Abstract

Title: $k-$SMOOTHNESS OF BOUNDED LINEAR OPERATORS AND RELATED TOPICS

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The study of $k-$smoothness of operators between Banach spaces is relatively new area of research in the geometry of Banach spaces. The study of $k-$smoothness and Birkhoff-James orthogonality plays an important role in the geometry of Banach spaces. One of the most interesting aspects of Birkhoff-James orthogonality is the relation between orthogonality of operators and that of norm attainment set in the ground space. We first characterize $k-$smoothness of an element on the unit sphere of a finite-dimensional polyhedral Banach space and k-smoothness of an operator $T \in \mathbb{L}(\ell_\infty^n, \mathbb{Y})$, where $\mathbb{Y}$ is a two-dimensional Banach space with the additional condition that T attains norm at each extreme point of the unit ball $B_{\ell_\infty^n}$. Then we characterize $k-$smoothness of an operator defined between ℓ_∞^3 and ℓ_1^3. Next we study $k-$smoothness of bounded linear operators defined between infinite-dimensional Hilbert spaces. We also characterize $k-$smoothness of operators on some particular spaces, namely $\mathbb{L}(\mathbb{X}, \ell_\infty^n)$, $\mathbb{L}(\ell_\infty^3, \mathbb{Y})$, where $\mathbb{X}$ is a finite-dimensional Banach space and $\mathbb{Y}$ is a two-dimensional Banach space. Study of $k-$smoothness is deeply related to extreme contractions, the characterization of which is still elusive, in the general setting of Banach spaces. As an application of the study of k-smoothness of operators, we characterize extreme contractions defined between ℓ_∞^3 and $\mathbb{Y}$, where $\mathbb{Y}$ is a two-dimensional polygonal Banach space. Then we obtain extreme contractions defined between finite dimensional polyhedral Banach spaces using $k-$smoothness of operators. We explicitly compute the number of extreme contractions in some special Banach spaces. Next we explore the connections between the numerical radius norm and operator norm under certain conditions. We characterize nu-smoothness of order k for a bounded linear operator defined on a finite-dimensional Banach space. Also we characterize nu-extreme contractions defined on two-dimensional polygonal Banach spaces. Next we obtain the structure of the set of extreme points in the dual of $\mathbb{L}(\mathbb{X})_w$, where $\mathbb{X}$ is a two-dimensional polygonal Banach space. Then we move our attention to the study of Birkhoff-James orthogonality of bounded linear operators. We explore the relation between the orthogonality of bounded linear operators in the space of operators and that of elements in the ground space. We continue exploring the validity of the BŠ (Bhatia-Šemrl) Property in the setting of different Banach spaces. We characterize the space ℓ_∞^3 among all 3-dimensional polyhedral Banach spaces whose unit ball have exactly eight extreme points.

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