better results though the gain adjustment with the electronics components is a really tough task

for the researcher. The initial undershoot, which is obvious in step response of the NMP system, has been reduced in software simulation but completely nullified by hardware simulation.

#### 14.2 Major Inferences

The crucial interpretation has drawn from the dissertation are listed below:

- i) Exact set point tracking using the inverse model technique is effective for MP systems, but it does not work on NMP systems.
- ii) In the case of a NMP system, a cascaded inverse, non-inverse transfer function model produces an unbounded oscillatory response in state of cancelling the poles and zeros of its numerator and denominator polynomials.
- iii) Adaptive feed-forward control technique is very effective as a tracking control system, but it needs inversion of the system model, whereas unstable zeros become unstable poles in the inverse transfer function model of the NMP system.
- iv) To circumvent this tracking control problem of the NMP system, Lyapunov Design technique based on MRAC as feed-forward and arbitrary pole placement based on SFB controller as feedback compensation in the 2DOF framework has been proposed as the tracking control system of an NMP system, where feedforward and feedback controller is decoupled with each other.
- v) The robustness of stability is ensured spontaneously while deriving the parameters of the control law of MRAC.
- vi) The suggested control technique has been verified by 2<sup>nd</sup> order, 3<sup>rd</sup> order and mechanical realized 4<sup>th</sup> order NMP system.
- vii) Then, the proposed control methodology has been validated by Op-amp based 2<sup>nd</sup> order NMP system by digital simulation using MATLAB SIMULINK toolbox and analogue simulation using hardware experimental set-up.
- viii) Finally, to conclude, researchers can take the freedom to claim the proposed control law is not only able to solve the tracking and stability problem of the NMP system theoretically, but also exhibits the robust performance of this control approach in a real hardware environment.

# 14.3 Future Scope

- The design of the effective controller to meet the tracking control problem of SISO NMP system has been suggested in this research work. Researchers may give attention to the designing of the controller for MIMO NMP Systems.
- As the settling time and magnitude of initial undershoot are inversely proportional to each other, here is an attractive opportunity for control designer to explore this area of research work.

#### **BIBLIOGRAPHY**

- S. Bose, Y. V. Hote, S. D. Hanwate, "Analysis of Effects due to Right Half Plane Zeros in PI Controller based Hydro Turbine", Conference on Advances in Proportional Integral-Derivative Control, Ghent, Belgium, pp. 633-638, May 2018.
- [2] P. Shah, R. Sekhar, S. Agashe, "Application of Fractional PID Controller to Single and Multi-Variable Non-Minimum Phase Systems", International Journal of Recent Technology and Engineering (IJRTE), vol. 8, no. 2, pp. 2801-2811, July 2019.
- [3] P. Patil, C. S. Rao, "Enhanced PID Controller for Non-minimum Phase Second Order Plus Time Delay System", Chemical Product and Process Modeling, vol. 14, no. 3, pp. 1-13, March 2019.
- [4] M. S. Triantafyllou, and M. A. Grosenbaugh, "Robust Control for Underwater Vehicle Systems with Time Delays", IEEE Journal of Oceanic Engineering, vol.16, no. 1, pp. 146-151, January 1991.
- [5] J. Hasan, T. Karmakar, M. I. Ahmed, "Mathematical Modeling and Simulation based System Identification of Non-Minimum Phase Electro-Hydraulic Servo (EHS) System", International Journal of Engineering Research, vol. 8, no. 09, pp. 189-195, September 2019.
- [6] P. Shah, R. Sekhar, S. Agashe, "Application of Fractional PID Controller to Single and Multi-Variable Non-Minimum Phase Systems", International Journal of Recent Technology and Engineering (IJRTE), vol. 8, no. 2, pp. 2801-2811, July 2019.
- [7] L. Qui and E. J. Davison, "Performance limitations of non-minimum phase systems in the servomechanism problem", Automatica, vol. 29, no. 2, pp. 337-349, 1993.
- [8] N. Getz, "Control of balance for a nonlinear non-holonomic non-minimum phase model of a bicycle", American Control Conference, vol. 1, pp. 148-151, June-July 1994.
- [9] K. J. Astrom, R. E. Klein, and A. Lennartsson, "Bicycle dynamics and control: adapted bicycles for education and research", Control Systems, IEEE, vol.25, no. 4, pp. 26-47, August 2005.
- [10] N. Kaznantzakis, "Fractional PID Controller design for non-minimum phase system (NMP)", Dept. of Instrumentation Control, Chapter 3, pp. 40-50.

- [11] C. H Menq and J. Z. XIA, "Characterization and compensation of discrete-time nonminimum-phase systems for precision tracking control", International Journal Systems SCI, vol. 24, no. 6, pp. 1189-1205, 1993.
- [12] M. Patil, B. Bandyupadhyay, A. Chalanga, and H. Arya, "Output Tracking of Nonminimum-Phase Systems via Reduced Order-Sliding Mode Design", IEEE/ASME Transactions on Mechatronics, vol.23, No.4, pp.1981-1992, August, 2018.
- [13] D. Chen, H. Peng, "Non-minimum Phase Phenomenon of PEM Fuel Cell Membrane Humidifiers", Journal of Dynamic Systems, Measurement, and Control, vol. 130, pp. 044501-1 – 044501-2, July 2008.
- [14] I. A. Shkolnikov, Y. B. Shtessel, "Aircraft Non-minimum phase control in Dynamic Sliding Manifolds", Journal of Guidance Control and Dynamics, vol. 24, no. 3, pp. 566-572, May-June 2001.
- [15] D. Kwon and W. J. Book, "A time-domain inverse dynamics tracking control of a single–link flexible manipulator", Journal of dynamic systems, measurement, and control, vol. 116, no. 2, pp. 193-200, 1994.
- [16] Y. B. Shtessel, A.S. Zinober, and I. A. Shkolnikov, "Sliding mode control of boost and buck-boost power converters using method of stable system care", Automatica, vol. 39, no. 6, pp. 1061-1067, 2003.
- [17] H. Sira-Ramirez, and M. Rios-Bolivar, "Sliding mode control of dc-to-dc power converters via extended linearization", IEEE Transactions on Circuits and Systems I: Fundamental Theory and Application, vol. 41, no. 10, pp. 652-661, 1994.
- [18] S. H. Nagarsheth, S.N.Sharma, "Control of non-minimum phase systems with dead time: a fractional system viewpoint", International Journal of Systems Science, pp. 1-24, 2020.
- [19] I. Horowitz and Y. Liao, "Limitation of non-minimum phase feedback systems", International Journal of Control, vol. 40, no. 5, pp. 1003-1013, 1984.
- [20] A. P. Aguiar, J. P. Hespanha, "Path-Following for Non-Minimum Phase Systems Removes Performance Limitations",IEEE Transactions on Automatic Control\_, Volume: 50, no. 2, pp. 234-239, Feb. 2005.

- [21] S. Bucz, A. Kozakova, V. Vesely, "Robust guaranteed performance PID controller Design for non-minimum phase plants", Journal of Electrical Engineering, vol. 69, no. 2, pp. 117-127, 2018.
- [22] M. N. Sahinkaya, P. Wang, and S. Akehurst, "Casual inversion of non-minimum phase systems for hardware-in-the-loop transmission testing", Journal of System and Control Engineering, vol. 227, no. 3, pp. 298-307, 2013.
- [23] J. Firman, A. Naiborhu, "Output tracking of some class non-minimum phase nonlinear systems via output redefinition", AIP Conference Proceedings, pp. 1-7, February 2016.
- [24] M. J. Patrick, N. Ishak, R. Adnan, "Trajectory E-Filter Zero Phase Error Tracking Controller for Non-Minimum Phase XY Table System", International Journal of Scientific & Engineering Research, vol. 3, no. 8, pp. 1-6, 2012.
- [25] L. Dong, "Output Tracking of Non-linear Non-minimum Phase Systems: an Engineering Solution", IEEE Conference on Decision and Control, Spain, pp. December 2005.
- [26] J. Rajput, Z. Weiguo, "Fundamental methodologies for control of nonlinear nonminimum phase systems: An overview", Journal on Systems and Control Engineering, vol. 228, no. 8, pp. 553-564, 2014.
- [27] O.Kaneko, H.T, Nguyen, Y. Wadaqaki, S.Yamamoto, "Fictitious Reference Iterative Tuning for Non-Minimum Phase Systems in the IMC Architecture: Simultaneous Attainment of Controllers and Models", SICE Journal of Control, Measurement, and System Integration, vol. 5, no. 2, 2012.
- [28] L. Wang and J. Cheng, "Robust disturbance rejection methodology for unstable nonminimum phase systems via disturbance observer", ISA Transactions, vol. 100, pp. 1-12, 2019.
- [29] A. Chemori, N.Marchand, "A prediction-based nonlinear controller for stabilization of a non-minimum phase PVTOL aircraft", International Journal ofRobust And Nonlinear Control, pp. 876-889, 2008.
- [30] N. Ikemoto, Y. Ishida, N. Matsumoto, "Active Disturbance Rejection Control with Linear Quadratic Regulator for non-minimum phase systems", Control and System Graduate Research Colloquium, Malaysia, pp. 89-93, August 2018.

- [31] M. Anil Kumar, B.Amarendra Reddy, "Robust PID Controller Design for Non-Minimum Phase Systems Using Magnitude Optimum and Multiple Integration and Numerical Optimization Methods", International Research Journal of Engineering and Technology, vol. 4, no. 10, pp. 1474-1479, October 2017.
- [32] W. Garcia-Gabin, E. F. Camacho, "Sliding mode model based predictive control for non-minimum phase systems", Europan Control Conference, Cambridge, UK, pp. 904-909, September 2003.
- [33] Q. Zoua, S. Devasi, "Precision preview-based stable-inversion for nonlinear nonminimum phase systems: The VTOL example", Automatica 43, pp. 117-127, 2007
- [34] Q. Quang, K. Cai, H. Lin, "Additive-state-decomposition-based tracking control framework for a class of non-minimum phase systems with measurable nonlinearities and unknown disturbances", International Journal of Robust and Nonlinear Control, vol. 25, no. 2, pp. 163-178, January 2015.
- [35] M.R. Buehner and P.M.Young, "Perfect Tracking for Non-minimum Phase Systems", American Control Conference, American Control Conference, Bultimore, MD, USA, pp. 4010-4015, June-July 2010.
- [36] A. W. Nasir, and A. K. Singh, "IMC based Fractional Order Controller for Nonminimum phase systems", Annual IEEE India Conference, New Delhi, pp. 1-6, December 2015.
- [37] P. P. Arya, S. Chakrabarty, "IMC based Fractional Order Controller Design for Specific Non-minimum Phase Systems", IFAC Paper OnLine, vol. 51, no. 4, pp. 847-852, January 2018.
- [38] O. Iskrenovic-Momcilovic, "Output Tracking in Causal Non-minimum Phase Systems Using Discrete-time Stable System Centre", IETE Journal of Research, vol.59, no. 5, pp. 535-540, Sep-Oct 2013.
- [39] T. Li, "Eliminating the internal instability in Iterative learning control for nonminimum Phase system", Thesis of Doctor of Philosophy in the Graduate school of Arts and Science, Colombia University, 2017.
- [40] W. Wang, S. Qui, and J. Xu, "Application of a multi-input multi-output (MIMO) nonlinear non-minimum phase system control method to hydro turbine unit", Journal of Physics Conference Series, vol. 1303, no. 1, pp. 1-10, August 2019.

- [41] K. Jouili, "Robust Stabilization of Non-minimum Phase Switched Nonlinear Systems with Uncertainty", Journal of Systems Science and Complex, vol.33, no. 2, pp. 289-311, April 2020.
- [42] T. Berger, "Funnel Control for linear non-minimum phase systems", Proc. Appl. Math. Mech, vol. 19, no. 1, pp. 1-2, November 2019.
- [43] T. Berger, "Tracking with prescribed performance for linear non-minimum phase systems", Automatica 115, pp. 1-11, 2020.
- [44] M. Araki and H. Taguchi, "Two-Degree-of-Freedom PID Controllers", International Journal of Control Automation, and Systems, vol. 1, no. 4, pp. 401-411, December 2003.
- [45] H. Taguchi, M. Araki, "Two-degree-of-freedom PID Controllers-their Functions and Optimal Tuning", IFAC Digital Control: Past, Present and Future of PID Control, Terrassa, Spain, pp. 91-96, 2000.
- [46] K. Lurang, C. Thammarat, S. Hlangnamthip, and D. Puangdownreong, "Optimal Design of Two-Degree-of-Freedom PIDA Controllers for Liquid-Level System by Bat-Inspired Algorithm", International Journal of Circuits, Systems and Signal Processing, vol.13, pp. 34-39, 2019.
- [47] H. A. Suthar, G. Jagrut, "Two Degree of Freedom PID Controller, Its Equivalent Forms and Special Cases", International Journal of Robotics and Automation, Vol. 4, No. 4, pp.269-283, December 2015.
- [48] M. Karlsson, F. B. Carlson, A. Robertsson, R. Johansson, "Two-Degree-of-FreedomControlfor Trajectory Tracking and Perturbation Recovery during Execution of Dynamical Movement Primitives", IFAC (International Federation of Automatic Control), pp. 1923-1930, 2017.
- [49] P. F. Beevi, T. K. Sunil Kumar, J. Jacob, "Two-Degree-of-Freedom Controller Design by AGTM\AGMP Matching Method for Time Delay Systems", Global Colloquium in Recent Advancement and Effectual Researches in Engineering, Science and Technology, pp. 20-27, 2016.
- [50] T. Asai, H. Okajima, and C. L. Ooi, "Pole Constraint of Reference Models in 2DOF Servo Systems Design for Non-Minimum Phase Systems", SICE Journal of Control, Measurement, and System Integration, vol. 1, no. 6, pp. 429-434, November 2008.

- [51] A. Poti, V. C. Pal, R. Negi, "Design of a 2-DOF Control and Disturbance Estimator for a Magnetic Levitation System", Engineering, Technology & Applied Science Research, vol. 7, no. 1, pp. 1369-1376, February 2017.
- [52] L. A. Emmanuel, C. C Mbaocha, and M. Olubiwe, "Design of Two-Degree-Of-Freedom (2DOF) Steering Control for Automated Guided Vehicle", International Journal of Scientific Engineering and Science, Vol. 3, No. 4, pp.57-64, 2019.
- [53] G. A. Hassaan, "Tuning of a Feedforward 2DOF PID Controller to Control Second Order-Like Processes", International Journal of Engineering and Techniques, Vol. 4, No. 4, pp. 132-142, 2018.
- [54] J. Sanchez, A.Visioli, "A two-degree-of-freedom PI controller based on events", Journal of Process Control, vol. 21, no. 4, pp. 639-651, April 2011.
- [55] B. G. Abdelaty, and A. H. Ahmed, "Fixed Set Point Weighting 2DOF PID Controller for Control Processes", Engineering Mathematics, vol. 2, no. 1, pp. 21-27, 2018.
- [56] T. Kinoshita and T. Yamamoto, "Design of a Data-Oriented PID controller for a Two-Degree-of-Freedom Control System", 3<sup>rd</sup> IFAC Conference on Advances in Proportional-Integral-Derivative Control, Belgium, pp. 412-415, May 2018.
- [57] Y. Ebihara, Y. Fujiwara, T. Hagiwara, and M. Sato, "2DOF Control System Design for Maneuverability Matching and Gust Disturbance Rejection in In-Flight Simulator MuPAL-α", SICE Annual Conference, Taiwan, pp. 524-528, August 2010.
- [58] M. Viteckova, A. Vitecek, "2DOF PI and PID Controllers Tuning", IFAC Proceedings, vol. 43, no. 2, pp. 343-348, June 2010.
- [59] H. Nemati and P. Bagheri, "A New Approach to Tune the Two-Degree-of –Freedom (2DOF)", IEEE International Symposium on Computer-Aided Control System Design, Yokohama, Japan, pp. 18199-1824, September 2010.
- [60] N. G. Adar and R. Kozan, "Comparison between Real Time PID and 2-DOF PID Controller for 6-DOf Robot Arm", International Conference on Computational and Experimental Science and Engineering (ICCESEC), vol. 30, no.1, pp. 269-271, 2015.

- [61] P. C. Nayak, U. C. Prusty, P.C Sahu, R. C. Prusty, "Design and Tuning of Flower pollination algorithm based 2DOF controller in AGC of multi area thermal wind system", International Journal of Advanced Research in Electronics and Communication Engineering(IJARECE), vol. 7, no. 2, pp. 118-126, February 2018.
- [62] C. B. Kadu and G.L. Deshmukh, "Design of two-degree-of-freedom PID controller for temperature control system", International Conference on Automatic Control and Dynamic Optimization Techniques (ICACDOT), Pune, pp. 586-589, 2016.
- [63] A. SeshagiriRao, and M. Chidambaram, "Enhanced Two-Degrees-of Freedom Control Strategy for Second-Order Unstable Processes with Time Delay", Ind. Eng. Chem. Res, vol. 45, no.10, pp, 3604-3614, 2006.
- [64] FebinaBeevi P, Sunil Kumar TK, J. Jacob, "Novel Two-Degree-of-Freedom Model Matching Controller for Desired Tracking and Disturbance Rejection", Studies in Informatics and Control, vol. 26, no.1, pp. 105-114, March 2017.
- [65] K. Bingi, R. Ibrahim, M. N. Karshiti, "Real-Time Control of Pressure Plant Using 2DOF Fractional-Order PID Controller", Arabian Journal for Science and Engineering, pp.1-12, June 2018.
- [66] V. M. Alfro, and R.Vilanova, "Robust Tuning and Performance Analysis of 2DOF PI Controllers for Integrating Controlled Processes", Industrial& Engineering Chemistry Research, vol. 51, pp. 13182-13194, 2012.
- [67] F. Padula and A. Visioli, "Set-point Filter Design for a Two-Degree-of-freedom Fractional Control System", IEEE/CAA Journal of AutomaticaSinica, vol. 3, no. 4, pp. 451-462, October 2016.
- [68] K. Sladka, J. Czebe, R. Wagnerova, "The 2DOF Controller Implementation in Control System with Single-chip Computer", International Carpathian Control Conference, Hungary, pp. 228-234, May 2018.
- [69] H. Ismail, N. Ishak, M, Tajjudin, M. H. F. Rahiman, and R. Adnan, "Model Identification and Feed-forward Control for Non-minimum Phase System", International Journal of Applied Physics and Mathematics, vol. 1, no. 3, pp. 185-190, November 2011.

- [70] K. Sharma, andR. Pradhan, "Design of A Novel Feed-Forward Control Strategy for A Non-Minimum Phase System", IOP Conference Series Materials Science and Engineering, vol. 225, no. 1, pp. August 2017.
- [71] M. J. Patrick, N. Ishak, and R. Adnan, "Contouring Control of Non-minimum Phase XY Table System using Trajectory ZPETC", International Journal of Scientific & Technology Research, vol. 1, no. 6, pp. 80-85, July 2012.
- [72] J. Mamboundou, N. Langlois, "Application of Feed-forward Model Inverse Control Using MPC on a Non- Minimum Phase System", IEEE International conference on Industrial Technology, pp. 899-905, March 2012.
- [73] A. Uthman, "Antenna Azimuth Position Control System Using Model Reference Adaptive Control Method Gradient Approach and Stability Approach", Journal of Engineering and applied sciences, vol.14, no. 16, pp. 5657-5664, 2019.
- [74] H. Perez, B. Ogunnaike, and S. Devasia, "Output Tracking Between Operating Points for Nonlinear Processes: Van deVusse Example", IEEE Transactions on Control Systems Technology, vol. 10, no. 4, pp. 611-617, August 2002.
- [75] Q. Zou, S. Devasia, "Preview-Based Stable-Inversion for Output Tracking, "Proceedings of the American Control Conference, San Diego, California, pp.3544-3548, June 1999.
- [76] K.Uren and G.V.Schoor, "Predictive PID Control of Non-minimum Phase Systems", Advances in PID Control, InTech,availablefrom:http://www.intechopen.com/books/advances-in-pidcontrol/predictive- pid-control-of-non- minimum phase systems, pp.1-22.
- [77] G. Gang, W. Jinzhi, "The Feed-forward Tracking Control for Linear Systems with Quadratic Performance", Proceedings of the 30th Chinese Control Conference, Yantai, China, pp. 3834-3839, July 2011.
- [78] P. Tomei, and C. M. Verrelli, "Adaptive Learning Control for Non-Minimum Phase Control Linear Systems", European Control Conference (ECC), Strssbourg, France, pp. 2066-2071, June, 2014.
- [79] W. S. Black, P. Haghi, K. B. Ariyur, "Adaptive Systems: History, Techniques, Problems and Perspectives", Systems, EISSN 2079-8954, MDPI AG, Basel, Switzerland, pp. 607-660, November, 2014.

- [80] J. B. Hoagg, "Model Reference Adaptive Control For Non minimum Phase Systems Using A Surrogate Tracking Error", IEEE Conference on Decision and Control and European Control Conference (CDC-ECC) Orlando, FL,USA, pp. 360-365, December 2011.
- [81] M. Boerlagey, M. Steinbuchy, P. Lambrechtsy, M. van de Wal, "Model-based feedforward for motion systems", Proceedings of IEEE Conference on Control Applications, pp.1-6, June 2003
- [82] G. Shan, Q. Wang, "The Design of Disturbances Rejection Controller for Decoupling Internal Model Control System of Non-Square Processes", International Workshop on Computer Science in sports, pp 1-6, 2013.
- [83] J. Dey and T. K. Saha, "Design and Performance Analysis of Two Degree-of-freedom
   (2 DOF) Control of DC-DC Boost Converter", Industrial Technology (ICIT), 2013
   IEEE InternationalConference, pp. 493-498, Feb. 2013.
- [84] F. Z. Baghli, L. E. bakkali, Y. Lakhal, A. Nasri, B. Gasbaoui, "The Efficiency of the Inference System Knowledge Strategy for the Position Control of a Robot Manipulator with Two degree of Freedom", International Journal of Research in Engineering and Technology, Vol. 2, No. 7, pp.138-145, 2013.
- [85] A. Petros., I. Jing. Sun, "Robust Adaptive Control", University of Southern California, pp. 434-436, 2003.
- [86] G. R. Cho, P. H. Chang, and Y. Jin, "Enhanced feed forward control of non-minimum phase systems for tracking predefined trajectory", Industrial Electronics (ISIE), 2010 IEEE International Symposium, pp. 167-172, July 2010.
- [87] H. Ismail, N. Ishak, M. Tajjudin, M. H. F. Rahiman, R. Adnan, "Model Identification and Feed-forward Control for Non-minimum Phase Systems", International Journal of Applied Physics and Mathematics, vol. 1, no. 3, pp. 185-190, November 2011.
- [88] P. Sukkarnkha, C. Panjapornpon, "Two-degree-of-freedom Controller Design for Uncertain Processes UsingInput/outputLinearization ControlTechnique", ASEAN Journal of Chemical Engineering, Vol. 11, No. 1, pp. 16-21, 2011.

- [89] P. C. Mishra, S. K. Nayak, S. K. Nayak, P. Pradhan, D. P. Ghosh, "Model based numerical state feedback control of jet impingement cooling of a steel plate by Poleplacement technique", International Journal of Computational Engineering Research (IJCER), vol. 3, no.8, pp. 37-44, 2013.
- [90] K. Ogata, "Modern Control Engineering", PHI Learning Private Limited, 3<sup>rd</sup> Edition, 2010.pp. 787-789.
- [91] J. E Slotine, W. Li, "Applied non-linear control", Prentice Hall, Englewood Cliffs, New Jersey 07632, pp. 94-96, 1991.
- [92] J. Nagrath and M. Gopal, "Control System Engineering", New Age Publication, pp.695-775, 2007.
- [93] R. Adnan, H. Ismail, N. Ishak, M. Tajjudin and M. H. F. Rahiman, "Adaptive Feedforward Zero Phase Error Tracking Control for Minimum Phase and Non-minimum Phase Systems – XY Table Real-Time Application", IEEE Control and System Graduate Research Colloquium, pp. 359-363, July 2012.
- [94] J. Mamboundou, N. Langlois, "Application of Feed-forward Model Inverse Control Using MPC on a Non-Minimum Phase System", IEEE International conference on Industrial Technology, pp. 899-905, March 2012.
- [95] R. Jafari and R. Mukherjee, "Intermittent Output Tracking for Linear Single-Input Single-Output Non-Minimum-Phase Systems", American Control Conference Fairmont Queen Elizabeth, Montréal, Canada, pp. 5942-5947, June 2012.
- [96] T. Schauer, K. J. Hunt, J.C. Kalkkuhl, and U. Korn, "Output Tracking of a Bioreactor with Non-minimum Phase Characteristic", American Control Conference, pp. 3338 – 3342, Jun 2000.
- [97] M. J. Patric, N. Ishak, M. H. F. Rahiman, M. Tajjudin, "Modeling and Controller Design for Non- Minimum Phase System with Application to XY-Table", Control and System Graduate Research Colloquium, pp.113-118, June 2011.
- [98] R. K.Barai, K. Nonami, "Optimal two-degree-of-freedom fuzzy control for locomotion control of a hydraulically actuated hexapod Robot", Information Sciences: an International Journal, Elsevier Science Inc. New York, NY, USA, vol. 177, pp. 1892-1915, 2007.
- [99] Z. Chi1 and Q. Xu1, "Recent Advances in the Control of Piezoelectric Actuators", International Journal of Advanced Robotic Systems, Review paper, International Journal of Advanced Robotic Systems, pp.1-11, November, 2014.
- [100] P.J.Antsaklis, and E.Garcia, "How to Obtain Desirable Transfer Functions in MIMO Systems under Internal Stability Using Open and Closed Loop Control", Technical Report of the ISIS Group, pp.1-11, June, 2013.
- [101] R. Ghazali, Y.M. Sam, M. F. Rahmat, and D. Hanafi, "Two-degree-of-freedom Robust Control for a Non-minimum Phase Electro-hydraulic System", 24th Chinese Control and Decision Conference (CCDC), pp.1184 – 1190, May, 2012.
- [102] R. Ghazali, Y.M. Sam, M.F. Rahmat, p. Zulfatman, "Point-to-point trajectory tracking with two-degree-of-freedom robust control for a non-minimum phase electro-hydraulic system," Intelligent Control and Automation (WCICA), pp. 2661-2668, July 2012.
- [103] M. Treuer, T. Weissbach, and V. Hagenmeyer, "Flatness-Based Feedforward in a Two-Degree-of Freedom Control of a Pumped Storage Power Plant", IEEE Transactions on Control Systems Technology, vol. 19, no. 6, pp.1540-1548, November 2011.
- [104] M.Gopal, "Digital control and state variable method", Tata McGraw-Hill Publishing Company Limited, Second Edition, pp. 567-568.
- [105] T. Minan, C. Jie, Y. Xiaoxiao, W. Xiaoming, G. Baohui, "Model reference adaptive control based on Lyapunov stability theory", Proceedings of the 26th Chinese Control and Decision Conference (2014 CCDC), pp. 1828-1833, June 2014.
- [106] X. Zhao, "Stabilization and tracking of the Vander Pol Oscillator" A thesis submitted in partial fulfilment of the requirements for the degree of Master of Science, in the Department of Electrical and Computer Engineering in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida, pp.1-65, 2005.
- [107] Y. Chen, J. Wu, Cheung. N.C. Cheung, "Lyapunov's stability theory-based model reference adaptive control for permanent magnet linear motor drives", Proceedings of First International Conference on Power Electronics Systems and Applications, pp. 260-266, November 2004.

- [108] A. Duka, S. E. Oltean, M. Dulau, "Model Reference Adaptive vs. Learning Control for the Inverted Pendulum. A Comparative case study", CEAL, Journal of Control Engineering and Applied Informatics, Vol. 9, No. 34, pp.67-75, 2007.
- [109] S.E. Oltean, M. Dulau, A.V. Duka, "Model reference adaptive control for slow processes: a case study on level process control", International Conference Interdisciplinary in Engineering, Romania, pp.629-636,
- [110] M. Pal, G. Sarkar, R. K. Barai, T. Roy, "Design of Adaptive Two-Degree-of-Freedom Controller for Inversion based Non-Minimum Phase System", Proceedings of Michel Faraday IET International Summit - 2015 (MFIIS-2015), Kolkata, India, Vol. 1, pp. 78-83, September 2015.
- [111] A. Duka, S. E. Oltean, and M. Dulau, "Model Reference Adaptive Control and Fuzzy Model Reference Learning Control for the Inverted Pendulum. Comparative Analysis", WSEAS International Conference on Dynamical Systems and Control, Venice, Italy, pp.168-173, November 2005.
- [112] P. Ioannou, "Robust adaptive control", University of Southern California, pp.313-315, 2003.
- [113] K. J. Astrom and B. Wittenmark, "Adaptive Control", Dover Publication, INC., 2nd edition,pp. 420-425.
- [114] A. Nasar, N E Jaffar, "Lyapnov rule based Model Reference Adaptive Controller Designed for Steam Turbine Speed", International Journal of Electrical Engineering & technology (IJEET) vol. 6, no. 7, pp. 13-22, July-August, 2015.
- [115] T.M. Do, Z. Man, J. Jin, C. Zhang, J. Zheng, Wang H. Wang, "Sliding mode learning control of non- minimum phase non-linear system", International Journal of Robust and Nonlinear Control, vol. 26, no. 11, pp. 2281-2298, 2015.
- [116] M. Duan, S.K. Ramani, C.E. Okwudire "Tracking control of non-minimum phase system using filtered basis functions: a nurbs-based approach", Proceedings of the ASME 2015 Dynamic Systems and Control Conference, Ohio, USA, pp.1-10, 2015.
- [117] Z.B. Wei, J.R. Ren, Q. Quan, "Further results on additive-state-decomposition-based output feedback tracking control for a class of uncertain non-minimum phase nonlinear systems", 28th Chinese Control and Decision Conference (CCDC), Yinchuan, China, pp.6793-6798, 2016.

- [118] M. Orsag, C. Korpela, Bogdan S. Bogdan, and P. Oh, "Lyapunov based model reference adaptive control for aerial manipulation", International Conference on Aircraft Systems, Atlanta, GA, USA, pp. 966-973, May 2013.
- [119] Y. R. Chen, J. Wu, and N.C. Cheung, "Lyapunov's stability theory based Model reference adaptive control for permanent magnet linear motor drives", First International Conference on Power Electronics System and Applications, Hong Kong, pp. 260-266, November 2004.
- [120] T.N. Trajkov, H. Koppe, U. Gabbert, "Direct model reference adaptive control (MRAC) design and simulation for the vibration suppression of piezoelectric smart structures", Communications in Nonlinear Science and Numerical Simulation, vol. 13. No. 9,pp. 1896–1909, November 2008.
- [121] J. Brufau-Penella, K. Tsiakmakis, T. Laopoulos, and M. Puig- Vidal, "Model reference adaptive control for an ionic polymer metal composite in underwater applications", IOP Publishing Ltd, Smart Materials and Structures, vol. 17, No. 4, pp.1-9, August 2008.
- [122] L. Yujie and C. Shaozhong, "Model reference adaptive control system simulation of permanent magnet synchronous motor", IEEE Advanced Information Technology, Electronic and Automation Control Conference (IAEAC), pp. 498-502, 2015.
- [123] A. Chelihi, M. Chemachema, "Model reference adaptive control for twin rotor multiple-input and multiple-output system via minimal controller synthesis", Journal of System and Control Engineering, Vol. 228, No. 6, pp.406-418, June 2014.
- [124] J.K. Tar, L. Kova'cs, A'. Taka'cs, B. Taka'cs, P. Zentay, T. Haidegger, I. Rudas, " Novel design of a model reference adaptive controller for soft tissue operations", IEEE International Conference on Systems, Man, and Cybernetics, San Diego, CA, USA, pp. 2446-2451, October 2014.
- [125] A. Antony, A. Nasar, "Model reference adaptive control on glucose regulation", International Journal of Science and Research (IJSR), Vol. 4, No. 9, pp. 120- 122, 2015.
- [126] G. Gabriel, P. Keerthana, J. Gnanasoundharam, "Comparison of pi controller, model reference adaptive controller and fuzzy logic controller for coupled tank system", Indian Journal of Science and Technology, Vol. 9, No. 12, pp. 1-5, 2016.

- [127] F. Merrikh-Bayat, "Practical and efficient method for fractional-order unstable polezero cancellation in linear feedback systems", Elsevier Science, pp. 1-17, December 2013.
- [128] S. Bose, Y.V.Hote, S.D, Hanwate, "Analysis of Effects due to Right Half Plane Zeros PI Controller based Hydro Turbine", 3rd IFAC Conference on Advances in Proportional-Integral-Derivative Control, Ghent, Belgium, pp. 633-638, May 9-11, 2018.
- [129] A. Bemardo, S. L. D. L. Barra, "On Undershoot in SISO System", IEEE Transaction on Automatic Controls, vol. 39, no. 3, pp. 578-581, March 1994.
- [130] L. Wang, J. Cheng, "Robust disturbance rejection methodologyfor unstable nonminimum phase systems via disturbance observer", ISA Transaction, pp. 1-12, 2019.
- [131] Li. S. Tzuu-Hseng, S. Ming-Yuan, "Design of a GA-based fuzzy PID controller for non-minimum phase systems", Fuzzy Sets and Systems111, pp. 183-197, December 2000.
- [132] Li. S. Tzuu-Hseng, S. Ming-Yuan, "PID Controller Design for Non- Minimum Phase Time-Delay Systems Using Optimization Methods", *Helix*, vol. 8, no. 2 pp. 3100-3106, February 2018.
- [133] J. Wang, Y. Zhang , and W. Wang, "Optimal acceleration autopilot design for nonminimum phase missiles using evolutionary algorithms", International Journal of Bio-Inspired Computation, vol.8, no. 4, pp. 221-227, 2016.
- [134] V. Bijani, A. Khosravi, P. Sarhadi, "Optimal design of PI/PD controller for nonminimum phase system", Transactions of the Institute of Measurement and Control, vol.28, no. 1, pp. 27-35, 2006.
- [135] V. Dudiki, "Feed-forward Compensation of Non-Minimum Phase Systems", Master of Engineering thesis, Wright State University, Dayton, Ohio, United State, November 2018.
- [136] K. Marek, B. Vladimir, B. Tomas, "Predictive Control Of Two-Input Two Output System with Non-Minimum Phase", Proceedings31st European Conference on Modelling and Simulation, pp. 1-6, May 2017.

- [137] R. A. Alphinas, H.H. Hansen, and T. Tambo, "Comparison of conventional closedloop controller with an adaptive controller for a disturbed thermodynamic system", Evolving and Adaptive Intelligent Systems (EAIS), Slovenia, pp. 1-8, May 2017.
- [138] S. Bose, Y.V. Hote, and S.D. Hanwate, "Analysis of Practical Non- minimum Phase Systems by Transfer Function Approach", 4<sup>th</sup> International Conference on Electrical Energy Systems(ICEES), pp. 500-505, 2018.
- [139] S. Tofighi, and F. Merrikh-Bayat, "A benchmark system to investigate the nonminimum phase behavior of multi-Input multi-output systems", Journal of Control and Dicision, vol. 5, no. 3. pp 300-317, 2018.
- [140] J. Stewart, D.E.Davison, "On Overshoot and Non-minimum Phase Zeros", IEEE Transactions on Automatic Control, vol.51, no.88, pp.1378-1382, 2006.
- [141] K. J. Åström, and R. D. Bell, "Drum-boiler dynamics", Automatica, vol. 36, no. 3, pp. 363-378, 2000.
- [142] K. Cohen, and D. E. Bossert, "Fuzzy logic non-minimum phase autopilot design", AIAA Guidance, Navigation, and Control Conference and Exhibit, Austin, Texas, Paper 2003-5550, August 2003.
- [143] J. Hauser, S. Sastry, and G. Meyer, "Nonlinear control design for slightly nonminimum phase systems: Application to V/STOL aircraft", Automatica, vol. 28, no. 4, pp. 665-679, 1992.
- [144] D. Kwan, and W. J Book, "Tracking Control of a Non-minimum Phase Flexible Manipulator", Modeling and
- Control of Compliant and Rigid Motion System, AMSE, pp. 27-35, 1991.
- [145] C. Kravaris and P. Daoutidis, "Nonlinear state feedback control of second-order nonminimum-phase nonlinear systems", Computer Chemical Engineering, 1990, vol.14, no. 5, pp. 439-449, 1990.
- [146] P. R. Gray and R. G. Meyer, "Analysis and Design of Analog Integrated Circuits", 3rd edition, New York: Wiley, 1993.

- [147] F. Merrikh-Bayat, and F. Bayat, "Method for Undershoot-Less Control of Non-Minimum Phase Plants Based on Partial Cancellation of the Non-Minimum Phase Zero: Application to Flexible-Link Robots", International Conference on Electrical and Computer Engineering, pp. 1-6, 2013.
- [148] W. H. Chen, and D. J. Balance, "QFT Design for Uncertain Non-Minimum Phase and Unstable Plants Revisited", International journal of control, vol. 74, pp.957-965, 2001.
- [149] K. S. Ramani, M. Duan, C. E. Okwudire, and A. G. Ulsoy, "Tracking Control of Linear Time-Invariant Non- minimum Phase Systems Using Filtered Basis", Journal of Dynamic Systems, Measurement, and Control, ASME, vol. 139, pp. 1-11, 2017.
- [150] L. Benvenuti, M.D.Di. Benedetto, and J.W.Grizzle, "Approximate output tracking for Non- linear Non-Minimum Phase System with an application to Flight Control", International Journal of Robust and Non-linear Control, vol-4, pp. 397-414,1994.
- [151] M. Duan, K. S. Ramani, and C. E. Okwudire, "Tracking Control of Non- minimum Phase System Using Filtered Basis Functions: A Nurbs-based Approach", Proceedings of the ASME 2015 Dynamic Systems and Control Conference, pp. 1-10,October 2015.
- [152] M. T. Do, Z. Man, J. Jin, C. Zhang, J. Zheng, and H. Wang, "Sliding mode learning control of non-minimum phase non-linear system", International Journal of Robust and Nonlinear Control, pp. 2281-2298, 2015.
- [153] H. Dimitar, and H. J. Carl, "Control of nonlinear non-minimum phase systems with input-output linearization", American Control Conference (ACC), pp. 4016-4023, 2015.
- [154] M. Pal, G. Sarkar, R. K. Barai, and T. Roy, "Reference Input tracking of inversionbased Non- Minimum Phase System using Adaptive Two-Degree-of-Freedom Control", IEEE First International Conference on Control, Measurement and Instrumentation, pp. 508- 513, 2016.
- [155] M. Araki, and H. Taguchi, "Two-Degree-of-Freedom PID Controllers", International Journal of Control, Automation, and Systems, vol. 1, no. 4, pp. 401-411, 2003.

- [156] N. N. Abdul Aziz, M. I. Yusoff, M.I.Solihin, and R. Akmeliawati, "Two Degrees of Freedom Control of a Ball and Beam System", International Conference on Mechatronics, pp. 1-12, 2013.
- [157] N. He, D. Shi, J. Wang, M. Forbes, J. Backström, and T. Chen, "Automated Two-Degree- of-Freedom Model Predictive Control Tuning", Ind. Eng. Chem. Res, vol. 54, no. 43, pp 10811–10824, 2015.
- [158] J. F. Quindlen, G. Chowdhary, and P. H. Jonathan, "Hybrid Model Reference Adaptive Control for Unmatched Uncertainties", American Control Conference (ACC), pp. 1125-1130, 2015
- [159] M. Pal, G. Sarkar, R. K.Barai& T. Roy, "Model Reference Adaptive Control of unstable system with different Reference Models", International Conference on Advances in Science and Technology, pp. 252-256,2017.
- [160] L. Yujie, and C. Shaozhong, "Model reference adaptive control system simulation of permanent magnet synchronous motor", IEEE Advanced Information Technology, Electronic and Automation Control Conference, pp. 498-502, 2015.
- [161] H. Tahersima, M. Saleh, A. Mesgarisohani, and M.Tahersima, "Design of Stable Model Reference Adaptive System via Lyapunov Rule for Control of a Chemical Reactor", Australian Control Conference, Proc. pp. 348-353,2013.
- [162] Q.Wang, S.Zhu, and Y.He, "Model Reference Adaptive Control for Active Trailer Steering of Articulated Heavy Vehicles", SAE Technical Paper, pp. 1-10,2015.
- [163] Y. Wei, S. Liang, Y. Hu and Y. Wang, "Composite Model Reference Adaptive Control for a Class of Nonlinear Fractional Order Systems", ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Proc. pp. 1-9,2015.
- [164] D. Roy Choudhury, "Modern Control Engineering", Prentice Hall of India private limited, pp. 530-645, 2005.
- [165] B. Orun, S. Necipoglu, C. Basdogan, andL. Guvenc, "State feedback control for adjusting the dynamic behaviour of a piezo actuated bimorph atomic force microscopy probe", 9th International Conference on Robotic, Vision, Signal Processing and Power Applications, Proc. pp. 419-426,2009.

- [166] D. Puangdownreong, "Optimal State-Feedback Design for Inverted Pendulum System by Flower Pollination Algorithm", International review of Automatic Control (IREACO), vol.9, no.5, 2016.
- [167] C. T. Freeman, P. L. Lewin & E. Rogers, "Experimental evaluation of iterative learning control algorithms for non-minimum phase plants", International Journal of Control, Taylors and Francis, 2007, Vol. 78, No. 11, pp. 826–846, 2005.
- [168] M.Borah, P.Roy and B.K.Roy, "Enhanced Performance in Trajectory Tracking of a Ball and Plate System using Fractional Order Controller", IETE Journal of Research , pp. 1-11, 2017.
- [169] M.S. AbouOmar, Y.X. Su, and H. Zhang, "Hybrid Feedback-Feedforward Fuzzy Control of PEM Fuel Cell Air Feed System with Electromagnetic Field Optimization", IETE Journal of Research, pp. 1-17, 2019.
- [170] K. Jouili, N. B. Braiek, "A gradient descent control for output tracking of a class of non-minimum phase nonlinear systems", Journal of Applied Research and Technology, pp. 383-395, 2016.
- [171] I.A. Shkolnikov, and Y.B. Shtessel, "Tracking in a class of non-minimum-phase systems within on linear internal dynamics via sliding mode control using method of system center", Automatica 38, pp. 837-842, 2002.
- [172] X. Geng, S. Hao, T. Liu, C. Zhong, "Generalized predictor based active disturbance rejection control for non-minimum phase systems", ISA Transactions, pp.1-12, 2018.
- [173] S. Zhoh, M. K. Helwa, and A. P. Schoellig, "An Inversion Based Learning Approach for Improving Impromptu Trajectory Tracking of Robots with Non-Minimum Phase Dynamics", IEEE Robotics and Automation, vol. 3, no.3, pp. 1663-1670, 2018.
- [174] R. A. Alphians, H. H. Hansen, T. Tambo, "Comparison of Conventional Closed Loop Controller with an Adaptive Controller for a Disturbed Thermodynamic System", Evolving and adaptive Intelligent systems (EAIS), Slovenia, pp. 1-8, May 2017.
- [175] K. Sharma, and R. Pradhan, "Design of A Novel Feed- Forward Control Strategy for A Non-minimum Phase System", ICMAEM-2017, IOP Conf. Series: Materials Science and Engineering, pp.1-7, 2017.

- [176] O. Kaneko, Y. Yamashina, and S. Yamamoto, "Fictitious Reference Tuning of the Feed- Forward in a Two-Degree-of-Freedom Control System", SICE Journal of Control, Measurement, and System Integration, vol. 4, no. 1, pp. 055-062, 2021.
- [177] S. R. Danaboyina, S. K. Mangipudi, and R. R. Manyala, "Two-Degree-of-Freedom Robust Controller Design Approach for Fuzzy Parametric Uncertain Systems using Particle Swarm Optimization", IETE Journal of Research, pp. 1-14, 2018.
- [178] G. Nandinipriyanka, S. Ishwarya, S. Janakiraman, C.T. Sekar, and P.Vaishali, "Design of Model Reference Adaptive Controller for Cylinder Tank System", International Journal of Pure and Applied Mathematics, vol. 118, pp. 2007-2013, 2018.
- [179] A. Tepljakov, B. B. Alagoz, E. Gonzalez, E. Petlenkov, and C. yeroglu, "Model Reference Adaptive Control Scheme for Retuning Method Based Fractional Order PID Control with Disturbance Rejection Applied to Closed Loop Control of a Magnetic Levitation System," Journal of Circuits, Systems, and Computers, vol. 27, no. 11, pp. 1-29, 2018.
- [180] X. Zuo, J. W. Liu, X. Wang, and H-q Liang, "Adaptive PID and Model Reference Adaptive Control Switch Controller for Nonlinear Hydraulic Actuator", Hindwaj Mathematical Problems in Engineering, pp.1-15, 2017.
- [181] D. Giribabu, S.P. Srivastava, and M.K. Pathak, "Modified Reference Model for Rotor Flux-Based MRAS Speed Observer Using Neural Network Controller", IETE Journal of Research, pp. 1-17, 2017.
- [182] S. Akhter, and D.S. Bernstein, "Lyapunov stable discrete time model reference adaptive control", International Journal of Adaptive Control and Signal Processing, vol. 19, pp. 745-767, 2005.
- [183] B. B. Alagoz, G. Kavuran, A. Atesh, and C. Yeroglu, "Reference-shaping Adaptive control by using gradient descent optimizers", PLOS ONE, pp. 1-20, November2017.
- [184] N. Nguyen, K. E. Hashemi, T. Yucelen, and E. Arabi, "Output Feedback Adaptive Control of Non-minimum Phase Systems Using optimal Control Modification", AIAA Guidance, Navigation, and Control Conference, pp. 1-26, 2018.
- [185] R.C. Klein, "Analog Simulation of Sampled Data Systems", IRE Transaction— Telemetry And RemoteComtrol, pp. 2-7, May 1955.

- [186] Λ Robert. Z Heart, General Electric Company, "Analog simulation techniques", IEEE spectrum, pp. 221-230, March1964.
- [187] S. Tofighi, F. Merrikh-Bayat, "A benchmark system to investigate the non-minimum phase behaviour of multi-input multi-output systems", Journal of Control and Dicision, pp.1-18, 2017.
- [188] A. Sikander, and R. Prasad, "A New Technique for Reduced-Order Modeling of Linear Time-Invariant System", IETE Journal of Research, PP. 1-9, 2016.
- [189] P. Jain, and M. J. Nigam, "Design of a Model Reference Adaptive Controller Using Modified MIT Rule for a Second Order System", Advanced Electronics and Electric Engineering, vol.3, no.4, pp. 477-484, 2013.
- [190] M P R V Rao, "New Design of Model Reference Adaptive Control Systems", Journal of Applied Mechanical Engineering, vol. 3, no.1, pp.1-3, 2014.
- [191] V.Kumar, E.J.Jerome, and S. Ayyppan, "Archives of Control Sciences", vol. 23 (LIX), no. 2, pp. 131-144, 2013.

## APPENDIX



Fig A.1. Realistic Op-amp based Simulink model of NMP system

0---0



Fig A. 2. Realistic Op-amp based Simulink model of inversed NMP system



Fig A.3. Realistic Op-amp based Simulink model of superimposed inversed NMP and non- inversed NMP System



Fig. A.4 Realistic Op-amp based Simulink model of cascaded inversed NMP and noninversed NMP system



Fig. A.5 Realistic Op-amp based Simulink model of reference model plant for MRAC structure



Fig A. 6 Realistic Op-amp based Simulink model of SFB controlled Op-amp based NMP

system



Fig. A.7 Realistic Op-amp based Simulink model of 2DOF controlled Op-amp based NMP system



Fig. A.8 Hardware experimental setup for 2DOF controlled NMP system