## A STUDY ON SOME ISSUES OF OPERATION AND PLANNING OF DISTRIBUTED GENERATION IN ELECTRIC POWER DISTRIBUTION SYSTEM

**Thesis Submitted** 

By

### SAMA SUDHAKAR REDDY

**Doctor of Philosophy (Engineering)** 

Department of Electrical Engineering Faculty Council of Engineering & Technology Jadavpur University Kolkata, India

2022

### A STUDY ON SOME ISSUES OF OPERATION AND PLANNING OF DISTRIBUTED GENERATION IN ELECTRIC POWER DISTRIBUTION SYSTEM

**Thesis Submitted** 

By SAMA SUDHAKAR REDDY

**Doctor of Philosophy (Engineering)** 

Department of Electrical Engineering Faculty Council of Engineering & Technology Jadavpur University Kolkata, India

2022

# JADAVPUR UNIVERSITY KOLKATA-700 032, INDIA

**INDEX NO. : 221/13/E** 

1. Title of the Thesis:

A Study on some issues of Operation

and Planning of Distributed Generation in Electric Power Distribution System 2. Name, Designation & Institution of the Supervisors:

### **Prof. (Dr.) Subrata Paul**

Professor,

Department of Electrical Engineering,

Jadavpur University, Kolkata,

India-700032

Email: speejupow@yahoo.co.in

## **Prof. (Dr.) Sunita Halder nee Dey**

Professor,

Department of Electrical Engineering,

Jadavpur University, Kolkata,

India-700032

Email: sunitaju@yahoo.com

### 3. List of Publications:

### • International Journals

 [1] Sudhakar Reddy Sama, Subrata Paul, and Sunita Halder Nee Dey, "Elimination of Partial Overloading of Generators Under Unbalanced Operating Conditions of Power Systems," CSEE Journal of Power and Energy Systems, IEEE, vol.2, no.1, pp. 81-87, Mar. 2016.
 DOI: 10.17775/CSEEJPES.2016.00012
 Indexed by SCIE/IEEE Explore Digital Library

### • International conferences

- S. Sudhakar Reddy, Subrata Paul, and S. Halder Nee Dey, "Optimal size and location of Distributed Generation and KVAR support in unbalanced 3-Φ Distribution system using PSO," in Proceeding of IEEE International Conference (ICETEEM-2012), pp. 77-83, Dec. 2012.
   DOI: 10.1109/ICETEEEM.2012.6494447
- [2] Sudhakar Reddy Sama, Subrata Paul, and Sunita Halder Nee Dey, "An improved 3-Phase load flow for DG integrated distribution system based on PSO," in Proceeding of IEEE International Conference (ICEFEET-2020), pp.1-6, Jul. 2020.
   DOI: 10.1109/ICEFEET49149.2020.9186975

4. List of Patents:

NIL

#### 5. List of Presentations in National/International/Conference/Workshop:

- S. Sudhakar Reddy, Subrata Paul, and S. Halder Nee Dey, "Optimal size and location of Distributed Generation and KVAR support in unbalanced 3-Φ Distribution system using PSO," in Proceeding of IEEE International Conference (ICETEEM-2012), pp. 77-83, Dec. 2012.
   DOI: 10.1109/ICETEEEM.2012.6494447
- [2] Sudhakar Reddy Sama, Subrata Paul, and Sunita Halder Nee Dey, "An improved 3-Phase load flow for DG integrated distribution system based on PSO," in Proceeding of IEEE International Conference (ICEFEET-2020), pp.1-6, Jul. 2020.
   DOI: 10.1109/ICEFEET49149.2020.9186975

### STATEMENT OF ORIGINALITY

I Sama Sudhakar Reddy registered on 23<sup>rd</sup> October, 2013 do hereby declare that this thesis entitled "A Study on some issues of Operation and Planning of Distributed Generation in Electric Power Distribution System" contains literature survey and original research work done by the undersigned candidate as part of Doctoral studies.

All information in this thesis have been obtained and presented in accordance with existing academic rules and ethical conduct. I declare that, as required by these rules and conduct, I have fully cited and referred all materials and results that are not original to this work.

I also declare that I have checked this thesis as per the "Policy on Anti Plagiarism, Jadavpur University, 2019", and the level of similarity as checked by iThenticate software is 4 %.

Signature of Candidate:

- Stal 1/ 25/3/22

Sama Sudhakar Reddy Date : 25/03/2022 *Certified by Supervisor:* 

Soutant Par 25/3/22

**Prof. (Dr.) Subrata Paul** *Professor, Department of Electrical Engi, Jadavpur University,* 

Sumla Halder 25/3/22

**Prof. (Dr.) Sunita Halder nee Dey** *Professor, Department of Electrical Engg., Jadavpur University,* 

### **CERTIFICATE FROM THE SUPERVISOR**

This is to certify that the thesis entitled "A Study on some issues of Operation and Planning of Distributed Generation in Electric Power Distribution System" submitted by Mr. Sama Sudhakar Reddy, who got his name registered on 23<sup>rd</sup> October, 2013 for the award of Ph.D. (Engg.) degree of Jadavpur University is under the supervision absolutely based upon his own work of Prof. Subrata Paul and Prof. Sunita Halder nee Dey, and that neither his thesis nor any part of the thesis has been submitted for any degree / diploma or any other academic award anywhere before

Suburt Park 25/3/22

Prof. Subrata Paul Supervisor Department of Electrical Engineering Jadavpur University Kolkata, India-700032

Sunta Halder 25/3/22

-----

Prof. Sunita Halder nee Dey Supervisor Department of Electrical Engineering Jadavpur University Kolkata, India-700032

#### ACKNOWLEDGEMENT

I feel it as a great honour to express my sincere thanks and gratitude to the esteemed people without whom this research work would not have been possible.

Firstly, I would like to express my sincere thanks and the deepest gratitude to my supervisor Prof. Subrata Paul for his passionate guidance, continuous support, useful advices and encouragement throughout my Ph.D. study. I express my sincere thanks for his unforgettable support during the struggling period of my research. Thank you so much sir.

Secondly, I would like to express my sincere thanks and the deepest gratitude to my supervisor Prof. Sunita Halder nee Dey without whom, I would not have been completed this research work. Her patience guidance, motivation and relentless encouragement helped me to make a walk towards the target. Her thorough review process has helped me a lot to improve the quality and scientific vision of research papers and the thesis. Thank you so much madam.

I would like to express my sincere thanks to All India Council for Technical Education (AICTE), New Delhi and Department of Technical Education, Andhra Pradesh for their financial support during the research period.

I take the privilege of thanking Prof. Saswati Mazumdar, Head of the Electrical Engineering Department, and also I extend my deep sense of gratitude to Prof. S.K. Goswami, Prof. D. Chatterjee, Prof. Sudipta Debnath, Mr. Ayan K. Tudu, Mrs. Madhumita Mandal and all the staff members of Electrical Engineering Department for their moral support during the research work.

I also take the opportunity to thank all the fellow researchers of Electrical Engineering Department for their cooperation during the active research period.

Finally, I my sincere thanks and deep gratitude goes to my family members who stood behind me and extended their unconditional and endless support, and encouragement throughout the period of my research work.

March, 2022

(Sama Sudhakar Reddy)

Jadavpur University, Kolkata.

Dedicated to

my Wife

Kavitha Reddy,

my Mother

Smt. Devakamma

and

my loving Sons

Rithvik Reddy & Sathvik Reddy

## LIST OF ABBREVIATIONS AND ACRONYMS

Abbreviation	Description
DG	Distributed Generation.
N-R	Newton Raphson
BIBC	Bus injected Branch Currents Matrix
BCBV	Branch Currents to Bus Voltages Matrix
DLF	Direct Load Flow Matrix
DALFA	Direct Approach Load Flow Analysis
PSO	Particle Swarm Optimization
Max	Maximum
kW	Kilo Watts
MW	Mega Watts
kVAR	Kilo Volt Ampere Reactive
MVAR	Mega Volt Ampere Reactive
INT	Generator Internal Node
URDN	Unbalanced Radial Distribution Network
p.u.	Per Unit
р	Subscripts $p$ is the phase indicator.
k	Super scripts $k$ is the iteration count
3	Tolerance
R	Generator internal resistance per phase
Х	Generator internal reactance per phase
α	Super scripts $\alpha$ is the phase indicators.
cr	Superscript cr indicates quantities associated with
	critical phase.
i	Subscript <i>i</i> indicates quantities associated with $i^{th}$ bus.
j	Subscript $j$ indicates quantities associated with $j$ <sup>th</sup> bus.
sp	Subscript sp indicates specified quantities.

Ε	Generator internal voltage.
$\Delta E$	Generator internal voltage mismatch.
G	Subscript $G$ indicates quantities associated with
	generator.
G	Conductance.
В	Susceptance.
8	Internal conductance of the generator.
b	Internal susceptance of the generator.
J	Jacobian matrix.
$\Delta M$	Mismatch vector.
N-R	Newton Raphson method.
n	Indicates total number of buses.
Р	Active power.
$\Delta P$	Active power mismatch.
Q	Reactive power.
$\Delta Q$	Reactive power mismatch.
V	Voltage magnitude.
$\Delta V$	Voltage mismatch.
Ι	Current magnitude
$\Delta X$	Correction vector.
$\theta$	Phase angle.
$\Delta \theta$	Phase angle mismatch.
δ	Generator internal phase angle.
$\Delta\delta$	Generator internal phase angle mismatch.

### List of Figures

Chapter 2
-----------

Fig.2.1	Flow chart for three phase load flow using PSO	20
Chapter 3		
Fig.3.1	The computational flowchart	38
Chapter 4		
Fig.4.1	Flow chart of the proposed methodology	58
Appendice	25	
Fig. A-1	Line diagram of IEEE 13 Node Test Feeder	92
Fig. A-2	Line diagram of 25 Node URDN Feeder	93
Fig. A-3	Line diagram of IEEE 37 Node Test Feeder	94

## Table of Contents

CHAI	PTER 1 INTRODUCTION,	1
1.1	Distributed Generation and its Role in Electric Power System	2
1.2	Different Types of DG	3
1.3	Modeling of DG in Load Flow Study	3
1.4	Three-phase Distribution Load Flow	4
1.5	Scope of work undertaken in this Thesis	9
CHAI	PTER 2 A PSO BASED 3-PHASE LOAD FLOW FOR DG INTEGRATED DISTRIBUTION SYSTEM	14
2.1	Introduction	14
2.2	The Proposed Methodology	15
2.3	The Flowchart	19
2.4	Test Results	21
2.5	Chapter Summery	30
CHAPTER 3 A MODIFIED APPROACH FOR INCLUSION OF		
	LOAD FLOW	31
3.1	Introduction	32
3.2	The Proposed Modifications	32
3.2.1	Proposed modifications for P-Q model of DG	32
3.2.2	Proposed modifications for P-V model of DG	35
3.2.3	The Flowchart	37
3.3	Case Studies	39
3.4	Chapter Summery	47

CHAF	TER 4 ELIMINATION OF PARTIAL OVERLOADING OF GENERATORS UNDER UNBALANCED OPERATING CONDITION OF POWER SYSTEMS	DNS
<i>A</i> 1	Introduction	49 50
7.1		50
4.2	Brief Review of N-R 3-Phase Load Flow	50
4.3	The Proposed Methodology	56
4.3.1	The Flowchart	57
4.4	Case studies	58
4.5	Chapter Summery	64
CHAF	PTER 5 ELIMINTION OF PARTIAL OVER LOADING IN DIRE	СТ
	APPROACH LOAD FLOW	65
5.1	Introduction	66
5.2	The Proposed Methodology	66
5.3	Case studies	67
5.5	Chapter Summery	70
CHAP	PTER 6 OPTIMAL SIZE AND LOCATION OF DG FOR MINIMIZATION OF NETWORK POWER LOSS	71
6.1	Introduction	72
6.2	Problem Formulation	72
6.2.1	The objective function	72
6.2.2	The Constraints	73
6.3	Solution methodology adopted	73
6.4	Case studies and results	74
6.5	Chapter Summery	82
CHAF	PTER 7 CONCLUSIONS AND FURTHER SCOPE OF STUDIES	83
7.1	Conclusions	84
7.2	Future scope of studies	85

APPE	NDICES	86
A-I	A Brief Introduction to Direct Approach load flow	87
A-II	DG Active Power Correction	89
A-III	A Brief Introduction to PSO Algorithm	91
A-IV	Line diagrams of the three phase Networks	92
A-IV( a	a) IEEE 13 Node Test Feeder	93
A-IV (l	b) 25 bus Unbalanced Radial Distribution Network (URDN)	93
A-IV <b>{</b>	c) IEEE 37 Node Test Feeder	94
BIBLI	OGRAPHY	95

# CHAPTER 1

## Introduction

### Outline of the chapter

- 1.1 Distributed Generation and its Role in Electric Power System
- 1.2 Different Types of DG
- 1.3 Modeling of DG in Load Flow Study
- 1.4 Three-phase Distribution Load Flow
- 1.5 Scope of work undertaken in this Thesis

#### 1.1 Distributed Generation and its Role in Electric Power System

Over the last two decades, the share of distributed generation (DG) in power systems has been gradually increasing. Now-a-days, DG (also known as dispersed generation or embedded generation) has become an important and integral part of power system. According to the definitions found in the available literature [1]-[3] DG ideally can be defined as generating units of small capacities (typically ranging from less than a kW to tens of MW) customarily installed at the consumers' site or in the distribution network. Introduction of DG in distribution system can provide a number of benefits [1]-[5] which can provide answers to the problems being faced by traditional power systems. In traditional power systems, power is usually generated in large, centralized power stations which are normally located far away from the consumers. The bulk amount of power generated in these power plants require to be transmitted through transmission networks to suitable load centers and then to be distributed among the consumers by distribution networks. Ever-increasing demand of electric power necessitates time to time expansion of generation and transmission capacity. This requires installation of new generating stations or capacity enhancement of the existing ones along with installation of new transmission lines for transmission capacity enhancement to evacuate the increased generated power. Majority of the large conventional power plants are fossil-fuel fired plants. Enhancement of fossil-fuel fired generation has been facing serious challenges due to two reasons. Firstly, the fossil reserve is alarmingly diminishing, and, in addition to that, these plants emits excessive amount of pollutant gases. Growing public awareness in environmental pollution and demand for pollution free atmosphere has resulted in severe restrictions on growth of this type of power plants. Moreover, transmission network expansion is also facing problem due to strong public objection against land acquisition by power sector to find right of way for installation of new lines. The, ever-increasing demand of electric power without sufficient transmission and generation enhancement adversely affects the reliability, security and power quality of a power system. DG can provide answers to all these problems. As DG units are placed directly in the distribution network, therefore close to the consumers, their deployment contributes to reduction in network power loss. In addition it improves the voltage profile, enhances the system reliability and security, improves power quality and relieves transmission and distribution congestion. It also gives

opportunity for deferral of new investments and thereby provides the most costeffective measure in power industry to enhance loading capability. These benefits along with liberalization of electricity market have been the major driving force behind the rapid advancement in DG technology and its application. Electricity market liberalization policy has opened up scopes for the privately owned small generation companies to participate in the market. In addition to that, significant advancement in renewable energy technology such as PV solar, wind turbine etc. and availability of these technologies for development of small capacity generating units has provided huge impetus for these private companies to come forward to play important roles in today's power system.

### 1.2 Different Types of DG

Some types of DG units employ synchronous generators while more widely used versions are based on various renewable resources as such units do not emit pollutant gases and therefore are environment friendly. Diesel generator, microturbine and gas turbine are examples of synchronous generator base DG. These units are directly connected to the grid. Solar Photovoltaic (PV), mini/micro hydro, wind turbine, fuel cell, geothermal and biomass are examples of renewable based DG units. These units require power electronic inverters for their connection to the power network. Majority of the renewable based DG units use 3-phase inverters for this purpose.

### 1.3 Modeling of DG in Load Flow Study

As DG has become an important and integral part of electric power generation, proper modeling of the DG units and integrating those models in distribution load flow algorithm have become extremely necessary. Modeling of DGs for 3-phase distribution load flow have been discussed in [6]-[10] in details. DG units are operated in one of the following modes:

- i) P-Q mode: Fixed (specified) real and reactive power outputs (i.e. fixed active power output at a specified power factor).
- ii) P-V mode: Fixed (specified) active power output at a specified terminal voltage.

In the context of load flow study, the DG nodes in the first case can be represented as P-Q nodes, whereas, in the second case, they are required to be represented as P-V nodes. The total 3-phase active and reactive power of a DG unit are specified when it is set to operate in P-Q mode. In case the DG unit is operated in P-V mode, total 3-phase active power, along with the positive sequence terminal voltage or the average of the terminal voltage magnitudes of the three phases are specified.

#### **1.4 Three-phase Distribution Load Flow**

Three-phase load flow study is essential in analysis, operation and planning of 3-phase power systems under unbalanced condition. Several 3-phase load flow methods based on traditional Newton-Raphson [11]-[12], Fast Decoupled [13] and sequence component [14]-[17] techniques have been reported in the literature. The loading of a distribution network is usually unbalanced due to the presence of a number of unequal single-phase loads connected to different phases. In addition to that, distribution networks generally possess special network structure. These systems are usually radial or weakly meshed, and possess high R/X ratio. These networks are inherently unbalanced due to their multiphase nature and nonsymmetrical spacing of the conductors of 3-phase line segments. Because of these factors, the traditional methods based on Newton-Raphson (N-R) or Fast decoupled formulation, which are suitable for the general meshed topology of a typical transmission network, generally encounter convergence difficulties for distribution network. The sequence component based methods, on the other hand, are not applicable for distribution systems because of unbalanced nature of these networks. Three-phase load flow programs capable of taking into consideration the above mentioned characteristics are therefore necessary for distribution systems.

Several 3-phase load flow methods, specifically designed for distribution system analysis, have been developed and reported in the literature[18]-[46]. The proposed methods can be broadly divided into the following categories:

- Newton-Raphson [N-R] based methods (including Fast Decoupled version).
- Gauss based methods.
- Direct iterative methods.

The conventional N-R or fast decoupled methods often fail to converge for distribution networks for the reasons already mentioned above. In addition to that a common shortcoming of these methods is that the Jacobian matrix is required to be updated in each iteration. Moreover, the Jacobian can not be decoupled because of high R/X ratio of the distribution lines. This necessitates modifications in the N-R and fast decoupled methods to make these methods suitable for solving load flow for radial networks. Some attempts have been reported [18] to overcome these difficulties.

A fast decoupled methodology has been reported [19] for unbalanced radial distribution networks. This method utilizes the radial structure of these networks to reduce the number of equations and unknowns, and the decoupling numerical property to reduce the amount of computation. In addition, the Jacobian matrix is approximated by a constant triangular matrix. All these factors make the proposed method significantly faster than the traditional N-R and fast decoupled method. However, the authors have not considered the case of P-V bus in their study. In addition to that, this methods orders the 'lateral' instead of the 'nodes' into layers to reduce the problem size. However, such ordering makes the method efficient for a given topology, but it may add some overhead if the topology of the network is changed frequently, which is quite common in case of distribution networks because of switching operation.

A three phase fast decoupled load flow for distribution systems is reported in [20] which also suggests some modifications in the conventional method to achieve faster solution and good convergence property.

A modified Newton method has been proposed in [21] for load flow analysis of radial distribution system. In this methodology the Jacobian matrix is expressed in the form of  $UDU^{T}$  which has identical topology to that of the nodal admittance matrix. U is a constant upper triangular matrix depending solely on system topology and D is a block diagonal matrix resulting from the radial structure and special characteristics of distribution networks. Only the elements of D are required to be updated in each iteration. With this formulation, LU factorization and forward/backward substitution is replaced by backward/forward sweeps on radial feeders with equivalent

impedances. Thus the possibility of occurrence of an ill-condition associated with the Jacobian matrix and its LU factors is avoided. The method is capable of achieving of robust convergence and high efficiency. The proposed method is aimed for single phase radial system. Also presence of DGs has not been considered. However, the authors have given some suggestions on possible steps for extending the method for systems with loops (meshed network), to include DGs (both P-Q and P-V model), and for unbalanced 3-phases systems.

An N-R based 3-phase load flow formulation is presented in [22] where the Jacobian matrix is presented in complex form, but some simplifications are introduced by neglecting the component resulting from voltage changes.

In [23] a new 3-phase N-R load flow formulation based on current injection has been proposed. The current injection equations are written in rectangular coordinate and bus admittance matrix  $Y_{bus}$  of order 6n (n is the number of buses in the system) consists of n number of 6×6 blocks. The off-diagonal elements are identical to the corresponding elements of  $Y_{bus}$ . Only a few elements of 6×6 diagonal blocks require to be updated in each iteration. For systems having no voltage specified bus except for the slack bus (Grid substation bus), only one-third elements of the diagonal blocks of the Jacobian matrix require to be recalculated. The elements of the off-diagonal blocks remain constant. When P-V buses are considered, in addition to the block diagonal elements corresponding P-Q buses, the half of the non-zero elements of the columns corresponding to the P-V buses are to be recalculated in each iteration.

In the above approaches, equations for all the three phases are solved in a coupled manner. This requires handling of large matrices, and therefore, large amount of computations are required for solving the 3-phase flow. To overcome this obstacle, a phase-decoupled load flow using branch currents in rectangular for as state variables and an N-R algorithm is proposed in [24]. In this approach, a constant Jacobian matrix is used to improve the execution speed of the load flow program.

Among the Gauss based methods, the most well-known and widely used approach is the implicit  $Z_{bus}$  Gauss method [25]. This approach uses the sparse bifactored  $Y_{bus}$  matrix and current injection model to solve the network equations and exhibits a good convergence rate. The solution method is optimally ordered triangular factorization  $Y_{bus}$  method (implicit  $Z_{bus}$  Gauss method) which not only takes advantages of the high sparsity of the distribution network equations, but also has a very good convergence characteristics for distribution load flow problems. The convergence rate is highly dependent on the number of P-V buses present in the system. If the only voltage specified bus is the slack bus, the rate of convergence is comparable to the N-R method.

In the above methods the full  $Y_{bus}$  matrix is required to be factorized into lower and upper triangular matrices, which is quite time consuming particularly for largescale distribution networks. In [26], a solution method is proposed in which, the LU factorization, Gaussian elimination, inverse of the full  $Y_{bus}$  matrix or Jacobian matrix, and complicated  $Z_{bus}$  building process can be avoided. By using the corresponding incidence matrix, the load flow can be solve without the need for the complex processes, still all advantages of the implicit  $Z_{bus}$  Gauss approach can be achieved. However, DG has not been considered in the study.

In [27], the authors have proposed a 3-phase load flow methodology in which each phase is modeled in a decoupled (i.e., phase decoupled) way and, therefore, each phase can be solved separately and independently. The solution method is implicit  $Z_{bus}$  Gauss with implicit factorization of the sparse  $Y_{bus}$  matrix for each phase. The approach significantly reduces the amount of computation. This approach exhibits vary good accuracy and convergence property.

In recent years, the direct iterative methods [26]-[39] have become the most popular load flow methods for distribution systems. These methods are specific to radial structure of distribution networks and takes maximum advantage of the special topology of these networks. One of the most commonly used technique uses the backward/forward sweep based iterative algorithms. In [28], the iterative algorithm consists of three steps, which are nodal current calculation, backward sweep to get the line section currents and forward sweep to update nodal voltages and power flow have been used different authors [29], [30] to improve the efficiency of the backward/forward sweep based load flow technique. In [31], the authors have applied a fact decoupled Newton update for backward sweep. The ladder solution process uses many sub-iterations on the laterals. Sophisticated numbering and ordering along with improved search technique have been used by some authors [32] for improvement of backward/forward load flow method. A compensation based technique with break point impedance matrix is proposed in [33] to make the sweep based method applicable to weakly meshed networks. In the first step, the radial portion of the network is solved using usual backward/forward sweep technique. Then, in the second step, the meshes are solved using nodal current injection compensation. Authors in [34] have presented an efficient formation of the compensation matrix and handling of P-V nodes. The method described in [33] is developed on a single phase model. In [35], the authors have extended the method of [33] from its single phase version to 3-phase one. The idea of breakpoint voltage compensation in employed successfully to eliminate the voltage mismatches at P-V nodes, thus making the method capable for handling DGs in P-V mode of operation.

In [36], the authors have presented a 3-phase unbalanced load flow algorithm with the choice of modeling DG as P-Q or P-V node. The method is based on backward/forward sweep technique and applicable for radial structure. This program is capable of handling multiple source nodes. The authors in [37] have introduced some revisions in the work reported in [36] to extend it for shipboard power system.

In some approaches, the authors have applied the direct iterative method in conjunction with basic graph theory and inject current technique to develop more systematic approach and to achieve better convergence, efficiency and reliability. Two such very similar approaches are reported in [38] and [39]. In [38], a direct approach for 3-phase distribution load flow solution has been proposed based on the bus-injection to bus-current (BIBC) matrix and the branch-current to bus-voltage (BCBV) matrix which can be formed simply from observations, to obtain the load flow solution, only the BIBC matrix, the BCBV matrix, and a simple matrix manipulation are necessary. As a result, the time consuming LU factorization and forward/backward sweep substitution will no longer necessary. The treatment of weakly meshed networks are also included in the study. However, treatments for P-V nodes have not been included in this study. However, the inclusion of P-V models of DG in this load flow formulation has been presented in [8]. The authors in [39] have used the branch frame of reference to apply direct iterative method. Because of the radial topology of distribution networks, the corresponding branch impedance matrix  $Z_{BR}$  is identical to the primitive impedance matrix and, therefore,  $Z_{BR}$  can be obtained directly as identical to the primitive impedance matrix and thus, the time consuming

 $Z_{BR}$  building process is avoided. By the proposed methodology, only the branch-path incidence matrix is required which can be obtained directly from the network topology. With the current injection compensation technique, the proposed method can be easily extended to weakly meshed networks. However, no consideration for inclusion of P-V nodes has been given in this study.

The above survey is not being claimed to be an exhaustive one on this subject. Many more papers in this area are available in the literature. However, the methodologies used in those papers are primarily similar to those discussed above, with some variations and modifications introduced as attempts for achieving various improvements. Some of the significant documents of these studies are available in [40]-[46].

### 1.5 Scope of work undertaken in this Thesis

The goal of this thesis is to perform studies on some issues related to proper planning and operation of DGs in distribution systems.

Load flow analysis is one of the most valuable part of planning and operation of power systems. Because of the reasons mentioned earlier in section 1.1, distribution networks are inherently unbalanced, and these systems usually operate at relatively high degree of unbalanced loading. This necessitates application of 3-phase load flow analysis for appropriate studies and investigations. To study the impacts of DGs on distribution system operation, DG models are required to be included in the load flow programs. A brief literature review on the available methods of 3-phase distribution load flow has been presented in section 1.4. There, in those studies, the authors have used the P-Q model and the P-V model of DGs as described in section 1.3, and accordingly, the DG-connected buses are represented as P-Q buses or as P-V buses (voltage specified buses) depending on the mode of operation of the DG.

All the load flow methods mentioned in section 1.4 require the equivalent current injection to be calculated for each phase of every node in the network. This, in turn, requires the values of P and Q be known for each phase of every node. For load buses, this is not a problem because these values are specified, and hence known for those buses. But for generator buses (DG-connected buses), the situation is different. The output P and Q of the individual phases of a 3-phase DG unit can neither be

controlled nor be specified separately. Only total 3-phase values of these quantities can be specified for such units. For a DG unit set to operate in P-Q mode, total 3phase values of P and Q are specified, and when it operates in P-V mode, total 3phase value of P along with the terminal voltage magnitude (average of the three phases or the positive sequence value) are specified. For the analysis of balanced operation it does not pose any difficulty as the total 3-phase output of a DG unit is known to be divided equally (1/3 of the 3-phase value) between its three phases. But, for analysis of unbalanced operation, this assumption is not valid because under such circumstance, the output power from the three phases of a DG unit are more likely to be unequal though their summation should be equal to the specified total 3-phase value. So, necessary steps are required to be taken in the solution process so that P and Q output of a DG unit can be properly distributed among its three phases. In the available load flow methods, any measure for inclusion of this factor in the solution process is not evident.

Another factor for concern is that, without some extension of the DG model and some additional computational steps, the presently available load flow solutions with the existing DG models can not ensure a balanced 3-phase internal voltage for the DG units. Such balance in the internal voltage is an essential operational requirement which must be fulfilled by a 3-phase load flow solution. If the DG unit is a synchronous machine, this requirement arises due to its design and the voltage control method adopted. For inverter based DG units, such requirement arises where symmetrical 3-phase switching is employed for the inverters.

In [9], the authors have presented a current injection model for co-generators based on Norton's equivalent. This model is inherently capable of maintaining balance in the internal voltage of the generators. But application of this model has not been found in the existing load flow methods. Moreover, this model is capable of handling P-Q type DGs only. No consideration for handling DGs operated in P-V mode has been presented in this study.

In this thesis, two different methods of load flow have been proposed for 3phase distribution systems to alleviate the above two problems. Both the methods apply an extended DG model that includes its internal impedance and internal node in addition to the terminal nodes usually represented as P-Q node or P-V node. Accordingly the network dimension is increased to incorporate the DG internal node.

In chapter 2, a Particle Swarm Optimization (PSO) [56] based method is presented in which the PSO loop is used to take necessary corrective actions for achieving internal voltage balance. In every PSO loop, direct approach load flow is solved with the input received from the PSO loop. The proposed method is capable of handling both P-Q model and P-V model of DG with provision for switching from P-V mode to P-Q mode when necessary.

In chapter 3, a modified approach is presented to include the DG models in the direct approach load flow program. In this approach, the iteration of the direct approach load flow starts by assigning a balanced voltage to the internal node along with P and Q to each phase of that node. In every iteration, the node voltages (including the DG internal nodes) are updated followed by corrections of P and Q at the internal nodes. The iteration criteria are modified accordingly. Like the previous method, this method also can handle both P-Q model and P-V model of DG with provision for switching from P-V mode to P-Q mode if necessary.

Both the methodologies presented in chapter 2 and chapter 3 are quite general and can be implemented with any of the direct iterative load flow methods mentioned in section 1.4.

Chapter 4 and chapter 5 deal with the problem of 'partial overloading' of generators in power systems. Partial overloading of a generator is a condition in which one or two of the three phases of the generator get overloaded even if the total 3-phase power output of the generator is within its rated capacity. In this context, per phase capacity is taken as 1/3 of the total 3-phase capacity of the generator. Power systems are normally designed based on the assumption of balanced operating conditions. However, in real situations, power systems are often subjected to unbalanced operating conditions. Such conditions in a power system may give rise to partial overloading of its generators. Partial overloading of generators beyond certain allowable limits is undesirable and must be avoided. This problem is of more serious concern for distribution systems as these systems frequently operate with high degree of unbalance in loading. Introduction of DGs into today's distribution systems has made these systems highly susceptible to the problem of partial overloading. Proper

attention to this issue is required in analysis, operation and planning of these systems. No evidence of such study is available in the existing literature.

In chapter 4, a load flow technique based on N-R algorithm has been proposed to deal with the problem. Necessary modifications are introduced in the N-R algorithm to impose constraint on the maximum loading limit of individual phases of the generators (DGs). However, as discussed earlier, N-R method is not reliable for distribution load flow solution. It is therefore necessary to develop distribution load flow program capable of taking into consideration the constraint on the maximum allowable per phase loading. Chapter 5 presents such methodology based on the load flow technique proposed in chapter 3.

It has already been mentioned in section 1.1 that introduction of DG in distribution system can provide several benefits. But to derive those benefits to the maximum extent, proper planning for DG location and size is necessary. Inappropriate selection of DG size and location may lead to deterioration of system performance. Reduction in network power loss is one of the most desirable benefits that can be achieved from proper DG planning. The problem of network power loss minimization by proper selection of DG size and location has been addressed by a number of researchers. Some significant among those studies can be found in [47]-[55]. In these studies, different optimization methods have been used to find optimal size and location of DG that minimizes network power loss. Different constraints such as network power balance, maximum DG size, maximum and minimum voltage limit etc. have been considered. But these studies do not consider partial overloading for finding the optimal solution. In chapter 6, an investigation has been done on the effect of considering the partial overloading constraints on the solution of the problem of DG sizing and sitting for distribution network loss minimization. PSO has been employed to find the minimum-loss solution whereas the load flow technique presented in chapter 5 has been applied to obtain the network power loss for different size and location of DG.

In chapter 7, a conclusion has been drawn on the findings from the investigations carried out under this thesis. A brief note on further scope of studies is also included.

The direct approach load flow has been repeatedly used in the studies performed in this thesis. Therefore, a brief introduction of it has been given in Appendix-I. Appendix-II presents a derivation of the expression for DG active power correction used in chapter 3. A brief introduction to PSO algorithm is furnished in Appendix-III as a ready reference to chapter 2 and chapter 6. Three different benchmark distribution networks- IEEE 13 node Test feeder, 25 node URDN Feeder, and IEEE 37 node Test feeder have been used to present the results of the studies performed in this thesis. The line diagram of the three networks are given in Appendix-IV while the network data and load data are furnished respectively in [58], [59] and [60].

## CHAPTER 2

# A PSO Based 3-Phase Load Flow for DG Integrated Distribution System

Outline of the chapter

- 2.1 Introduction
- 2.2 The Proposed Methodology
- 2.3 The Flowchart
- 2.4 Test Results
- 2.5 Chapter Summery

#### **2.1 Introduction**

As mentioned in section 1.4, the existing methods of incorporating DG models in 3-phase distribution load flow suffer from limitations as these method can neither properly take into account the unequal loading of the three phases of the DG units under unbalanced operating condition nor can ensure the balance in the 3-phase internal voltage of the generators. This chapter presents a PSO based methodology for the solution of 3-phase load flow for distribution system incorporating DG units. Direct approach load flow (Appendix-I) has been coordinated with PSO (Appendix-III) to mitigate the above mentioned problems.

#### 2.2 The Proposed Methodology

The proposed 3-phase load flow methodology consists of two computation loops - an inner loop embedded in an outer loop. These two loops work in the following manner:

The inner loop: The inner loop receives multiple set of input data from the outer loop, solves load flow with each set, and sends the results back to the outer loop. Each set of input data consists of active power (P) and reactive power (Q) for the DG units modeled as P-Q nodes, and active power (P) and terminal voltage magnitude (|V|) for the DG units modeled as P-V nodes for the three phases of each DG unit. The values of P and Q for the three phases of each load bus are specified, and hence, known. The inner loop uses the direct approach method (Appendix-I) to solve the load flow in the following manner:

For  $k^{\text{th}}$  iteration of the inner loop,

$$\left[\Delta V\right]^{k} = \left[DLF\right]\left[I\right]^{k} \tag{2.1}$$

and

$$\begin{bmatrix} V \end{bmatrix}^k = \begin{bmatrix} V_0 \end{bmatrix} - \begin{bmatrix} \Delta V \end{bmatrix}^k \tag{2.2}$$

where, [V] is the node voltage vector.

 $[V_0]$  is a vector whose every element is equal to the grid (substation) bus voltage.

[*I*] is the node injected current vector.

$$[DLF] = [BCBV][BIBC]$$

[BCBV] and [BIBC] are defined in Appendix-I.

The injected current for each of the three phases of the  $i^{\text{th}}$  node can be obtained in  $k^{\text{th}}$  iteration as follows.

$$I_{i,p}^{k} = \left(\frac{P_{i,p}^{j} + jQ_{i,p}^{j}}{V_{i,p}^{k-1}}\right)^{*}$$
(2.3)

where, p=a, b, c indicates the phase.

For the load buses,  $P_{i,p}^{j}$  and  $Q_{i,p}^{j}$  are specified, whereas, these values for DG units modeled as P-Q nodes are set by the outer loop in its  $j^{\text{th}}$  iteration, and supplied to the inner loop to solve the load flow. For DG units modeled as P-V nodes, the values of  $P_{i,p}^{j}$  and  $|V_{i,p}^{j}|$  are set by the outer loop. However, to calculate  $I_{i,p}^{k}$ ,  $Q_{i,p}^{j}$  is required. Therefore, for the P-V nodes,  $Q_{i,p}^{j}$  is required to be calculated in every iteration of the inner loop to obtain  $I_{i,p}^{k}$ . Denoting the value of  $Q_{i,p}^{j}$  in the  $k^{\text{th}}$  iteration by  $Q_{i,p}^{j,k}$ , in the first iteration of the inner loop (k = 1),  $Q_{i,p}^{j,k}$  is set equal to zero for each phase of those DG units, or, can be set to any arbitrarily chosen values. The starting voltages for the P-Q buses are taken as balanced 1.0 p.u. For the P-V buses, balanced voltages with magnitude  $|V_{i,p}^{j}|$  are taken. At the end of each iteration, the values of  $Q_{i,p}^{j}$  are updated (to be used in the next iteration) as

$$Q_{i,p}^{j,k+1} = Q_{i,p}^{j,k} + \Delta Q_{i,p}^{j,k}$$
(2.4)

where,  $\Delta Q_{i,p}^{j,k} = V_{i,p}^{mis} \left( 2 \left[ X_g \right] \right)^{-1}$  is the required correction [8].

$$V_{i,p}^{mus} = \left( \left| V_{i,p}^{J} \right| \right) - \left( \left| V_{i,p}^{J,\kappa} \right| \right)$$
$$\left[ X_{g} \right] = \operatorname{Imag} \left( \operatorname{diag} \left[ DLF_{ii} \right] \right)$$

 $DLF_{ii}$  is the (3×3) element of DLF corresponding to the *i*<sup>th</sup> bus.  $V_{i,p}^{j,k}$  is the latest value of  $V_{i,p}$  calculated using equations (2.1), (2.2) and (2.3).

The inner loop iteration continues until  $\Delta V_{max} = Max \{ \|V_{i,p}^{j}\| - \|V_{i,p}^{j,k}\| \} \le \varepsilon$ ,  $\forall i \in g_v$ , where  $g_v$  is the set of P-V nodes, and at the same time, convergence in the values of

all other node voltages is obtained according to the criteria,  $|V_{i,p}^k - V_{i,p}^{k-1}| \le \varepsilon$ . Here,  $\varepsilon$  is a pre-specified tolerance. The final values of the node voltages for every set of input data are passed on to the outer loop.

The outer loop: The outer loop, in each iteration, runs a PSO program (Appendix-III) to set the values of the elements of each particle of the swarm. In this problem, a particle represents a set of input data for the load flow to be executed by the inner loop, and consists of the following elements:

$$\begin{array}{ll} P_{i,p} & ; & \forall i \in g \\ Q_{i,p} & ; & \forall i \in g_p \subset g \\ \left| V \right|_{i,p} & ; & \forall i \in g_v \subset g \end{array}$$

where, g is the set of nodes connected to DG units,

 $g_p$  is the set of nodes connected to DG units operating in P-Q mode,

 $g_v$  is the set of P-V nodes.

 $g_n \bigcup g_v = g$ 

and

In each iteration of the outer loop, the values of all the particles of the swarm (for a swarm size S, S no. of such particles) are passed on to the inner loop which generates load flow solutions with each particle. In the first iteration of the outer loop, the elements of the particles are chosen as follows:

 $P_{i,p}$  and  $Q_{i,p}$  are selected randomly such that

$$P_{spi} = \sum_{p=a,b,c} P_{ip}; \quad \forall i \in g$$
(2.5)

$$Q_{spi} = \sum_{p=a,b,c} Q_{ip}; \quad \forall i \in g_p$$
(2.6)

where,  $P_{spi}$  and  $Q_{spi}$  respectively, represent the specified total 3-phase active and reactive power output of the corresponding DG unit.

 $|V_{i,p}|$ ,  $\forall i \in g_v$ , are chosen randomly so that

$$|V|_{spi} = \frac{\sum\limits_{p=a,b,c} |V_{i,p}|}{3}$$
 (2.7)

if the average of the magnitudes is specified,

and

otherwise, 
$$|V|_{spi} = \frac{1}{3} ||V_{i,a}| + a |V_{i,b}| + a^2 |V_{i,c}||$$
 (2.8)

if the positive sequence voltage is specified.

After getting back the load flow solutions from the inner loop, the outer loop first calculates the internal voltages of the DG units corresponding to every load flow solution, and updates the local and global best as per the PSO methodology. The updation of local bests and global best is done on the basis of the following objective function:

$$f = k_1 f_1 + k_2 f_2 \tag{2.9}$$

where,

$$f_{1} = \sum_{i=1}^{8} \left( \left\| E_{i,a} \right\| - \left| E_{i,b} \right\| + \left\| E_{i,b} \right| - \left| E_{i,c} \right\| \right)$$
(2.10)

$$f_2 = \sum_{i=1}^{g} \left( \left| \phi_{i,a} - \phi_{i,b} - 120^0 \right| + \left| \phi_{i,c} - \phi_{i,a} - 120^0 \right| \right)$$
(2.11)

and  $k_1, k_2$  are appropriate weighting factors ( $k_1$ =0.02,  $k_2$ =0.98).

 $f_1$  is to ensure that  $|E_{i,a}| = |E_{i,b}| = |E_{i,c}|$  and  $f_2$  is to ensure that  $\phi_{i,b} = \phi_{i,a} - 120^{\circ}$ and  $\phi_{i,c} = \phi_{i,a} + 120^{\circ}$  for each *i*; i = 1, 2, 3, ..., g. Under these desired conditions, both  $f_1$  and  $f_2$  should be equal to zero.

The internal voltages  $E_{i,a} = |E_{i,a}| \angle \phi_{i,a}$ ,  $E_{i,b} = |E_{i,b}| \angle \phi_{i,b}$  and  $E_{i,c} = |E_{i,c}| \angle \phi_{i,c}$  of the three phases of the *i*<sup>th</sup> DG unit, are calculated from the load flow results in the following manner:

$$E_{i,p} = V_{i,p} + Z_i I_{i,p}$$
(2.12)

where,  $I_{i,p} = (P_{i,p} - jQ_{i,p})/V_{i,p}^*$  is the current in each phase of the *i*<sup>th</sup> DG unit (i=1,2,3,...,g) and  $Z_i$  is the impedance per phase of the DG unit.

Thus, the minimum value that f (and hence, the global best) can attain, is zero when a balanced 3-phase internal voltage is achieved.

If the global best obtained is not within some prescribed tolerance, the elements of the particles are updated by the PSO, and passed on to the inner loop to generate new load flow results. The PSO parameters used for the updation are w=0.8,  $c_1=0.12$ ,  $c_2=1.2$ . The quantities w,  $c_1$  and  $c_2$  are defined in [57].

However, before passing the updated values of the elements to the inner loop, those values are normalized in the following manner so that the updated elements satisfy the requirement given in (2.5)-(2.8).

$$P_{i,p} = \frac{P_{i,p}}{\sum_{p} P_{i,p}} \times P_{spi}$$
(2.13)

$$Q_{i,p} = \frac{Q_{i,p}}{\sum_{p} Q_{i,p}} \times Q_{spi}$$
(2.14)

$$\left|V_{i,p}\right| = \frac{\left|V_{i,p}\right|}{\sum_{p} \left|V_{i,p}\right|} \times 3\left|V\right|_{spi}$$
(2.15)

if the average of the magnitudes is specified.

$$|V_{i,p}| = \frac{|V_{i,p}|}{||V_{i,a}| + a|V_{i,b}| + a^2 |V_{i,c}||} \times 3|V|_{spi}$$
(2.16)

if the positive sequence voltage is specified.

### 2.3 The Flowchart

The computational flowchart for the PSO based proposed load flow is shown in Fig.2.1


Fig.2.1 Flow chart for the proposed 3-phase load flow.

#### **2.4 Test Results**

The proposed methodology has been tested on IEEE 13 node distribution network, 25 node URDN and IEEE 37 node distribution network. The schematic single line diagrams of the systems are shown respectively in Fig.A-1, Fig.A-2 and Fig.A-3 in Appendix-IV. The network and load data of the systems are available respectively in [58], [59] and [60]. In this study, the voltage regulator at bus 650 in IEEE 13 node and bus 799 in IEEE 37 node distribution feeder has been excluded. The networks are augmented by connecting the internal nodes of the DG units to the respective phases of the DG terminal buses through an internal impedance of realistic value ( $Z_{int}$ = 0.01+0.5  $\Omega$ /Phase). The DG internal buses (3-phase) are denoted in Fig.A-1, Fig.A-2 and Fig.A-3 as INT. Bus no. 650, 1 and 799 are the grid-buses of the IEEE 13node, 25 node URDN and IEEE 37 feeders respectively and has been taken as the slack buses for the solution of load flow problem.

The distributed load along line 632-671 of the IEEE 13 bus network has been considered as lumped loads in the following manner [36]:

1. At one-fourth length of the line from sending node, i.e. node 632, two third of the load is connected. For this, a dummy node is created.

2. One third load is connected at the receiving node, i.e. node 671.

The following case studies have been undertaken. The detailed results are shown only for the bus voltages for Case 1, and given in Table 1 and Table 2. For other results, only the relevant parts have been shown. The phase angles have been expressed with respect to the grid bus voltages.

Case 1: A 3-phase DG unit of 1.5 MW rating is connected at bus no. 671of IEEE 13 node feeder and operated in P-V (voltage control) mode with a specified positive sequence terminal voltage magnitude of 1.0 p.u.. No Q-limit is considered.

Table 1 shows the results obtained with previously suggested approach [8], [37] for including DG model in the load flow in which the total 3-phase power output of the DG unit is assumed equally divided among the three phases of the unit. From the results it is found that the solution does not furnish a balanced internal voltage of the generating unit. Also the total 3-phase output power of the DG unit is equally shared

by the three phases which is not natural in the prevailing unbalanced operating condition.

Table 2 shows the corresponding values obtained when the proposed modification was applied. From the table it is found that the internal voltage of the DG unit is balanced. Moreover, the total 3-phase power output is unequally distributed over the three phases of the unit which is quite natural in the prevailing operating condition.

**Table 1** Results with previously suggested approach for including DG model in the load flow

Dece Ma	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	Va	V <sub>b</sub>	V <sub>c</sub>	$\theta_{\mathrm{a}}$	$\theta_{b}$	$\theta_{\rm c}$	
650	1	1	1	0	-120	120	
632	0.988	1.0134	0.9785	-2.13	-120.79	118.236	
633	0.9849	1.0115	0.9758	-2.199	-120.84	118.232	
634	0.9601	0.9926	0.9562	-2.923	-121.32	117.712	
645		1.0042	0.9765		-120.98	118.264	
646		1.0025	0.9744		-121.05	118.311	
671	0.9861	1.0455	0.9684	-4.414	-120.44	117.001	
692	0.9861	1.0455	0.9684	-4.414	-120.44	117.001	
675	0.9798	1.0477	0.9664	-4.673	-120.6	117.005	
684	0.9841		0.9663	-4.437		116.893	
611			0.9643			116.738	
652	0.9783			-4.359			
680	0.9861	1.0455	0.9684	-4.414	-120.44	117.001	
INT	1.0335	1.0901	1.0167	-2.028	-118.31	119.471	

a) The voltage brome	a)	The	voltage	profile
----------------------	----	-----	---------	---------

b) Power balance

		Active Por	wer (MW)	)	Reactive Power (MVAR)			
	Pa	$P_b$	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	0.7344	0.4609	0.8187	2.0139	0.0798	-0.1589	0.0492	-0.0298
DG Power	0.5000	0.5000	0.5000	1.5000	0.519	0.519	0.519	1.557
Total Gen.	1.2344	0.9609	1.3187	3.5139	0.5988	0.3601	0.5682	1.5272
Total load	1.2164	0.9621	1.2875	3.466	0.5406	0.3326	0.5288	1.402
Loss	0.018	-0.0012	0.0312	0.0479	0.0581	0.0274	0.0393	0.1249
Total Power consumption	1.2344	0.9609	1.3187	3.5139	0.5987	0.36	0.5681	1.5269

**Table 2** Results with the proposed modification applied for considering DG model in the load flow

Dere Ma	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	Va	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{\rm c}$	
650	1	1	1	0	-120	120	
632	0.9937	0.9929	0.9937	-1.26	-121.65	118.323	
633	0.9906	0.9909	0.991	-1.328	-121.7	118.318	
634	0.9659	0.9716	0.9717	-2.043	-122.21	117.815	
645		0.9834	0.9919		-121.84	118.346	
646		0.9817	0.9898		-121.92	118.391	
671	0.9962	1.0056	0.9982	-2.681	-122.11	117.223	
692	0.9962	1.0056	0.9982	-2.681	-122.11	117.223	
675	0.9899	1.0078	0.9963	-2.931	-122.28	117.23	
684	0.9942		0.9962	-2.705		117.122	
611			0.9942			116.977	
652	0.9885			-2.629			
680	0.9962	1.0056	0.9982	-2.681	-122.11	117.223	
INT	1.0159	1.0159	1.0159	-1.725	-121.73	118.275	

a) The voltage profile

#### b) Power balance

		Active P	ower (MV	W)	Reactive Power (MVAR)			
	Pa	$P_b$	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Q <sub>c</sub>	Total Q
Grid Power	0.637	0.725	0.645	2.006	-0.089	0.009	-0.033	-0.113
DG Power	0.598	0.246	0.656	1.500	0.663	0.356	0.594	1.613
Total Gen.	1.234	0.971	1.301	3.506	0.574	0.365	0.56	1.499
Total load	1.216	0.962	1.287	3.466	0.541	0.333	0.529	1.402
Loss	0.018	0.009	0.013	0.04	0.033	0.032	0.032	0.097
Total Power consumption	1.234	0.971	1.301	3.506	0.574	0.365	0.560	1.499

Case 2: A 3-phase DG unit of 1.5 MW and 1.2 MVAR rating is connected at bus no. 671 of IEEE 13 node feeder and operated in P-Q mode.

Table 3 shows the results obtained with previously suggested approach for including DG model in the load flow. From the results it is found that the internal voltage of the generator is not balanced and the total 3-phase output power of the DG unit is equally shared by the three phases.

Table 4 shows the corresponding values obtained when the proposed modification was applied. It is found that the internal voltage of the DG unit is

balanced and the total 3-phase power output is unequally distributed over the three phases of the unit which is quite expected in the prevailing situation.

**Table 3** Results with previously suggested approach for including DG model in the load flow

Deer Me	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	$\mathbf{V}_{\mathrm{a}}$	$V_b$	$V_{c}$	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
650	1.0000	1.0000	1.0000	0.000	-120.00	120.00	
671	0.9769	1.0368	0.9583	-4.218	-120.39	117.19	
680	0.9769	1.0368	0.9583	-4.218	-120.39	117.19	
INT	1.0142	1.0719	0.9964	-1.751	-118.19	119.75	

a)	The	voltage	profile
a)	THC	vonage	prome

		Active Power (MW)				Reactive Power (MVAR)			
	Pa	$P_b$	Pc	Total P	Qa	$Q_b$	Qc	Total Q	
Grid Power	0.7317	0.4618	0.8196	2.0132	0.1993	-0.0465	0.17	0.3228	
DG Power	0.5000	0.5000	0.5000	1.5000	0.400	0.400	0.400	1.200	
Total Gen.	1.2317	0.9618	1.3196	3.5132	0.5993	0.3535	0.57	1.5228	
Total load	1.2164	0.9621	1.2875	3.466	0.5406	0.3326	0.5288	1.402	
Loss	0.0153	-0.0003	0.0321	0.0472	0.0586	0.0208	0.0411	0.1205	
Total Power consumption	1.2317	0.9618	1.3196	3.5132	0.5991	0.3534	0.5699	1.5225	

### b) Power balance

**Table 4**Results with the proposed modification applied for considering DG in theload flow

# a) The voltage profile

	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	Va	$V_b$	V <sub>c</sub>	$\theta_{\mathrm{a}}$	$\theta_{\rm b}$	$\theta_{\rm c}$	
650	1.0000	1.0000	1.0000	0.00	-120.00	12000	
671	0.9962	1.0056	0.9982	-2.68	-122.11	117.22	
680	0.9962	1.0056	0.9982	-2.68	-122.11	117.22	
INT	1.0159	1.0159	1.0159	-1.73	-121.73	118.28	

#### b) Power balance

	Active Power (MW)				Reactive Power (MVAR)			
	Pa	$P_b$	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	0.637	0.725	0.645	2.006	-0.089	0.009	-0.033	-0.113
DG Power	0.598	0.246	0.656	1.5	0.663	0.356	0.594	1.613
Total Gen.	1.234	0.971	1.301	3.506	0.574	0.365	0.56	1.499
Total load	1.216	0.962	1.287	3.466	0.541	0.333	0.529	1.402
Loss	0.018	0.009	0.013	0.04	0.033	0.032	0.032	0.097
Total Power consumption	1.234	0.971	1.301	3.506	0.574	0.365	0.56	1.499

Case 3: A 3-phase DG unit of 1000 kW rating is connected at bus no. 12 of the 25 node URDN feeder and operated in P-V mode. A voltage regulation scheme is assumed to maintain the average magnitude of the terminal voltages of the three phases at a specified value of 1.0 p.u..

Table 5 shows the results obtained with previously suggested approach for including DG model in the load flow and Table 6 shows the corresponding values obtained when the proposed modification was applied. The results are now self-explanatory.

**Table 5** Results with previously suggested approach for including DG model in the load flow

Due Me	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	$V_{a}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{\rm c}$	
1	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
12	1.0000	1.0000	1.0000	-1.55	-121.50	118.68	
25	0.9653	0.9676	0.9719	-0.72	-120.59	119.22	
INT	1.0385	1.0402	1.0371	0.00	-119.95	120.24	

a) The voltage	profile
----------------	---------

		Active Po	wer (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	767.31	777.91	771.67	2316.89	387.5	373.77	410.12	1171.38
DG Power	333.33	333.33	333.33	1000.00	433.14	453.07	416.63	1302.84
Total Gen.	1100.65	1111.25	1105	3316.89	820.64	826.85	826.74	2474.22
Total load	1073.3	1083.3	1083.3	3239.9	792.0	801.0	800.0	2393.0
Loss	27.35	27.95	21.7	76.99	28.64	25.85	26.74	81.22
Total Power consumption	1100.65	1111.25	1105	3316.89	820.64	826.85	826.74	2474.22

b) Power balance

**Table 6** Results with the proposed modification applied for considering DG model in

 the load flow

Due Ne	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	$\mathbf{V}_{\mathrm{a}}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{\rm c}$	
1	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
12	1.0004	0.9990	1.0006	-1.49	-121.47	118.57	
25	0.9653	0.9673	0.9721	-0.70	-120.59	119.20	
INT	1.0386	1.0386	1.0386	0.09	-119.91	120.09	

a) The voltage profile

# b) Power balance

		Active Po	wer (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	761.65	776.93	777.71	2316.29	389.96	381.18	398.95	1170.10
DG Power	338.58	334.04	327.39	1000.00	430.26	445.45	428.00	1303.71
Total Gen.	1100.23	1110.96	1105.10	3316.29	820.22	826.63	826.95	2473.81
Total load	1073.30	1083.30	1083.30	3239.90	792.00	801.00	800.00	2393.00
Loss	26.93	27.66	21.80	76.39	28.22	25.63	26.95	80.81
Total Power consumption	1100.23	1110.96	1105.10	3316.29	820.22	826.63	826.95	2473.81

Case 4: Two 3-phase DG units rated at 800 kW & 600kW and the average terminal voltages (average of the voltages of the three phases) specified at 0.98 p.u. & 0.99 p.u. are connected at bus nos. 10 & 24 of 25 node URDN feeder.

Table 7 shows the results obtained with previously suggested approach for including DG model in the load flow whereas Table 8 shows the corresponding values

obtained when the proposed modification was applied. The results are now self-explanatory.

**Table 7** Results with previously suggested approach for including DG model in the load flow

Decement	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degree)			
Bus no.	Va	$V_b$	$V_{c}$	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
1	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
10	0.9797	0.9784	0.9819	-0.66	-120.53	119.32	
24	0.9886	0.9889	0.9924	-0.76	-120.70	119.24	
INT1	1.0030	1.0017	1.0051	0.66	-119.20	120.63	
INT2	1.0111	1.0114	1.0149	0.21	-119.73	120.20	

# a) The voltage profile

		Active Po	Reactive Power (kVAR)					
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	622.43	633.44	630.04	1885.91	302.31	310.24	310.20	922.74
DG1 Power	266.67	266.67	266.67	800.00	254.85	254.82	254.90	764.56
DG2 Power	200.00	200.00	200.00	600.00	251.02	251.02	251.08	753.12
Total Gen.	1089.10	1100.11	1096.71	3285.91	808.18	816.08	816.18	2440.42
Total load	1073.30	1083.30	1083.30	3239.90	792.00	801.00	800.00	2393.00
Loss	15.80	16.80	13.41	46.01	16.17	15.08	16.18	47.43
Total Power consumption	1089.10	1100.10	1096.71	3285.91	808.17	816.08	816.18	2440.43

#### b) Power balance

**Table 8** Results with the proposed modification applied for considering DG model in

 the load flow

# a) The voltage profile

Bus No.	Voltage magnitude (p.u.)			Voltage Phase angle (Degrees)			
	V <sub>a</sub>	V <sub>b</sub>	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
1	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
10	0.9801	0.9790	0.9809	-0.65	-120.60	119.38	
24	0.9894	0.9894	0.9912	-0.76	-120.72	119.28	
INT1	1.0033	1.0033	1.0033	0.70	-119.30	120.70	
INT2	1.0125	1.0125	1.0125	0.23	-119.77	120.23	

		Active Po	wer (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	611.74	641.02	633.18	1885.95	296.53	292.48	332.87	921.88
DG1 Power	271.39	261.83	266.78	800	253.52	265.74	245.59	764.85
DG2 Power	205.74	197.26	197	600	257.85	257.79	237.92	753.56
Total Gen.	1088.88	1100.11	1096.96	3285.95	807.9	816.01	816.38	2440.29
Total load	1073.3	1083.3	1083.3	3239.9	792	801	800	2393
Loss	15.58	16.81	13.66	46.05	15.9	15.01	16.38	47.29
Total Power consumption	1088.88	1100.11	1096.96	3285.95	807.9	816.01	816.38	2440.29

b) Power balance

Case 5: A 3-phase DG unit of 1000 kW is connected at bus no. 12 of 25 node URDN feeder and operated in P-V mode with the max Q-limit of 650 kVAR and an average voltage specified at 1.0 p.u.

Table 9 shows the results with the proposed modified approach. From the results it is found that as Q-limit exceeds, the unit is run in P-Q mode with Q value set at 650 kVAR. It can now be found in the Table that the terminal voltage (average of the three phases) of the generator can no longer be maintained at the specified value of 1.0 p.u. and its magnitude is now 0.9834 p.u. The balance in the internal voltage is maintained in this case.

**Table 9** Results with the proposed modification applied for considering DG model in

 the load flow

Deep Ma	Volta	ge magnitude	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	$V_{a}$	$V_b$	$V_{c}$	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
1	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
12	0.9832	0.9825	0.9844	-0.23	-120.18	119.78	
25	0.9613	0.9637	0.9683	-0.55	-120.43	119.33	
INT	1.0035	1.0035	1.0035	1.45	-118.55	121.45	

ar the voltage brothe
-----------------------

		Active Po	Reactive Power (kVAR)					
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	761.84	783.18	767.62	2312.64	603.34	600.52	623.20	1827.06
DG Power	337.17	326.55	336.28	1000.00	217.99	226.92	205.09	650.00
Total Gen.	1099.01	1109.73	1103.90	3312.64	821.33	827.44	828.29	2477.06
Total load	1073.30	1083.30	1083.30	3239.90	792.00	801.00	800.00	2393.00
Loss	25.71	26.43	20.60	72.74	29.33	26.44	28.29	84.06
Total Power consumption	1099.01	1109.73	1103.90	3312.64	821.33	827.44	828.29	2477.06

b) Power balance

Case 6: A 3-phase DG unit of 1350 kW rating is connected at bus no. 708 of IEEE 37 bus network. A voltage regulation scheme is assumed to maintain the average magnitude of the terminal voltages of the three phases at a specified value of 1.0 p.u. (P-V mode of operation). Results with the proposed modification applied for considering DG model in the load flow have been presented in Table 10.

**Table 10** Results with the proposed modification applied for considering DG model

 in the load flow

Due Me	Volta	age Magnitude (	(p.u.)	Voltage Phase angle (Degrees)			
Bus No.	Va	$V_b$	$V_{c}$	$\theta_{a}$	$\theta_{b}$	$\theta_{\rm c}$	
INT	1.0170	1.0170	1.0170	-28.57	-148.57	91.43	
799	1.0000	1.0000	1.0000	-30.00	-150.00	90.00	
708	0.9970	1.0010	1.0030	-30.27	-149.99	89.65	

a) The voltage profile

b)	Power	generation
$\overline{\mathbf{v}}_{j}$	1000	Semeration

		Active P	ower (kW)	Reactive Power (kVAR			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc
Grid Power	395.37	277.19	443.40	1115.95	252.54	127.07	89.31
DG Power	466.87	394.09	489.05	1350.00	298.17	233.75	202.70
Total Gen.	862.23	671.28	932.44	2465.95	550.71	360.81	292.01

### 2.5 Chapter Summary

This chapter has proposed a PSO based methodology for 3-phase distribution load flow with DG units present in the system to eliminate the limitations of the existing methods as mentioned in section 1.4. The results of application of the proposed load flow on benchmark distribution systems have been presented. The results validate the claim made by the present researcher.

# CHAPTER 3

# A Modified Approach for Inclusion of Distributed Generator Models in 3-phase Distribution Load Flow

Outline of the chapter

- 3.1 Introduction
- 3.2 The Proposed Modifications
- 3.2.1 Proposed modifications for P-Q model of DG
- 3.2.2 Proposed modifications for P-V model of DG
- 3.2.3 The Flowchart
- 3.3 Case Studies
- 3.4 Chapter Summery

#### **3.1 Introduction**

In this chapter a new methodology is proposed for inclusion of DG models in 3-phase distribution load flow to eliminate the limitations (section 1.4) of the existing methods of incorporating DG models in distribution load flow. The proposed methodology utilizes the direct approach load flow method, a brief description of which is presented in Appendix-I. The direct approach load flow is originally formulated for conventional distribution networks where only load buses are present. The quantities  $P_{i,p}$  and  $Q_{i,p}$  in equation (A-5) of Appendix-I, are specified for the load buses. But, with DG units present in the network, the situation will not be the same because output of 3-phase DG units cannot be specified for individual phases under unbalanced operating condition. Modifications have been proposed in this work for both P-Q model and P-V model of DG. In this context, it may be noted that the convergence criteria are required for DG buses. The criteria to be used for PQ model and PV model are presented. The load flow iteration continues until all the convergence criteria are satisfied simultaneously.

#### 3.2 The Proposed Modifications

The formulation of the problem requires that the network be augmented by connecting the DG internal nodes to the respective terminal nodes through respective internal impedances. The sizes of [B], [I], [V],  $[V_0]$ , [BCBV] and [BIBC], defined in Appendix-I, therefore, need to be augmented accordingly to include the internal nodes.

#### 3.2.1 Proposed modifications for P-Q model of DG

In the proposed approach, instead of considering the DG output as the injected power at its terminal node, the internal power generated in the DG unit is assigned as injected power at its internal node. The DG terminal node is treated as a load node with the connected load as the injected power at that node. If no load is present, the injected power is taken as zero. The iteration of the direct approach load flow is started with allocating the specified 3-phase active power and reactive power equally to each phase of the DG internal node.

Thus, for the  $1^{st}$  iteration (k=1),

$$P_{G,p}^k = P_{sp}$$
 and  $Q_{G,p}^k = Q_{sp}$ 

where, p = a, b, c indicates the phase.

k is the iteration number.

 $P_{G,p}^{k}$  and  $Q_{G,p}^{k}$  indicate, respectively, the active and reactive power at the different phases of the DG internal node at the start of  $k^{th}$  iteration.

$$P_{sp} = P_{3sp} / 3$$
 and  $Q_{sp} = Q_{3sp} / 3$ 

 $P_{3sp}$  and  $Q_{3sp}$  are the total 3-phase active and reactive power specified for the DG unit.

In every iteration, after updating the node voltages (including the internal node) as described in Appendix-I, the following convergence criteria are to be checked:

(i) 
$$\underset{p}{Max}\left\{\left\|E_{p}^{k+1}\right\| - \left|E_{pos}^{k+1}\right\|\right\} \le \varepsilon_{1}$$
  
(ii) 
$$Max\left\{A, B\right\} \le \varepsilon_{2}$$
  
(iii) 
$$Max\left\{C, D\right\} \le \varepsilon_{3}$$

where,  $\mathcal{E}_1$ ,  $\mathcal{E}_2$  and  $\mathcal{E}_3$  are the respective tolerances.

 $|E_p^{k+1}|$  is the magnitude of the updated internal voltage and  $|E_{pos}^{k+1}|$  is that of the corresponding positive sequence voltage.

$$A = \left| \delta_a^{k+1} - \delta_b^{k+1} - 120 \right| \text{, and } B = \left| \delta_b^{k+1} - \delta_c^{k+1} - 120 \right|.$$

 $\delta_p^{k+1}$  indicates the angle of the updated internal voltage of different phases.

$$C = \left| P_{3sp} - P_{3g}^{k+1} \right|$$
 and  $D = \left| Q_{3sp} - Q_{3g}^{k+1} \right|$ .

 $P_{3g}^{k+1} = \sum_{p} P_{g,p}^{k+1}$  and  $Q_{3g}^{k+1} = \sum_{p} Q_{g,p}^{k+1}$  are respectively the total 3-phase active and reactive power output of the DG, and  $P_{g,p}^{k+1}$  and  $Q_{g,p}^{k+1}$  indicate respectively the active and reactive power output of different phases of the DG, obtained with the updated values of its terminal voltage  $V_{g,p}^{k+1}$  in the following manner:

$$P_{g,p}^{k+1} = \operatorname{Re}\left\{ \left( V_{g,p}^{k+1} \right)^* I_{G,p}^k \right\}$$
(3.1a)

$$Q_{g,p}^{k+1} = \operatorname{Im}\left\{ \left( V_{g,p}^{k+1} \right)^* I_{G,p}^k \right\}$$
(3.1b)

where,  $I_{G,p}^{k}$  is the generator current per phase.

Criteria (i) and (ii) are checked to achieve a balanced internal voltage, and (iii) is used to ensure that the specified 3-phase output,  $P_{3sp}$  and  $Q_{3sp}$ , are obtained at the generator terminal.

Before going to the next iteration (if necessary), the values of active power and reactive power at the internal node are updated as follows to include the effect of internal active and reactive power losses of the DG unit, and also to include the correction terms necessary to obtain a balance in the 3-phase internal node voltages.

$$P_{G,p}^{k+1} = P_{g,p}^{k+1} + C / 3 + R \left( I_{G,p}^{k} \right)^{2} + \Delta P_{G,p}^{k+1}$$
(3.2)

$$Q_{G,p}^{k+1} = Q_{g,p}^{k+1} + D/3 + X \left( I_{G,p}^{k} \right)^2 + \Delta Q_{G,p}^{k+1}$$
(3.3)

where, *R* is the generator internal resistance per phase.

*X* is the generator internal reactance per phase.

 $\Delta P_{G,p}^{k+1}$  and  $\Delta Q_{G,p}^{k+1}$  are additional correction terms.

 $\Delta P_{G,p}^{k+1}$  is obtained from the angle mismatch of internal voltage using equation (A-11) of Appendix-II as follows:

$$\Delta P_{G,p}^{k+1} = \overline{P}_p^{k+1} \Delta \delta_p^{k+1} \tag{3.4}$$

where,

 $\Delta \delta_p^{k+1} = \left(\delta_{pos}^{k+1} - \delta_p^{k+1}\right)$ 

 $\delta_{pos}^{k+1}$  is angle of the positive sequence component of the updated internal voltage.

 $\overline{P}_{p}^{k+1}$  is the synchronizing power co-efficient.

 $\Delta Q_{G,p}^{k+1}$  is calculated from the mismatch in the magnitudes of the internal voltages as follows [8]:

$$\Delta Q_{G,p}^{k+1} = \Delta E_p^{k+1} \left( 2X_e \right)^{-1}$$
(3.5)

where,

$$\Delta E_p^{k+1} = \left| E_{pos}^{k+1} \right|^2 - \left| E_p^{k+1} \right|^2.$$

 $X_e$  is formed by retaining only the imaginary part of the diagonal terms of the *DLF* matrix corresponding to the internal node.

#### 3.2.2 Proposed modifications for P-V model of DG

For a DG unit operated in P-V mode, 3-phase active power and either the magnitude of the positive sequence terminal voltage or the average magnitude of the three phase voltages at the terminal are required to be specified. In the present work, the specified active power is allocated equally to each phase of the DG internal node at the start of load flow iteration. A starting voltage equal to the specified voltage may be taken. An initial guess for the reactive power (allocated equally to three phases of the internal node) is required to start the iteration. Any suitable arbitrary value may be chosen for the initial guess. However, if any reactive power limit is specified then the arbitrary value chosen should be within that limit.

In each iteration, after the node voltages are updated, it is checked whether  $Q_{3g}^{k+1}$  is within the specified limit, i.e.,  $Q_{\min} \leq Q_{3g}^{k+1} \leq Q_{\max}$ , where  $Q_{\min}$  and  $Q_{\max}$  are the minimum and maximum limit respectively. This step is, however, required only when such limit exists.

If  $Q_{3g}^{k+1}$  is within the specified limit or if no such limit is specified, the following convergence criteria are checked:

- (i)  $Max \left\{ Max_{p} \left\| E_{p}^{k+1} \right\| \left| E_{pos}^{k+1} \right\|, \left\| V_{g}^{k+1} \right\| V_{sp} \right\| \right\} \leq \varepsilon_{1}$
- (ii)  $Max\{A, B\} \leq \varepsilon_2$
- (iii)  $\left| P_{sp} P_{3g}^{k+1} \right| \leq \varepsilon_3$

 $V_{sp}$  is the specified terminal voltage magnitude.  $V_g^{k+1}$  is the magnitude of the positive sequence component of the updated terminal voltage when positive sequence voltage is specified. When the average of the voltage magnitude is specified,  $V_g^{k+1}$  is the average of the magnitudes of the three phase voltages at the terminal. All the remaining quantities in the above three criteria are defined in Section 3.2.1.

Before going to the next iteration, the active power and reactive power at the internal node are updated. The updation of active power takes place exactly in similar manner as it is done for P-Q model, i.e., using equation (3.2) and (3.4). The updation of reactive power is done with the help of two correction terms as given in equation (3.6).

$$Q_{G,p}^{k+1} = Q_{G,p}^{k} + \Delta Q_{g,p}^{k+1} + \Delta Q_{G,p}^{k+1}$$
(3.6)

 $\Delta Q_{g,p}^{k+1}$  is the correction term required to compensate for the mismatch between the magnitudes of the updated terminal voltage and the specified voltage, and is given by

$$\Delta Q_{g,p}^{k+1} = \Delta V^k \left( 2X_g \right)^{-1} \tag{3.7}$$

where,  $\Delta V^{k} = V_{sp}^{2} - \left|V_{g}^{k+1}\right|^{2}$  is the mismatch voltage.

 $X_g$  is formed by taking the imaginary part of the diagonal terms of the DLF matrix corresponding to generator terminal node.

 $\Delta Q_{G,p}^{k+1}$  can be obtained from equation (3.5).

If  $Q_{3g}^{k+1}$  is not within the specified limit, the step for convergence checking is skipped, and  $P_{G,p}^{k+1}$  and  $Q_{G,p}^{k+1}$  are determined for the next iteration.  $P_{G,p}^{k+1}$  is calculated with the help of equations (3.2) and (3.4)as before, but equation (3.5), (3.6) and (3.7)are not used to calculate  $Q_{G,p}^{k+1}$  in this case. Instead,  $Q_{G,p}^{k+1}$  is set equal to  $Q_{lim}/3$ , where  $Q_{lim} = Q_{min}$  or  $Q_{max}$  depending on the limit violated. In the next and subsequent iterations, the DG unit is treated as a DG in P-Q mode with  $Q_{3sp}$  set equal to  $Q_{lim}$ , but with a difference. In this case, an additional step has to be introduced in every iteration for checking whether  $|V_g^{k+1}| > V_{sp}$ , if maximum Q-limit ( $Q_{max}$ ) violation had taken place or whether  $|V_g^{k+1}| < V_{sp}$ , if minimum Q-limit ( $Q_{min}$ ) violation had taken place. If either of the two conditions occur, then, from the next iteration, the DG is again converted into P-V mode.

#### 3.2.3 The flowchart

The computational flowchart for the load flow incorporating the proposed DG models is presented in Fig.3.1. All the symbols used in the flowchart have been defined in Section 3.2.1 and 3.2.2.



Fig.3.1. The computational flowchart.

#### 3.3 Case Studies

The developed methodology has been tested on IEEE 13 node distribution network, 25 node URDN and IEEE 37 node distribution network. The schematic single line diagrams of the systems are shown respectively in Fig.A-1, Fig.A-2 and Fig.A-3 in Appendix-IV. The network and load data of the systems are available respectively in [58], [59] and [60]. The voltage regulator at bus 650 in IEEE 13 node and bus 799 in IEEE 37 node distribution feeder has not been considered in the study. The networks are augmented by connecting the DG internal nodes to the respective phases of the generator terminal buses through the internal impedances (i.e.  $Z_{int}$ = 0.01+0.5  $\Omega$ /Phase). A realistic value for  $Z_{int}$  has been chosen. The DG internal buses (3-phase) are denoted in Fig.A-1, Fig. A-2 and Fig.A-3 as INT. Bus no. 650, 1 and 799 are the grid-buses of the IEEE 13 node, 25 node URDN and IEEE 37 feeders respectively and has been taken as the slack buses in the load flow studies. The distributed load along line 632-671 of the IEEE 13 bus network has been considered as lumped loads in the following manner [36]:

1. At one-fourth length of the line from sending node, i.e. node 632, two third of the load is connected. For this, a dummy node is created.

2. One third load is connected at the receiving node, i.e. node 671.

The following case studies have been done. The detailed results are shown only for the voltage profile for Case 1 and given in Table 1 and Table 2. For other results, only the relevant parts have been shown. The phase angles have been expressed with respect to the grid bus voltages.

Case 1: A 3-phase DG unit with its internal node designated as INT, is connected at bus no. 671 of IEEE 13 node network. It is operated in P-Q mode with a specified power output of 1890 kW and 915 kVAR.

Table 1 shows the results of load flow executed with previously reported method [8], [37] for including DG models in the study. From the results it is found that the active and reactive power outputs of the generating unit are same for all three phases which is unrealistic in unbalanced operating condition. Moreover, balance in the internal voltage of the DG unit is not obtained in the solution.

Table 2 shows the corresponding values obtained when the proposed modification was applied. From the table it is found that the DG output power (both active and reactive) is unevenly distributed over the three phases of the unit while internal voltage of the DG unit is balanced.

**Table 1** Results with previously suggested approach for considering DG in the load

 flow

Due No	Volta	ge Magnitude	(p.u.)	Volta	ige Phase Angle	(Degree)
DUS INO.	Va	V <sub>b</sub>	V <sub>c</sub>	$\theta_a$	$\theta_{b}$	$\theta_{\rm c}$
650	1.0000	1.0000	1.0000	0.00	-120.00	120.00
632	0.9826	1.0058	0.9720	-1.64	-120.47	118.73
633	0.9795	1.0038	0.9692	-1.71	-120.51	118.72
634	0.9545	0.9848	0.9495	-2.44	-121.00	118.20
645		0.9965	0.9700		-120.65	118.76
646		0.9948	0.9679		-120.73	118.80
671	0.9747	1.0305	0.9550	-3.45	-119.78	118.00
692	0.9747	1.0305	0.9550	-3.45	-119.78	118.00
675	0.9683	1.0327	0.9530	-3.71	-119.95	118.00
684	0.9726		0.9529	-3.47		117.89
611			0.9509			117.73
652	0.9668			-3.39		
680	0.9747	1.0305	0.9550	-3.45	-119.78	118.00
INT	1.0303	1.0827	1.0119	1.58	-115.26	123.22

a) The voltage profile

# b) Power balance

		Active Po	ower (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	595.97	335.73	682.79	1614.48	279.80	42.61	254.89	577.30
DG Power	630.00	630.00	630.00	1890.00	305.00	305.00	305.00	915.00
Total load	1216.41	962.11	1287.48	3466.00	540.57	332.60	528.82	1402.00
Loss	9.56	3.62	25.30	38.48	44.22	15.01	31.07	90.30

**Table 2** Results with the proposed modification applied for considering DG in the load flow

Due No	Volta	ge Magnitude	(p.u.)	Voltage Phase Angle (Degree)			
Dus Ino.	Va	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
650	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
632	0.9869	0.9856	0.9864	-0.80	-121.22	118.78	
633	0.9837	0.9836	0.9837	-0.87	-121.27	118.77	
634	0.9589	0.9641	0.9643	-1.60	-121.79	118.26	
645		0.9761	0.9845		-121.42	118.80	
646		0.9743	0.9825		-121.50	118.85	
671	0.9860	0.9864	0.9849	-1.04	-121.23	118.57	
692	0.9823	0.9909	0.9833	-1.78	-121.25	118.13	
675	0.9759	0.9932	0.9814	-2.03	-121.43	118.14	
684	0.9803		0.9813	-1.80		118.03	
611			0.9793			117.88	
652	0.9745			-1.72			
680	0.9823	0.9909	0.9833	-1.78	-121.25	118.13	
INT	0.9950	0.9950	0.9950	-0.58	-120.58	119.42	

a) The voltage profile

### b) Power balance

		Active Po	ower (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	$Q_b$	Qc	Total Q
Grid Power	513.6	566.47	526.72	1606.8	153.28	223.78	175.65	399.43
DG Power	716.11	403.83	770.06	1890	410.58	130.62	373.8	915
Total load	1216.41	962.11	1287.48	3466	540.57	332.6	528.82	1401.99
Loss	13.31	8.2	9.3	30.8	23.29	21.8	20.63	65.72

Case 2: The DG unit considered in Case 1 is operated in P-V mode with specified power output of 1890 kW and a specified positive sequence terminal voltage magnitude of 1.0 p.u.. No Q-limit is considered.

Table 3 shows the results with previously suggested approach for considering DG in the load flow, whereas, Table 4 shows the same obtained with the proposed modified approach. The results are now self-explanatory.

Table 3 Results with previously suggested approach for considering DG in the load flow

Bus No	Volta	ge Magnitude	(p.u.)	Voltage Phase Angle (Degree)			
DUS NO.	$\mathbf{V}_{\mathrm{a}}$	$V_b$	$V_{c}$	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
650	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
671	0.9885	1.0429	0.9697	-3.74	-119.85	117.70	
INT	1.0333	1.0848	1.0151	-0.72	-117.13	120.83	

# a) The voltage profile

# b) Power balance

		Active P	ower (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	599.12	334.06	680.9	1614.08	101.9	-121.17	77.146	57.88
DG Power	630.0	630.0	630.0	1890.0	480.99	476.91	478.67	1436.56
Total load	1216.41	962.11	1287.48	3466	540.57	332.6	528.82	1402
Loss	12.71	1.95	23.42	38.08	42.32	23.13	26.99	92.44

**Table 4** Results with the proposed modified approach for considering DG in the load

 flow

# a) The voltage profile

Bus No.	Volta	ge Magnitude	(p.u.)	Voltage Phase Angle (Degree)			
	$V_a$	V <sub>b</sub>	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
650	1	1	1	0	-120	120	
671	0.9926	1.0145	0.9931	-32.28	-151.13	87.72	
INT	1.0541	1.0541	1.0541	-29.38	-149.38	90.62	

# b) Power balance

		Active P	ower (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	508.00	516.63	578.50	1603.13	16.02	-16.75	10.45	9.73
DG Power	719.83	450.24	719.93	1890.00	545.55	368.11	540.06	1453.72
Total load	1216.41	962.11	1287.48	3466.00	540.57	332.60	528.82	1402.00
Loss	11.42	4.76	10.95	27.13	21.00	18.76	21.69	61.45

Case 3: Two 3-phase DG units, DG1 and DG2, are connected at bus no. 12 and 15 respectively of URDN 25 node network. DG1 is operated in P-Q mode with a specified output of 500 kW and 200 kVAR, whereas DG2 is operated in P-V mode

with specified power output of 1000 kW and a specified positive sequence terminal voltage magnitude of 1.0 p.u.. No Q-limit is considered for this unit.

Table 5 shows the results obtained with proposed modified approach. It is to be observed that for DG2 (operating in P-V mode), the terminal voltage of the three phases result in a specified positive sequence magnitude of 1.0 p.u.. Other aspects are now self-explanatory.

**Table 5** Results obtained with the proposed modified approach for including DG in

 the load flow

Bus No	Volta	ge Magnitude	(p.u.)	Voltage Phase Angle (Degree)			
Dus No.	$V_{a}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_b$	$\theta_{c}$	
1	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
12	0.9894	0.9886	0.9899	-0.56	-120.53	119.45	
15	1.0003	0.9997	1.0000	-1.07	-121.06	118.97	
INT1	0.9981	0.9981	0.9981	0.23	-119.77	120.23	
INT2	1.0471	1.0471	1.0471	0.08	-119.92	120.08	

a) The voltage profile

# b) Power balance

		Active Po	wer (kW)		Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	584.80	600.59	601.15	1786.54	274.59	264.55	282.04	821.19
DG1 Power	168.50	165.51	165.99	500.00	64.80	75.06	60.14	200.00
DG2 Power	336.53	334.35	329.12	1000.00	470.57	477.78	474.76	1423.12
Total load	1073.30	1083.30	1083.30	3239.90	792.00	801.00	800.00	2393.00
Loss	16.53	17.15	12.96	46.64	17.96	16.39	16.95	51.30

Case 4: Both the DG units mentioned in Case 3 are operated in P-V mode with a specified positive sequence terminal voltage magnitude of 1.0 p.u.. DG1 has a specified power output of 500 kW with a Q limit of 300 kVAR and DG2 has a specified power output of 1000 kW with a Q limit of 1500 kVAR.

Table 6 shows the results obtained with proposed modified approach. It is to be observed that, for DG2, the reactive power output is within limit; hence, the specified terminal voltage (positive sequence value) is maintained. But in case of DG1, the reactive power output crosses its upper limit to maintain the specified terminal

voltage. So the reactive power output is fixed at the upper limit, and the specified terminal voltage can no longer be maintained.

**Table 6** Results obtained with the proposed modified approach for including DG in

 the load flow

Bus No.	Volta	ge Magnitude	(p.u.)	Voltage Phase Angle (Degree)			
	$\mathbf{V}_{\mathrm{a}}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
1	1.0000	1.0000	1.0000	0.00	-120.00	120.00	
12	0.9912	0.9902	0.9915	-0.69	-120.66	119.33	
15	1.0003	0.9997	1.0001	-1.06	-121.05	118.97	
INT1	1.0027	1.0027	1.0027	0.06	-119.94	120.06	
INT2	1.0451	1.0451	1.0451	0.11	-119.89	120.11	

a) The voltage profile	a)	The	voltage	profile
------------------------	----	-----	---------	---------

		Reactive Power (kVAR)						
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	584.34	599.63	601.16	1785.13	264.79	254.43	271.45	790.67
DG1 Power	168.62	166.19	165.20	500.00	97.62	108.39	93.99	300.00
DG2 Power	336.40	334.13	329.47	1000.00	447.15	454.20	451.11	1352.45
Total load	1073.30	1083.30	1083.30	3239.90	792.00	801.00	800.00	2393.00
Loss	16.06	16.65	12.52	45.23	17.57	16.02	16.54	50.13

### b) Power balance

Case 5: A DG unit of 1350 kW rating is connected at bus no. 708 of IEEE 37 bus network. A voltage regulation scheme is assumed to maintain the average magnitude of the terminal voltages of the three phases at a specified value of 1.0 p.u. (P-V mode of operation). No Q limit is considered.

Table 7 and Table 8 show the results obtained with proposed modified approach along with that obtained by the conventional N-R 3-phase load flow method and also the PSO based method presented in the previous chapter. From the Tables presented for the three different approaches it is seen that the three methods produces almost identical results.

**Table 7** Voltage magnitude & phase angle obtained by the three different load flow

 methods

Due No	Volta	ige magnitude	(p.u)	Voltage Phase Angle (Degree)		
Dus No.	$\mathbf{V}_{\mathrm{a}}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$
INT	1.0170	1.0170	1.0170	-28.57	-148.57	91.43
799	1.0000	1.0000	1.0000	-30.00	-150.00	90.00
708	0.9960	1.0010	1.0030	-30.28	-150.01	89.65

# a) Load Flow with the proposed modified approach

b) N-R Load Flow
------------------

Bus No.	Volta	ige magnitude	(p.u)	Voltage Phase Angle (Degree)			
	Va	V <sub>b</sub>	$V_a$	V <sub>b</sub>	$V_{a}$	V <sub>b</sub>	
INT	1.0169	1.0169	1.0169	-28.57	-148.57	91.43	
799	1.0000	1.0000	1.0000	-30.00	-150.00	90.00	
708	0.9964	1.0009	1.0027	-30.28	-150.01	89.65	

# c) Load flow with PSO based approach

Bue No	Volta	ige magnitude	(p.u)	Voltage Phase Angle (Degree)			
Dus No.	$V_a$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
INT	1.0170	1.0170	1.0170	-28.57	-148.57	91.43	
799	1.0000	1.0000	1.0000	-30.00	-150.00	90.00	
708	0.9960	1.0010	1.0030	-30.28	-150.01	89.65	

Table 8 Power balance obtained by the three different load flow methods

a) Load flow with the proposed modified approach

		Active po	wer (kW)	Reacti	ve power (l	(VAR)	
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc
Grid Power	399.04	281.91	447.01	1127.96	249.66	125.14	89.38
DG Power	467.50	394.07	488.43	1350.00	301.91	237.07	206.24
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52
Loss	7.48	5.40	8.08	20.96	4.95	3.29	6.08

		Active po	wer (kW)	Reacti	ve power (l	(VAR)	
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc
Grid Power	399.69	281.8	447.08	1128.55	276.8	144.39	113.4
DG Power	467.1	394.38	488.49	1350.0	296.72	233.4	202.14
Total load	859.06	670.59	927.35	2457.0	548.58	360.9	291.52
Loss	7.73	5.59	8.22	21.54	24.93	16.89	24.02

c) Load flow with PSO based approach

		Active po	wer (kW)	Reacti	ve power (l	(VAR)	
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc
Grid Power	399.04	281.91	447.01	1127.96	249.66	125.14	89.38
DG Power	467.50	394.07	488.43	1350.00	301.91	237.07	206.24
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52
Loss	7.48	5.40	8.08	20.96	4.95	3.29	6.08

Case 6: The generator considered in Case 5 is operated in P-Q mode with P and Q specified at 1350 kW and 745.22kVAR respectively and connected at bus no. 708 of IEEE 37 bus network.

Table 9 and Table 10 show the results obtained with the load flow methods considered in Case 5.

Table 9 Voltage magnitude & phase angle obtained by different load flow methods

Bus No	Volta	ge magnitude	(p.u.)	Voltage Phase Angle (Degree)			
Dus No.	$V_a$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
INT	1.0170	1.0170	1.0170	-28.57	-148.57	91.43	
799	1.0000	1.0000	1.0000	-30.00	-150.00	90.00	
708	0.9960	1.0010	1.0030	-30.28	-150.01	89.65	

a) Load Flow with proposed modified approach

b) N-R Load Flow

Bus No	Volta	ige magnitude	(p.u)	Voltage Phase Angle (Degree)			
Dus No.	$V_a$	V <sub>b</sub>	V <sub>c</sub>	$\theta_{a}$	$\theta_b$	$\theta_{c}$	
INT	1.01690	1.01690	1.01690	-28.57	-148.57	91.43	
799	1.00000	1.00000	1.00000	-30.00	-150.00	90.00	
708	0.99640	1.00090	1.00270	-30.28	-150.01	89.65	

Bus No	Volta	ge magnitude	(p.u)	Voltage Phase Angle (Degree)			
Dus No.	$\mathbf{V}_{\mathrm{a}}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
INT	1.0170	1.0170	1.0170	-28.57	-148.57	91.43	
799	1.0000	1.0000	1.0000	-30.00	-150.00	90.00	
708	0.9960	1.0010	1.0030	-30.28	-150.01	89.65	

	c)	Load	flow	with	PSO	based	approach
--	----	------	------	------	-----	-------	----------

Table 10 Power balance obtained by the three different load flow methods

		Active p	Reacti	ve power (l	(VAR)		
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc
Grid Power	399.04	281.91	447.01	1127.96	249.66	125.14	89.38
DG Power	467.50	394.07	488.43	1350.00	301.91	237.07	206.24
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52

a) Load flow with proposed modified approach

# b) N-R Load Flow

20.96

4.95

3.29

6.08

8.08

		Active po	ower (kW)	Reactive power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc
Grid Power	399.69	281.80	447.08	1128.55	276.80	144.39	113.40
DG Power	467.10	394.38	488.49	1350.00	296.72	233.40	202.14
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52
Loss	7.73	5.59	8.22	21.54	24.93	16.89	24.02

c) Load flow with PSO based approach

		Active po	ower (kW)	Reactive power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Q <sub>c</sub>
Grid Power	399.04	281.91	447.01	1127.96	249.66	125.14	89.38
DG Power	467.50	394.07	488.43	1350.00	301.91	237.07	206.24
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52
Loss	7.48	5.40	8.08	20.96	4.95	3.29	6.08

# 3.4 Chapter Summary

7.48

Loss

5.40

This chapter has proposed an improved methodology for inclusion of 3-phase DG models in distribution load flow formulations. Both P-Q model and P-V model of DG units have been considered and the corresponding convergence criteria have been established. The uneven division of loads among the three phases of a DG unit in unbalanced operating condition can be properly taken into consideration by the

suggested method and the internal voltage balance of the generator is also maintained in the solution. Results for a number of case studies with different operating conditions for IEEE 13 node, IEEE 37 node and 25 node URDN networks have been presented. The results show that the proposed methodology can provide realistic solution of 3-phase distribution load flow in the presence of both P-Q model and P-V model of DG units.

# **CHAPTER** 4

# Elimination of Partial Overloading of Generators under Unbalanced Operating Conditions of Power Systems

Outline of the chapter

- 4.1 Introduction
- 4.2 Brief Review of N-R 3-Phase Load Flow
- 4.3 The Proposed Methodology
- 4.3.1 The Flowchart
- 4.4 Case studies
- 4.5 Chapter Summery

#### **4.1 Introduction**

In this chapter, a methodology is proposed for more effective planning and operation of power systems subjected to unbalanced operating conditions. With the help of the proposed method, occurrence of any partial overloading of the generators in the system can be avoided and the generators can be operated within their safe loading limits at all conditions of unbalanced operation. The problem of partial overloading of generators has been introduced in section 1.4 of chapter 1. The presented methodology utilizes conventional N-R 3-phase load flow and is based on a modification in the necessary/ suitable/ appropriate constraint Equation.

#### 4.2 Brief Review of N-R 3-Phase Load Flow

N-R based 3-phase load flow is well documented in the literature [11], [12].As the present work is based on the formulation suggested in the above references, this section presents a brief review of N-R 3-phase load flow, particularly highlighting the formulation adopted in those references.

The main objective of 3-phase load flow is to obtain the magnitudes and angles of voltages of all the three phases at each bus in a 3-phase network corresponding to specified system conditions. Typically, twelve variables are attached to each 3-phase bus. They include injected active and reactive power, as well as voltage magnitude and the angle of each of its three phases. Six of these variables should be specified or be known, leading to six constraints in the form of six equations to be solved for the remaining six unknowns for each 3-phase bus. An iterative solution is required since the constraint equations are non-linear. The N-R load flow requires mismatches between the specified values and the calculated values in each iteration is to be related linearly with the required changes or corrections in the values of the unknown quantities with the help of Jacobian matrix as follows

$$\Delta M = J \Delta X \tag{4.1}$$
$$\Delta X = \left[ J^{inv} \right] \Delta M$$

where, for a network having *n* number of 3-phase buses,

$$\Delta M = \left[\Delta M_1, \Delta M_2, \dots, \Delta M_n\right]^T \tag{4.2}$$

is the mismatch vector or error vector, and

$$\Delta X = \left[\Delta X_1, \Delta X_2, \dots, \Delta X_n\right]^T \tag{4.3}$$

is the vector of changes or correction vector.

The constraints, however, are different for different types of buses. For the slack bus, the voltage magnitude and phase angle are specified and the angle is usually taken as zero, and the angles of all other voltages are given with respect to the voltage phasor of this bus. No constraints are attached to this bus.

For the load buses, the injected active and reactive power for each of the three phases of this type of bus is specified separately. The six quantities to be determined for these buses are the voltage magnitudes and phase angles of the three phases. The following active and reactive power balance equations constitute the six constraints for these buses.

Assuming the  $i^{th}$  bus to be a load bus,

$$P^{\alpha}_{_{\mathrm{sp}_{i}}} = \sum_{j=1\neq i}^{n} P^{\alpha}_{ij} \tag{4.4}$$

$$Q_{_{\mathfrak{sp}_i}}^{\alpha} = \sum_{j=1\neq i}^n Q_{ij}^{\alpha}$$
(4.5)

where,

and

$$P_{ij}^{\alpha} = V_{i}^{\alpha} \sum_{k=a,b,c} \left[ V_{i}^{k} \left( G_{ij}^{\alpha k} \cos\left(\theta_{i}^{\alpha} - \theta_{i}^{k}\right) + B_{ij}^{\alpha k} \sin\left(\theta_{i}^{\alpha} - \theta_{i}^{k}\right) \right) - V_{j}^{k} \left( G_{ij}^{\alpha k} \cos\left(\theta_{i}^{\alpha} - \theta_{j}^{k}\right) + B_{ij}^{\alpha k} \sin\left(\theta_{i}^{\alpha} - \theta_{j}^{k}\right) \right) \right]$$

$$(4.6)$$

and

$$Q_{ij}^{\alpha} = V_{i}^{\alpha} \sum_{k=a,b,c} \left[ V_{i}^{k} \left( G_{ij}^{\alpha k} \sin\left(\theta_{i}^{\alpha} - \theta_{i}^{k}\right) - B_{ij}^{\alpha k} \cos\left(\theta_{i}^{\alpha} - \theta_{i}^{k}\right) \right) - V_{j}^{k} \left( G_{ij}^{\alpha k} \sin\left(\theta_{i}^{\alpha} - \theta_{j}^{k}\right) - B_{ij}^{\alpha k} \cos\left(\theta_{i}^{\alpha} - \theta_{j}^{k}\right) \right) \right]$$

$$(4.7)$$

( $\alpha$ =a, b, c for three phases).

The six mismatches are as follows,

$$\Delta P_i^{\alpha} = P_{\mathrm{sp}_i}^{\alpha} - \sum_{j=1\neq i}^n P_{ij}^{\alpha} \quad , \tag{4.8}$$

and  $\Delta Q_i^{\alpha} = Q_{\mathrm{sp}_i}^{\alpha} - \sum_{j=1 \neq i}^n Q_{ij}^{\alpha}$ .

Then, for the  $i^{\text{th}}$  bus, the (6×1) mismatch vector is as follows

$$\Delta M_{i} = \left[\Delta P_{i}^{a}, \Delta P_{i}^{b}, \Delta P_{i}^{c}, \Delta Q_{i}^{a}, \Delta Q_{i}^{b}, \Delta Q_{i}^{c}\right]^{T}$$

$$(4.10)$$

(4.9)

and the  $(6\times 1)$  correction vector is given as

$$\Delta X_{i} = \left[\Delta \theta_{i}^{a}, \Delta \theta_{i}^{b}, \Delta \theta_{i}^{c}, \frac{\Delta V_{i}^{a}}{V_{i}^{a}}, \frac{\Delta V_{i}^{b}}{V_{i}^{b}}, \frac{\Delta V_{i}^{c}}{V_{i}^{c}}\right]^{T}$$
(4.11)

The above-mentioned formulation for slack bus and load bus is quite general for N-R 3-phase load flow, and has been used in [11], [12].

For the generator buses, the total injected 3-phase active power is specified. A specification on voltage magnitude is also imposed depending on the voltage control scheme adopted for the generator. These two specified quantities give rise to two constraints only. The remaining four constraints are derived, making use of the fact that the three internal voltages of the generator are balanced in magnitude and phase because of the balanced design of its windings. The same is also true for the 3-phase inverter based on DGs where due to symmetrical switching, the three phase voltages of the inverter remains balanced. Traditionally, the six unknowns for generator bus are the 3-phase terminal voltage magnitudes and their phase angles.

In the formulation adopted in [12] in addition to these six quantities, the magnitude of the internal induced voltage and the angle of any one of its phases are introduced as unknown quantities to meet the requirement for maintaining balanced condition of the internal voltages. This results in a total of eight unknowns assigned to each generator bus requiring eight quantities to be specified for generating eight constraints. These specified quantities are active and reactive loads (both will be zero if no load is connected to the bus) at each phase of the generator terminal, the total 3-phase active power output of the generator, and the generator terminal voltage magnitude (i.e., the voltage regulator setting).

Accordingly, the eight constraints for the generator buses can be expressed as follows: Assuming that the  $i^{th}$  bus is a generator bus,

$$P_{_{\rm sp}_i}^{\alpha} = \sum_{j=1 \neq i}^{n} P_{ij}^{\alpha} - P_{G_i}^{\alpha}$$
(4.12)

$$Q_{_{\mathfrak{S}_{i}}}^{\alpha} = \sum_{j=1\neq i}^{n} Q_{ij}^{\alpha} - Q_{G_{i}}^{\alpha}$$
(4.13)

$$P_{G_{\mathrm{sp}_i}} = P_{G_i} \tag{4.14}$$

$$V_{\rm sp_i} = f\left(V_i^{\rm a}, V_i^{\rm b}, V_i^{\rm c}\right) \tag{4.15}$$

where  $P_{ij}^{\alpha}$  and  $Q_{ij}^{\alpha}$  are as defined in (4.6) and(4.7), and

$$P_{G_i}^{\alpha} = V_i^{\alpha} E_i \left[ g_i \cos\left(\delta_i^{\alpha} - \theta_i^{\alpha}\right) - b_i \sin\left(\delta_i^{\alpha} - \theta_i^{\alpha}\right) \right] - \left(V_i^{\alpha}\right)^2 g_i \qquad (4.16)$$

$$Q_{G_i}^{\alpha} = V_i^{\alpha} E_i \Big[ b_i \cos\left(\delta_i^{\alpha} - \theta_i^{\alpha}\right) + g_i \sin\left(\delta_i^{\alpha} - \theta_i^{\alpha}\right) \Big] - \left(V_i^{\alpha}\right)^2 b_i \qquad (4.17)$$

$$P_{G_i} = \sum_{\alpha = a,b,c} P_{G_i}^{\alpha}$$
(4.18)

 $f(V_i^{a}, V_i^{b}, V_i^{c})$  depends on the voltage regulation scheme adopted in the generator. The corresponding mismatches are as follows:

$$\Delta P_{i}^{\alpha} = P_{\mathrm{sp}_{i}}^{\alpha} - \sum_{j=1\neq i}^{n} P_{ij}^{\alpha} + P_{G_{i}}^{\alpha}$$
(4.19)

$$\Delta Q_i^{\alpha} = Q_{\mathrm{sp}_i}^{\alpha} - \sum_{j=1\neq i}^n Q_{ij}^{\alpha} + Q_{G_i}^{\alpha}$$

$$(4.20)$$

$$\Delta P_{G_i} = P_{G_{sp_i}} - P_{G_i} \tag{4.21}$$

and

and

$$\Delta V_i = V_{\mathrm{sp}_i} - f\left(V_i^{\mathrm{a}}, V_i^{\mathrm{b}}, V_i^{\mathrm{c}}\right).$$
(4.22)

Then, for the  $i^{th}$  bus, the (8×1) mismatch vector is given by

$$\Delta M_{i} = \left[\Delta P_{i}^{a}, \Delta P_{i}^{b}, \Delta P_{i}^{c}, \Delta Q_{i}^{a}, \Delta Q_{i}^{b}, \Delta Q_{i}^{c}, \Delta P_{G_{i}}, \Delta V_{i}\right]^{T}$$
(4.23)

and the correction vector will be as follows:

$$\Delta X_{i} = \left[\Delta \theta_{i}^{\mathrm{a}}, \Delta \theta_{i}^{\mathrm{b}}, \Delta \theta_{i}^{\mathrm{c}}, \frac{\Delta E_{i}}{E_{i}}, \frac{\Delta V_{i}^{\mathrm{b}}}{V_{i}^{\mathrm{b}}}, \frac{\Delta V_{i}^{\mathrm{c}}}{V_{i}^{\mathrm{c}}}, \Delta \delta_{i}, \frac{\Delta V_{i}^{\mathrm{a}}}{V_{i}^{\mathrm{a}}}\right]^{T}$$
(4.24)

The Jacobian J comprises block matrices  $J_{ij}$  of proper dimensions.

When  $i^{\text{th}}$  bus is a load bus,  $J_{ij}$  is  $a(6\times6)$  block (shown below as  $J_1$ ) if j is also a load bus, and  $J_{ij}$  is a (6×8) block (shown below as  $J_2$ ) if j is a generator bus.

$$J_{1} = \begin{bmatrix} \frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}} & \frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}} & \frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}} & \frac{\partial P_{ij}^{a}}{\partial V_{i}^{a}} V_{i}^{a} & \frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}} & \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}} & \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}} & \frac{\partial P_{ij}^{c}}{\partial V_{i}^{a}} V_{i}^{a} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}} & \frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}} & \frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}} & \frac{\partial P_{ij}^{c}}{\partial V_{i}^{a}} V_{i}^{a} & \frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}} & \frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}} & \frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}} & \frac{\partial Q_{ij}^{a}}{\partial V_{i}^{a}} V_{i}^{a} & \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}} & \frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}} & \frac{\partial Q_{ij}^{a}}{\partial V_{i}^{a}} V_{i}^{a} & \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}} & \frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}} & \frac{\partial Q_{ij}^{a}}{\partial V_{i}^{a}} V_{i}^{a} & \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}} & \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}} & \frac{\partial P_{ij}^{a}}{\partial P_{i}^{c}} & \frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}} & \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}} & \frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}} & \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}} & \frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}} & \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}} & \frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}} V_{i}^{b} & \frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}} V_{i}^{c} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$$

If  $i^{th}$  bus is a generator bus, then  $J_{ij}$  is a (8×8) block (shown below as  $J_3$ ) if j is also a generator bus, and  $J_{ij}$  is a (8×6) block (shown below as  $J_4$ ) if j is a load bus.  $J_{ij}$  will be zero block of proper dimension if bus i and bus j are not directly connected.

	$\partial P^a_{ij}$	$\partial P^a_{ij}$	$\partial P^a_{ij}$	$\frac{\partial P_{ij}^a}{E_i^a}$	$\frac{\partial P_{ij}^a}{\partial V_i^b}$	$\frac{\partial P_{ij}^a}{\partial V_i^c}$	$\partial P^a_{ij}$	$\frac{\partial P^a_{ij}}{\partial V_i^a}$
	$\partial \theta^a_i$	$\partial  heta_i^b$	$\partial \theta_i^c$	$\partial E_i^a = E_i^a$	$\partial V_i^{b}$	$\partial V_i^c$	$\partial \delta_i$	$\partial V_i^{a}$
	$\frac{\partial P_{ij}^b}{\partial P_{ij}^a}$	$\frac{\partial P_{ij}^b}{\partial P_{ij}^b}$	$\frac{\partial P_{ij}^b}{\partial P_{ij}^c}$	$\frac{\partial P_{ij}^b}{\partial r_a}E_i^a$	$\frac{\partial P_{ij}^b}{\partial U_i^b} V_i^b$	$\frac{\partial P_{ij}^b}{\partial W^c} V_i^c$	$\frac{\partial P_{ij}^b}{\partial P_{ij}^b}$	$\frac{\partial P_{ij}^b}{\partial V_i^a} V_i^a$
	$\partial \theta_i^a$	$\partial \theta_i^{\nu}$	$\partial \theta_i^c$	$\partial E_i^a$	$\partial V_i^{b}$	$\partial V_i^c$	$\partial \delta_i$	$\partial V_i^{a}$
	$\frac{\partial P_{ij}^{\epsilon}}{\partial Q^{a}}$	$\frac{\partial P_{ij}^{c}}{\partial \rho^{b}}$	$\frac{\partial P_{ij}^c}{\partial \rho^c}$	$\frac{\partial P_{ij}^c}{\partial E^a}E^a_i$	$\frac{\partial P_{ij}^{c}}{\partial U^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^c}{\partial V_i^c} V_i^c$	$\frac{\partial P_{ij}^{c}}{\partial S}$	$\frac{\partial P_{ij}^{c}}{\partial V_{a}^{a}}V_{i}^{a}$
	$\partial \Theta_i$	$\partial \theta_i$	$\partial \theta_i$	$\partial E_i$	$\partial V_i$	$\partial V_i$	$\partial O_i^a$	$\partial V_i$
<b>,</b>	$\frac{\partial Q_{ij}}{\partial \theta_i^a}$	$\frac{\partial \mathcal{Q}_{ij}}{\partial \theta_i^b}$	$\frac{\partial \mathcal{Q}_{ij}}{\partial \theta_i^c}$	$\frac{\partial Q_{ij}}{\partial E_i^a} E_i^a$	$\frac{\partial Q_{ij}}{\partial V_i^b} V_i^b$	$\frac{\partial Q_{ij}}{\partial V_i^c} V_i^c$	$\frac{\partial \mathcal{Q}_{ij}}{\partial \delta_i}$	$\frac{\partial Q_{ij}}{\partial V_i^a} V_i^a$
$J_{3} =$	$\partial Q^b_{ii}$	$\partial Q^b_{aa}$	$\partial Q^b_{aa}$	$\partial Q^b_{ii}$	$\partial Q^b_{ii}$	$\partial Q^b_{ii}$	$\partial Q^b_{ii}$	$\partial Q^b_{ii}$
	$\frac{-\omega_{ij}}{\partial \theta_i^a}$	$\frac{-\omega_{ij}}{\partial \theta^b_i}$	$\frac{-\omega_{ij}}{\partial \theta_i^c}$	$\frac{\omega_{ij}}{\partial E_i^a} E_i^a$	$\frac{\omega_{ij}}{\partial V_i^b}V_i^b$	$\frac{\omega_y}{\partial V_i^c} V_i^c$	$\frac{-\omega_{ij}}{\partial \delta_i}$	$\frac{\omega_y}{\partial V_i^a}V_i^a$
	$\partial Q^c_{ij}$	$\partial Q^c_{ij}$	$\partial Q^c_{ij}$	$\partial Q^c_{ij} = {}_{{m F}^a}$	$\partial Q^c_{ij} = V^b$	$\partial Q_{ij}^{c}$	$\partial Q^c_{ij}$	$\partial Q^c_{ij} V^a$
	$\overline{\partial  heta_i^a}$	$\overline{\partial  heta_i^b}$	$\partial \theta_i^c$	$\overline{\partial E_i^a}^{L_i}$	$\overline{\partial V_i^b}^{v_i}$	$\overline{\partial V_i^c}^{v_i}$	$\partial \delta_i$	$\overline{\partial V_i^a}^{v_i}$
	$\frac{\partial P_{Gi}}{\partial \theta^a}$	$\frac{\partial P_{Gi}}{\partial A^b}$	$\frac{\partial P_{Gi}}{\partial A^c}$	$\frac{\partial P_{Gi}}{\partial F^a} E^a_{ij}$	$\frac{\partial P_{Gi}}{\partial V^b}V_i^b$	$\frac{\partial P_{Gi}}{\partial V^c}V_i^c$	$\frac{\partial P_{Gi}}{\partial \delta}$	$\frac{\partial P_{Gi}}{\partial V^a}V_i^a$
	$\partial V_i$	$\partial V_i$	$\partial V_i$	$\partial V_i$	$\partial V_i$	$\partial V_i$	$\partial V_i$	$\partial V_i$
	$\overline{\partial \theta_i^a}$	$\overline{\partial \theta^b_i}$	$\overline{\partial \theta_i^c}$	$\frac{i}{\partial E_i^a} E_i^a$	$\frac{1}{\partial V_i^b}V_i^b$	$\frac{1}{\partial V_i^c} V_i^c$	$\overline{\partial \delta_i}$	$\frac{1}{\partial V_i^a} V_i^a$
	$\partial P^a$	$\partial P^a$	$\partial P^a$	$\partial P^a$	$\partial P^a$	$\partial P^a$		
	$\left  \begin{array}{c} \partial P^a_{ij} \\ \overline{\partial  heta^a_i} \end{array}  ight $	$rac{\partial P^a_{ij}}{\partial  heta^b_i}$	$rac{\partial P^a_{ij}}{\partial  heta^c_i}$	$rac{\partial P^a_{ij}}{\partial E^a_i}E^a_i$	$rac{\partial P^a_{ij}}{\partial V^b_i}V^b_i$	$\left[ \frac{\partial P_{ij}^a}{\partial V_i^c} V_i^c \right]$		
	$\left  egin{array}{c} \partial P^a_{ij} \ \overline{\partial  heta^a_i} \ \partial P^b_{ii} \end{array}  ight $	$rac{\partial P^a_{ij}}{\partial  heta^b_i} \ rac{\partial P^b_{ij}}{\partial P^b_{ii}}$	$rac{\partial P^a_{ij}}{\partial  heta^c_i} \ rac{\partial P^a_{ij}}{\partial P^c_{ii}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}}E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial P_{ij}^{b}}=\pi^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial P_{ij}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial P_{ij}^{b}}$		
	$rac{\partial P^a_{ij}}{\partial  heta^a_i} \ rac{\partial P^b_{ij}}{\partial  heta^a_i} \ rac{\partial P^b_{ij}}{\partial  heta^a_i}$	$rac{\partial P^a_{ij}}{\partial  heta^b_i} \ rac{\partial P^b_i}{\partial  heta^b_i}$	$rac{\partial P^a_{ij}}{\partial  heta^c_i} \ rac{\partial P^b_{ij}}{\partial  heta^c_i}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}}E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}}E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$		
	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}}E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}}E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}}E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{b}$	$egin{array}{c} rac{\partial P^a_{ij}}{\partial V^c_i} V^c_i \ rac{\partial P^b_{ij}}{\partial V^c_i} V^c_i \ rac{\partial P^b_{ij}}{\partial V^c_i} V^c_i \ rac{\partial P^c_{ij}}{\partial V^c_i} V^c_i \end{array}$		
	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}} \\ \frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}} \\ \frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}} \\ \frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}} $	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$		
	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$		
$J_4 =$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$		
$J_4 =$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$		
$J_4 =$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$	$ \frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c} \\ \frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c} \\ \frac{\partial P_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c} \\ \frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c} \\ \frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c} \\ \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c} \\ \frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c} \\ \frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c} $		
$J_4 =$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$		
$J_4 =$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$		
$J_4 =$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{Gi}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{Gi}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{Gi}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{Gi}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$		
$J_4 =$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{a}}$ $\frac{\partial P_{Gi}}{\partial \theta_{i}^{a}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{b}}$ $\frac{\partial P_{Gi}}{\partial \theta_{i}^{b}}$ $\frac{\partial V_{i}}{\partial \theta_{i}^{b}}$	$\frac{\partial P_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{a}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{b}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial Q_{ij}^{c}}{\partial \theta_{i}^{c}}$ $\frac{\partial P_{Gi}}{\partial \theta_{i}^{c}}$ $\frac{\partial V_{i}}{\partial \theta_{i}^{c}}$	$\frac{\partial P_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{b}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{a}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial Q_{ij}^{c}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial P_{Gi}}{\partial E_{i}^{a}} E_{i}^{a}$ $\frac{\partial V_{i}}{\partial E_{i}^{a}} E_{i}^{a}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial P_{Gi}}{\partial V_{i}^{b}}V_{i}^{b}$ $\frac{\partial V_{i}}{\partial V_{i}^{b}}V_{i}^{b}$	$\frac{\partial P_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{a}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{b}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial Q_{ij}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial P_{Gi}}{\partial V_{i}^{c}}V_{i}^{c}$ $\frac{\partial V_{i}^{c}}{\partial V_{i}^{c}}V_{i}^{c}$		
#### 4.3 The Proposed Methodology

As already mentioned the proposed method is based on a modification in the conventional formulation of N-R 3-phase load flow. The suggested modification is associated with the constraint equation (4.18), and must be introduced if during planning or operation any partial overloading is detected from the results of load flow by any existing method.

On detection of partial overloading, the generators in a system are divided into two categories—one category includes the generators, which are loaded within safe limits, and the other category includes those that violate such limits. The generators belonging to the second category will be called "critical generators" and the phase of a critical generator that exceeds the maximum limit of active power per phase will be termed as "critical phase."

When two phases of a generator are simultaneously overloaded, the phase with higher loading will be considered as the critical phase. If two phases are equally overloaded, then any one of them should be selected as the critical phase. After detection of the critical generators and their critical phases, the constraint on total 3-phase active power output Equation (4.18) attached to each of the critical generators will be replaced by the following equation (assuming the  $i^{\text{th}}$  bus to be a critical generator bus):

$$P_{G_{\mathrm{sp}_i}}^{\mathrm{cr}} = P_{G_i}^{\mathrm{cr}} \tag{4.25}$$

where,  $P_{G_i}^{cr} = \max\left[P_{G_i}^{\alpha}\right]$ , ( $\alpha$ =a,b,c indicates 3 phases).  $P_{G_i}^{cr} = V_i^{cr} E_i \left[g_i \cos\left(\delta_i^{cr} - \theta_i^{cr}\right) - b_i \sin\left(\delta_i^{cr} - \theta_i^{cr}\right)\right] - \left(V_i^{cr}\right)^2 g_i \quad (4.26)$ 

The superscript "*cr*" indicates critical phase and  $P_{G_{spi}}^{cr}$  is the specified active power output of the critical phase. This means that on detection of any partial overloading, the constraint on the total 3-phase active power output associated with each critical generator should be replaced by the constraint on the output of the critical phase, which should now be specified at its maximum limit of loading per phase. In other words, instead of specifying the total 3-phase active power output, the output of the critical phases of the critical generators is specified. Thus, the active power output of the critical phases can be restricted to their respective upper limits if the specified values are set to those limits. The other two phases of the critical generators will remain under-loaded in such conditions. However, if two phases of a critical generator are simultaneously overloaded, the proposed load flow with the one with higher loading as the critical phase may generate a solution where the other phase still remains overloaded. In that case, the modified load flow has to be run again with the overloaded phase taken as the critical phase.

The corresponding mismatch equation will be given by

$$\Delta P_{G_i}^{\rm cr} = P_{G_{\rm sp_i}}^{\rm cr} - P_{G_i}^{\rm cr}.$$
(4.27)

It is to be noted that as per reference [12], the 7<sup>th</sup> row of  $J_{ij}$  is a full row if the *i*<sup>th</sup> bus is a generator bus, and corresponds to the mismatch in the total 3-phase power output of the generator. In the proposed method, the corresponding row represents the mismatch in the power output of the critical phase of the generator. In this case, it is not a full row. Only the elements corresponding to  $V_i^{cr}$ ,  $\theta_i^{cr}$ ,  $\delta_i$ , and  $E_i$  will be nonzero.

#### 4.3.1 The Flowchart

The methodology is presented in the form of a flow chart shown in Fig.4.1.



Fig.4.1. Flow chart of the proposed methodology.

#### 4.4 Case studies

The efficacy of the proposed methodology has been tested on the IEEE 37 node distribution feeder network shown in Appendix IV. In this study, the voltage regulator at bus 799 (the grid bus) of the network has not been considered. The network is augmented by employing DG units and connecting the DG internal nodes to the respective phases of the generator terminal buses through the internal impedances.

The  $Y_{bus}$  is modified accordingly. The DG internal node (3-phase) is denoted as INT. The grid bus has been taken as the slack or swing bus for the load flow solutions. The phase angles are expressed with respect to the slack bus line-to-line voltages in all cases. For conciseness, only the relevant parts of the results have been shown for all cases.

Three case studies have been conducted as follows:

Case 1: A DG unit (3-phase synchronous generator) of 1350 kW rating is connected at bus 708 of the IEEE 37 network.

Table 1 shows the results obtained with conventional N-R 3-phase load flow. For brevity, the results pertaining only to the relevant buses (799, 708, and INT) have been shown. To maximize the utilization of DG power, its total 3-phase active power output is specified at 1350 kW (rated capacity). A voltage regulation scheme is assumed to maintain the average magnitude of the terminal voltages of the three phases at a specified value of 1.0 p.u. From the results we find that both phase-a and phase-c of the DG unit are overloaded ( $P_a$ =467.10kW,  $P_c$ =485.49kW) as they exceed the per phase kW rating (which is 1350/3=450kW) of the DG, though the total 3phase output ( $P_a$ + $P_b$ + $P_c$ ) of the DG unit is almost 1350kW.

**Table 1** Results of conventional N-R 3-phaseload flow with total 3-phase poweroutput specified at 1350 kW (its total 3-phase rating)

Bus No.	Voltag	ge magnitude	e (p.u.)	Voltage Phase angle(Degrees)				
	$\mathbf{V}_{\mathrm{a}}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$		
799	1.0000	1.0000	1.0000	-30.000	-150.000	90.000		
708	0.9965	1.0009	1.0027	-30.264	-149.994	89.662		
INT	1.0169	1.0169	1.0169	-28.566	-148.566	91.434		

a)	The	voltage	profile
~			p101110

0) I Ower Durance
-------------------

		Active Po	ower (kW	)	Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q
Grid Power	394.42	277.73	441.08	1113.22	251.03	127.98	90.52	469.53
DG Power	467.10	394.38	488.49	1349.98	296.72	233.40	202.14	732.26
Total Gen.	861.52	672.11	929.57	2463.20	547.74	361.38	292.66	1201.79
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00
Loss	2.46	1.52	2.22	6.20	-0.83	0.48	1.15	0.79

Table 2 shows the corresponding values obtained when the proposed modification was applied. The load flow was run taking phase-c of the DG unit as the critical phase as it is the maximum loaded phase as obtained from the conventional load flow. The kW output of this phase of the DG unit was specified at 450kW (per phase kW rating of the DG unit). A similar specification on voltage magnitude was assumed. From Table 2 it is found that the maximum loaded phase (phase-c) is delivering 450kW. The total 3-phase output of the DG unit is 1234.4kW, which should be considered as the maximum allowable 3-phase kW output under the operating condition considered. The DG unit must be run with its output setting fixed at 1234.4kW to get the maximum kW output under the given operating condition without overloading any of its phases. Table 3 shows the results (obtained by conventional load flow) of running the DG unit at the given operating condition with its output set at 1234.4kW. From Table 2 and Table 3, it is found that the various voltages and active and reactive powers are same in the two Tables.

**Table 2** Results of the proposed method with the critical phase power output of the DG unit specified at 450 kW

Bus No.	Volta	ge Magnitude	e (p.u)	Voltage Phase angle (Degrees)			
	$V_a$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
799	1.000	1.000	1.000	-30.00	-150.00	90.00	
708	0.996	1.001	1.003	-30.50	-150.22	89.43	
INT	1.0198	1.0198	1.0198	-28.94	-148.94	91.06	

a) The voltage profile

D) Power balance	b)	Power	bal	lance
------------------	----	-------	-----	-------

		Active P	ower (kW	)	Reactive Power (kVAR)				
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q	
Grid Power	431.12	318.62	480.19	1229.93	203.28	80.43	47.11	330.82	
DG Power	430.92	353.48	450.00	1234.40	344.86	281.03	246.17	872.06	
Total Gen.	862.04	672.11	930.19	2464.33	548.14	361.46	293.28	1202.88	
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00	
Loss	2.98	1.52	2.83	7.33	-0.44	0.56	1.76	1.88	

**Table 3** Results of conventional load flow with total 3-phase power of the DG unitspecified at 1234.4 kW

Bus No.	Volta	ige Magnitude	e (p.u)	Voltage Phase angle (Degrees)				
	Va	V <sub>b</sub>	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{\rm c}$		
799	1.000	1.000	1.000	-30.00	-150.00	90.00		
708	0.996	1.001	1.003	-30.50	-150.22	89.43		
INT	1.0198	1.0198	1.0198	-28.94	-148.94	91.06		

#### a) The voltage profile

		Active Po	ower (kW	)	Reactive Power (kVAR)				
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q	
Grid Power	431.14	318.65	480.21	1230.00	203.27	80.42	47.10	330.82	
DG Power	430.90	353.47	449.98	1234.40	344.86	281.04	246.18	872.06	
Total Gen.	862.04	672.12	930.19	2464.40	548.13	361.46	293.28	1202.88	
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00	
Loss	2.98	1.53	2.84	7.40	-0.45	0.56	1.76	1.88	

#### b) Power balance

Case 2: Two DG units of rating 600kW (DG1) and 750kW (DG2) are installed at bus 720 and bus 708 of the IEEE 37 network, respectively. Results are shown in Table4, Table5 and

Table 6.

Table 4 shows the results of conventional load flow with both DG units operating at their full 3-phase kW capacity. At this condition, phase-b and phase-c of DG1 are overloaded, while for DG2, phase-a and phase-c have become overloaded. For both DG units, phase-c is taken as the critical phase (the overloaded phase with higher loading) for the load flow with the proposed modification. The c-phase output of DG1 was specified at 200kW and that of DG2 was specified at 250kW. From the corresponding results, given in Table 5, it is found that the maximum allowable 3-phase output for DG1 and DG2 are 562.13kW and 643.43kW, respectively under the given operating condition. Table 6 shows the results of running the system at the given operating condition with the set point of DG1 and DG2 fixed at 562.13 kW and 643.43 kW, respectively. From Table 5 and Table 6, it is found that the various voltages and active and reactive powers are the same in the two Tables.

		Power balance										
	Active Power (kW)				Reactive Power (kVAR)							
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q				
Grid Power	401.11	285.35	435.01	1121.47	16.86	-88.60	-132.55	-204.29				
DG1 Power	183.32	204.24	212.44	600.00	126.39	103.56	132.66	362.61				
DG2 Power	280.21	184.21	285.57	750.00	407.25	346.31	294.43	1047.99				
Total Gen.	864.65	673.81	933.02	2471.47	550.50	361.27	294.54	1206.31				
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00				
Loss	5.59	3.22	5.67	14.47	1.92	0.36	3.02	5.31				

**Table 4** Results of conventional load flow with two DG units with their 3-Phasepower output specified at 600 kW and 750 kW

**Table 5** Results of the proposed method with the critical phase power output of thetwo DG units specified at 200 kW and 250 kW

		Power balance										
	Active Power (kW)				Reactive Power (kVAR)							
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q				
Grid Power	447.84	338.32	484.81	1270.96	-41.40	-146.03	-184.25	-371.68				
DG1 Power	171.69	190.44	200.00	562.13	141.08	117.79	145.07	403.95				
DG2 Power	247.02	146.41	250.00	643.43	452.23	390.73	335.49	1178.44				
Total Gen.	866.56	675.16	934.81	2476.53	551.91	362.48	296.32	1210.71				
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00				
Loss	7.50	4.58	7.45	19.53	3.33	1.58	4.80	9.71				

**Table 6** Results of conventional load flow with 3-phase power output of the two DGunits specified at 562.13 kW and 643.43kW

		Power balance										
		Active P	ower (kW	7)	Reacti							
	Pa	P <sub>b</sub>	Pc	Total P	Qa	Q <sub>b</sub>	Qc	Total Q				
Grid Power	447.84	338.32	484.81	1270.96	-41.40	-146.03	-184.25	-371.69				
DG1 Power	171.70	190.44	200.00	562.13	141.10	117.81	145.09	404.00				
DG2 Power	247.02	146.41	250.00	643.43	452.21	390.71	335.47	1178.39				
Total Gen.	866.56	675.16	934.80	2476.53	551.90	362.48	296.32	1210.70				
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00				
Loss	7.50	4.58	7.45	19.53	3.33	1.58	4.80	9.70				

Case 3: Conventional load flow is run with the 3-phase output of DG-1 specified at 500kW (much lower than its rating) and that of DG2 specified at its rated value 750kW connected at bus 720 and bus 708 of the IEEE 37 network.

From the corresponding results, shown in Table 7, it is found that while the kW loading of each of the three phases of DG1 are within the limit of 200kW, phase-a and phase-c of DG2 are overloaded. DG2 was then taken as the critical generator with its c-phase as the critical phase for which the output was specified at 250kW. A hybrid load flow was run with the proposed modification applied for DG2 only while retaining the conventional formulation for DG1. Table 8 shows the corresponding results. From the results, it is found that the maximum allowable 3-phase output of DG2 under the given condition is only 642.52kW. The conventional load flow was then run with the output of DG1 set at 500kW and that of DG2 set at 642.52kW. The corresponding results shown in Table 9 corroborates with those in Table 8.

**Table 7** Results of conventional load flow with 3-phase power output of the two DG

 units specified at 500 kW and750 kW

		Power balance							
	Active Power (kW)				Reactive Power (kVAR)				
	Pa	$P_b$	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Q <sub>c</sub>	Total Q	
Grid Power	432.91	321.36	469.16	1223.43	-26.25	-131.35	-171.38	-328.98	
DG1 Power	151.95	169.60	178.45	500.00	176.85	154.65	180.29	511.79	
DG2 Power	280.36	183.58	286.06	750.00	400.35	338.63	286.34	1025.31	
Total Gen.	865.22	674.54	933.67	2473.43	550.94	361.93	295.25	1208.12	
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00	
Loss	6.16	3.95	6.31	16.43	2.37	1.02	3.73	7.12	

**Table 8** Results of hybrid Load flow with the 3-phase power output of DG-1 specifiedat 500 kW and phase-c output of DG2 specified at 250 kW

		Power balance							
	Active Power (kW)				R	Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q	
Grid Power	467.98	361.11	506.42	1335.51	-68.53	-172.92	-208.66	-450.11	
DG1 Power	152.21	168.91	178.88	500.00	172.46	149.56	174.69	496.70	
DG2 Power	246.83	145.69	250.00	642.52	448.34	386.35	330.83	1165.52	
Total Gen.	867.02	675.71	935.29	2478.03	552.27	362.99	296.86	1212.11	
Total load Loss	859.06 7.97	670.59 5.13	927.35 7.94	2457.00 21.03	548.58 3.69	360.90 2.09	291.52 5.34	1201.00 11.11	

		Power balance							
	Active Power (kW)				Reactive Power (kVAR)				
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q	
Grid Power	467.99	361.12	506.42	1335.53	-68.54	-172.93	-208.67	-450.14	
DG1 Power	152.21	168.91	178.88	500.00	172.46	149.56	174.69	496.70	
DG2 Power	246.83	145.69	250.00	642.52	448.35	386.36	330.84	1165.54	
Total Gen.	867.02	675.71	935.29	2478.03	552.27	362.99	296.85	1212.11	
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00	
Loss	7.97	5.13	7.94	21.03	3.69	2.09	5.34	11.11	

**Table 9** Results of conventional load flow with 3-phase power output of the two DGunits specified at 500 kW and 642.52 kW

### 4.5 Chapter Summary

This chapter has presented a methodology for a more effective operation and planning of power systems under unbalanced operating conditions. The methodology is based on a modification in the N-R based 3-phase load flow. With the introduction of the suggested modification, limits on the output of the maximum loaded phase of a generator can be imposed. As a result, safe loading limit (total 3-phase) and the corresponding output set point of the generators can be determined so that partial overloading of the generators under unbalanced operating conditions can be avoided. Results for a number of case studies demonstrate the efficacy of the suggested methodology.

# **CHAPTER 5**

# Consideration of Partial Overloading Elimination Issue in Direct Approach Load Flow

Outline of the chapter

- 5.1 Introduction
- 5.2 The Proposed Methodology
- 5.3 Case studies
- 5.4 Chapter Summery

#### **5.1 Introduction**

In the previous chapter, a methodology based on conventional N-R 3-phase load flow was proposed to determine safe loading limit of generators in the network to eliminate partial overloading of them under unbalanced operating condition. The problem of partial overloading of generators has been introduced in section 1.5 of chapter 1. Conventional N-R load flow method frequently encounters convergence difficulties in case of distribution networks due to the special characteristics of such networks as mentioned in section 1.4 of chapter 1. It is therefore necessary to device a methodology to consider partial overloading issue in distribution load flow problem. In this chapter, a methodology is suggested based on a modification in the formulation presented in the previous chapter for incorporating DG models indirect approach load flow.

#### 5.2 Proposed Methodology

A methodology to incorporate DG models in direct approach load flow has been presented in Chapter 3 where, in every iteration, equation (3.2) is used to update the active power at the internal node of the DG unit before going to the next iteration in order to include the effect of internal active power losses of the unit, and also to include the correction terms necessary to obtain a balance in the 3-phase internal node voltages. Equation (3.2) is reproduced below for ready-reference and given in equation no. (5.1).

$$P_{G,p}^{k+1} = P_{g,p}^{k+1} + C/3 + R \left( I_{G,p}^{k} \right)^2 + \Delta P_{G,p}^{k+1}$$
(5.1)

In which  $C = \left| P_{3sp} - P_{3g}^{k+1} \right|$ 

 $P_{3sp}$  is the total 3-phase active power specified for the DG unit.

 $P_{3g}^{k+1} = \sum_{p} P_{g,p}^{k+1}$  is the total 3-phase active power output of the DG unit, and  $P_{g,p}^{k+1}$  indicates the active power output of different phases of the DG unit, obtained with the updated values of its terminal voltage  $V_{g,p}^{k+1}$  at the end of  $k^{\text{th}}$  iteration.

The approach suggested in this chapter modifies the expression of C to make it applicable for dealing with the issue of partial overloading. The modified expression is given in equation (5.2).

$$C = \left| P_{3sp} - 3 \times \max\left( P_{g,p}^{k+1} \right) \right| \tag{5.2}$$

#### **5.3 Case Studies**

The efficacy of the proposed methodology has been tested on the IEEE 37 node distribution feeder network and 25 node URDN. The schematic single line diagrams of the two systems are shown in Appendix IV. In this study, the voltage regulator at bus 799 of the IEEE 37 node network has not been considered. The two networks are augmented by employing DG units and connecting the DG internal nodes to the respective phases of the generator terminal buses through the internal impedances. The DG internal bus (3-phase) is denoted as INT. Bus 799 and bus 1 are the grid buses of the two systems, and have been taken as the slack buses for the load flow solutions.

Three case studies have been conducted as follows:

Case 1: A DG unit of 1350 kW rating is connected at bus 708 of IEEE 37 Feeder and operated in P-V mode with the average magnitude of the terminal voltages of the three phases maintained at a specified value of 1.0 p.u.

From Table 1of the previous chapter it is found that if the DG unit is operated with its total 3-phase capacity of 1350 kW, the maximum loaded phase (phase-c) of the DG unit delivers 488.59 kW which exceeds the per phase limit of (1350/3)=450 kW. Here, Table 1(furnished below) shows the results of incorporating the proposed modification (equation (5.5)) in the direct approach load flow with phase-c output set at 450 kW. The total 3-phase output of the DG unit is now 1234.4 kW, which should be considered as the maximum allowable 3-phase kW output of the DG unit under the operating condition considered. The DG unit must be run with its output setting fixed at 1234.4 kW to get the maximum kW output under the given operating condition without overloading any of its phases.

**Table 1** Results of incorporating the suggested modification in the direct approach
 load flow with the critical phase power output of the DG unit specified at 450kW

Bus No.	Volta	ge Magnitude	(p.u.)	Voltage Phase Angle (degrees)			
	Va	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
INT	1.0198	1.0198	1.0198	-28.94	-148.94	91.06	
799	1.0000	1.0000	1.0000	-30.00	-150.00	90.00	
708	0.9963	1.0009	1.0029	-30.51	-150.22	89.42	

w) interventer promise
------------------------

	Active Power (kW)				Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	$Q_a$	Q <sub>b</sub>	Qc	Total Q
Grid Power	431.12	318.63	480.19	1229.93	203.28	80.43	47.11	330.82
DG Power	430.92	353.49	450.00	1234.40	344.86	281.03	246.17	872.06
Total Gen.	862.04	672.11	930.19	2464.33	548.14	361.46	293.28	1202.88
Total load	859.06	670.59	927.35	2457	548.58	360.9	291.52	1201
Loss	2.98	1.52	2.84	7.33	-0.44	0.56	1.76	1.88

b) Power balance

Case 2: Two DG units of total 3-phase rating of 600 kW (DG1) and 750 kW (DG2) are installed respectively at bus 720 and bus 708 of IEEE 37 node Feeder.

It has been shown previously (Table 4 of chapter 4)that when the two DG units operate at their full 3-phase kW capacity phase-b and phase-c of DG1 are overloaded, while for DG2, phase-a and phase-c have become overloaded. For both DG units, phase-c is taken as the critical phase. Table 2 given below shows the results of adopting the proposed modification in the load flow with c-phase output of DG1 specified at 200 kW and that of DG2 specified at 250 kW. To keep focus on the main aspect of the results, only active and reactive power output of the generators are shown. It is found that the maximum allowable 3-phase output for DG1 and DG2 are 562.13 kW and 643.43 kW respectively, under the given operating condition. Hence, to get the maximum kW output from the two generating units without causing any partial overloading at the given operating condition, the system should run with set point of DG1 and DG2 fixed at 562.13 kW and 643.43 kW, respectively.

	Power balance								
		rower balance							
	Active Power (kW)				Reactive Power (kVAR)				
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc	Total Q	
Grid Power	447.84	338.32	484.81	1270.96	-41.40	-146.0	-184.3	-371.68	
DG1 Power	171.70	190.44	200.00	562.13	141.08	117.79	145.08	403.95	
DG2 Power	247.03	146.41	250.00	643.43	452.23	390.73	335.49	1178.44	
Total Gen.	866.56	675.16	934.81	2476.53	551.91	362.49	296.31	1210.71	
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00	
Loss	7.50	4.57	7.46	19.53	3.33	1.59	4.79	9.71	

**Table 2** Results of adopting the suggested modification in the direct approach loadflow with the critical phase power output of DG1 and DG2 specified respectively at200 kW and 250 kW

Case 3: The system considered in case 2 is operated with the 3-phase output of DG1 set at 500 kW (much lower than its rating) and that of DG2 specified at its rated value 750 kW.

In Table 7 of the previous chapter it is found that while the kW loading of each of the three phases of DG1 are within the limit of 200 kW, phase-a and phase-c of DG2 are overloaded. DG2 was then taken as the critical generator with its c-phase as the critical phase for which the output was specified at 250 kW. A hybrid load flow was run with the proposed modification applied for DG2 only while the formulation used in chapter 3 is retained for DG1. Table 3 given below shows the corresponding results. From the results, it is found that the maximum allowable 3-phase output of DG2 under the given condition is only 642.52 kW. That means, under the given operating condition with DG1 operated at 500 kW, DG2 should be operated with its set point fixed at 642.52 kW to get the maximum possible output from it without causing partial overloading of the unit.

	Power balance							
	Active Power (kW)				Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	$Q_{a}$	Q <sub>b</sub>	Qc	Total Q
Grid Power	467.98	361.11	506.42	1335.51	-68.53	-172.92	-208.66	-450.11
DG1 Power	152.21	168.91	178.88	500.00	172.46	149.56	174.69	496.70
DG2 Power	246.83	145.69	250.00	642.52	448.34	386.35	330.83	1165.52
Total Gen.	867.03	675.71	935.30	2478.03	552.27	362.99	296.86	1212.12
Total load	859.06	670.59	927.35	2457.00	548.58	360.90	291.52	1201.00
Loss	7.97	5.12	7.94	21.03	3.69	2.09	5.34	11.11

**Table 3** Results with the 3-phase power output of DG-1 specified at 500 kW and thatof DG-2 critical phase power specified at 250 kW

## 5.4 Chapter Summary

This chapter has dealt with the problem of taking into consideration the issue of elimination of generator partial overloading while solving direct approach load flow. Modification necessary in the formulation given in chapter 3 to incorporate DG models in direct approach load flow has been presented and applied in case studies. Three case studies have been undertaken. The results demonstrate the effectiveness of the proposed method.

# CHAPTER 6

# Optimal size and location of Distributed Generation for Minimization of Network Power Loss

Outline of the chapter

- 6.1 Introduction
- 6.2 Problem Formulation
- 6.3 Solution methodology adopted
- 6.4 Case studies and results
- 6.5 Chapter Summery

#### **6.1 Introduction**

In this chapter the problem of optimal sitting and sizing of DG has been addressed. The objective is to find the proper size and location of DG in distribution network that minimizes the power loss in the network. The problem has been formulated as an optimization problem where PSO technique [56] is used to find the optimal solution that furnishes the required size and bus location of the DG. The PSO in this application utilizes load flow program as ancillary for the computation of network power loss. Two different types of studies have been undertaken. In one type, kVAR support along with the DG has been considered. The load flow used in this study for calculating the network power loss employs the approach suggested by earlier authors [8], [37] for inclusion of DG models. No consideration of constraint on partial overloading limit has been considered. In the second type, modified approach for load flow proposed in this thesis has been applied for loss calculation. Imposition of partial overloading limit was considered. The main objective of this study was to investigate the effect of introducing any limit on partial overloading of DG on the solution of the sitting and sizing problem.

#### **6.2 Problem Formulation**

The problem is posed as an optimization problem where the power loss in a distribution network is minimized by manipulating the DG size and location. So, the network power loss is the objective function in the problem, and DG size and location constitute the control (manipulating) variables.

#### 6.2.1 The objective function

Bus voltages and branch currents in a distribution network vary with the size and location of DG units in the network. Consequently the total network power loss which is equal to the sum of the branch power losses, is a function of DG size and location.

Thus,

$$total \_ loss = f(P_g, Q_g, N_g)$$
 for DG operating in P-Q mode (6.1)

$$total\_loss = f(P_g, N_g)$$
 for DG operating in P-V mode (6.2)

where,

total\_loss indicates total network power loss

$$total\_loss = \sum Branch\_losses \tag{6.3}$$

$$Branch_loss = \left| real \{ (Branch_input) - (Branch_output) \} \right|$$
(6.4)

$$Branch\_input = (V_{sa} \times b_a^* + V_{sb} \times b_b^* + V_{sc} \times b_c^*)$$
(6.5)

$$Branch\_output = (V_{ra} \times b_a^* + V_{rb} \times b_b^* + V_{rc} \times b_c^*)$$
(6.6)

 $V_{sa}$ ,  $V_{sb}$  and  $V_{sc}$  are sending end voltages and  $V_{ra}$ ,  $V_{rb}$  and  $V_{rc}$  are receiving end voltages of the three phases a, b, c of the branch.

 $b_a$ ,  $b_b$  and  $b_c$  are the branch currents.

 $P_g$  and  $Q_g$  are respectively the active and reactive power output of DG

 $N_g$  is bus location of the DG.

For a given loading condition of a network, and specified values of  $P_g$ ,  $Q_g$ ( $Q_g$  is specified for P-Q bus only and not for P-V bus) and  $N_g$ , the total network power loss can be obtained from distributed load flow results. So, the function f in equation (6.1) is an implicit function of  $P_g$ ,  $Q_g$ ,  $N_g$  (or simply  $P_g$ ,  $N_g$ ).

#### **6.2.2** The Constraints

The constraints reflect the limits on physical devices in the power system. They are given by

$$P_{g\min} \le P_g \le P_{g\max} \tag{6.7}$$

$$Q_{g\min} \le Q_g \le Q_{g\max} \tag{6.8}$$

where  $P_{gmin}$  and  $P_{gmax}$  are, respectively, the minimum and maximum limits of DG active power.  $Q_{gmin}$  and  $Q_{gmax}$  are, respectively, the minimum and maximum limits of DG reactive power.

#### 6.3 Solution methodology adopted

The proposed methodology uses PSO technique (Appendix-III) to solve the optimization problem. To calculate the loss for different values of the manipulating variables ( $P_g$ ,  $Q_g$ ,  $N_g$ ), the PSO algorithm calls a distribution load flow routine. In this study, the direct approach load flow (Appendix-I) has been applied. The proposed approach consists of two computing loops – an inner loop embedded in an outer loop. The outer loop, in each iteration, runs a PSO program to set the values of the elements of each particle of the swarm. In this study, a particle represents a set of input data (the manipulating variables) for the load flow to be executed by the inner loop, and consists of

DG size:  $P_g$  (within range of its limits) for PV mode

 $P_{g}$ ,  $Q_{g}$  (within range of their limits) for PQ mode

DG location:  $N_g$  the bus at which the DG is placed

In each PSO loop, the values of all the particles of the swarm are passed on to the inner loop which generates load flow solution with each particle. After getting back the load flow solutions from the inner loop, the outer loop calculates the network power loss corresponding to every load flow solution. The network power loss is used as fitness function (objective function) for the present study. The purpose of the PSO program is to find  $\min(f(P_g, Q_g, N_g))$ .

At the end of each PSO loop, the position (the last local best) and velocity of each particle along with the global best position are updated by the process given in Appendix-I for the next iteration. The process continues until convergence is reach as per the predefined criteria.

### 6.4 Case studies and results

The proposed method has been tested on the IEEE-37 node test feeder (Appendix-IV). Two types of studies have been undertaken. In the first type, the effect of DG placement along with kVAR support on loss minimization is investigated. In the related studies, previously proposed method [8],[37] has been followed to consider DG models in load flow solution. Only P-Q model of DG has been considered in the study. In the first case, only the effect of balanced and unbalanced kVAR support is investigated without the presence of any DG in the network. Table 1 shows the results with balanced support. The optimal balanced kVAR support for each bus are given in the Table. The result shows that bus no. 709 is the location for kVAR support to obtain minimum loss (maximum loss saving) and bus no. 740 is the location for getting best loss saving per unit kVAR support.

<b>Table 1</b> Optimal reduction in losses by balanced 3-phase kVAR support at different	
buses	

Bus No.	Optimal kVAR support to each phase $(Q_a=Q_b=Q_c)$	Optimal kVAR pport to each phaseloss with kVAR support (kW)loss sa kVAl $(Q_a=Q_b=Q_c)$ support (kW)(0)		loss saving per unit kVAR support
799	No support	60.575		
701	419.878	55.347	5.228	0.004151
702	368.716	53.551	7.024	0.006350
705	207.442	56.341	4.234	0.006803
712	164.352	57.157	3.418	0.006931
742	149.901	57.308	3.267	0.007264
713	285.838	54.585	5.990	0.006985
704	222.287	55.399	5.176	0.007762
714	202.537	55.735	4.840	0.007966
718	134.711	57.194	3.381	0.008365
720	169.264	56.175	4.400	0.008664
707	113.073	57.206	3.368	0.009930
724	84.125	57.963	2.612	0.010348
722	107.517	57.295	3.280	0.010168
706	138.055	56.948	3.627	0.008757
725	118.592	57.429	3.146	0.008843
703	295.934	53.353	7.221	0.008134
727	230.702	54.665	5.910	0.008539
744	204.414	55.217	5.358	0.008736
729	161.821	56.228	4.347	0.008955
728	175.681	55.879	4.696	0.008909
730	241.599	53.241	7.334	0.010119
709	230.718	53.226	7.349	0.010618
731	183.189	54.656	5.919	0.010770
708	218.874	53.311	7.264	0.011062
732	175.721	54.688	5.887	0.011167
733	203.758	53.321	7.254	0.011867
734	185.422	53.451	7.123	0.012806
710	146.456	54.749	5.826	0.013260
736	94.951	56.664	3.911	0.013730
735	135.361	55.142	5.433	0.013378
737	164.484	53.806	6.769	0.013718
738	153.318	54.141	6.433	0.013987
711	141.989	54.539	6.036	0.014170
740	132.128	54.910	5.665	0.014291
741	131.148	54.954	5.621	0.014287
775	105.070	57.339	3.236	0.010267

Table 2 illustrates the effect of unbalanced kVAR support and shows that the bus no. 730 is DG location for minimum loss (maximum loss saving) whereas bus no.

740 is the position for best loss saving per unit kVAR support. The corresponding optimal kVAR sizes are therefore Qa=316.33,  $Q_b$ =247.80,  $Q_c$ =158.72 and  $Q_a$ =168.16,  $Q_b$ =150.71,  $Q_c$ =79.04.

**Table 2** Optimal reduction in losses by unbalanced 3-phase kVAR support at different

 buses

	Optimal	size of kVAF	R Support	Loss with	Loss saving	Loss saving
Bus no.	0	0	0	KVAK	with kVAR	
	$Q_a$	(1-V(A D))	$Q_c$	support	support (kW)	KVAR
700		(KVAR)		(KW)		suport
799	607.25	No support	261.05	00.373		
701	607.23 505.20	391.20	201.95	54.555	0.040	0.0048
702	303.29	333.73	248.70	52.704	/.8/1	0.0071
705	288.29	183.87	150.27	55.841	4./34	0.0076
712	232.83	142.91	11/./0	56.703	3.872	0.0078
742	201.25	127.21	118.00	57.003	3.5/1	0.0080
/13	3/9.51	264.93	213.42	54.026	6.549	0.0076
704	282.39	201.43	182.99	55.076	5.499	0.0082
714	254.12	183.54	168.31	55.460	5.115	0.0084
718	164.96	126.96	110.91	57.039	3.536	0.0088
720	210.58	142.54	153.75	55.943	4.632	0.0091
707	134.27	84.13	119.48	57.021	3.554	0.0105
724	96.79	60.61	93.92	57.803	2.772	0.0110
722	125.50	78.95	116.55	57.108	3.467	0.0108
706	170.80	115.30	127.52	56.762	3.813	0.0092
725	146.83	97.60	110.94	57.266	3.309	0.0093
703	399.62	303.77	188.74	52.409	8.166	0.0092
727	310.21	237.60	147.72	53.917	6.658	0.0096
744	270.58	212.05	133.74	54.585	5.990	0.0097
729	212.59	167.15	106.35	55.737	4.837	0.0100
728	232.49	181.15	115.92	55.341	5.234	0.0099
730	316.33	247.80	158.72	52.384	8.191	0.0113
709	299.41	238.68	152.41	52.395	8.180	0.0118
731	234.07	185.59	128.17	54.092	6.483	0.0118
708	284.08	235.83	139.96	52.413	8.162	0.0124
732	231.15	187.59	110.78	53.925	6.650	0.0126
733	261.01	221.34	130.09	52.439	8.136	0.0133
734	236.46	206.78	115.66	52.529	8.046	0.0144
710	190.23	159.60	91.06	53.987	6.588	0.0149
736	121.86	98.51	63.57	56.266	4.309	0.0152
735	177.43	146.74	83.23	54,404	6.171	0.0151
737	206.09	186.81	101.70	52.918	7.657	0.0155
738	191.78	176.93	93.39	53.260	7.315	0.0158
711	179.13	162.56	86.04	53 690	6 885	0.0161
740	168.16	150.71	79.04	54.087	6.487	0.0163
741	166.09	147 58	80.25	54,174	6.401	0.0162
775	148.51	109.11	59.30	56.845	3.730	0.0118

Table 3 exhibits the effect of using both DG (with balanced active and reactive power output) and balanced kVAR support. The results shows that bus no. 709 is the

location where DG should be placed along with kVAR source for maximum loss reduction and bus no.741 is the location fo rbest loss saving per unit kVA support.

**Table 3** optimal reduction in losses by both DG and balanced 3-phase kVAR support

 at different buses

	Optimal	DG size	Loss with	Loss	Loss
DG placed at bus no.	$P_a = P_b = P_c$	$Q_a = Q_b = Q_c$	DG	saving with DG	saving per
ut ous no.	(kW)	(kVAR)	(kW)	(kW)	unit kVA
799	No s	upport	60.575		
701	873.89	247.93	34.732	26.439	0.0097
702	776.66	216.78	25.373	35.799	0.0148
705	442.36	123.57	39.050	22.121	0.0161
712	351.58	97.27	43.171	18.001	0.0164
742	310.89	89.83	43.953	17.219	0.0177
713	605.19	169.25	30.363	30.809	0.0163
704	472.84	134.34	34.260	26.912	0.0182
714	424.29	121.44	35.994	25.178	0.0190
718	279.79	80.12	43.304	17.868	0.0205
720	360.23	105.34	38.106	23.066	0.0205
707	241.42	75.72	43.100	18.072	0.0238
724	172.69	55.88	46.967	14.204	0.0261
722	224.65	71.46	43.570	17.602	0.0249
706	294.53	86.35	42.000	19.172	0.0208
725	253.41	74.78	44.418	16.753	0.0211
703	636.43	170.44	23.552	37.620	0.0190
727	497.85	132.48	30.271	30.901	0.0200
744	441.86	116.92	33.098	28.074	0.0205
729	344.92	91.61	38.262	22.910	0.0214
728	380.45	100.80	36.465	24.707	0.0209
730	501.60	132.61	23.275	37.897	0.0243
709	478.59	125.88	23.202	37.970	0.0256
731	382.59	101.38	30.178	30.994	0.0261
708	474.10	121.92	23.503	37.669	0.0257
732	383.32	98.22	30.350	30.822	0.0260
733	431.84	110.28	23.686	37.486	0.0280
734	402.82	101.30	24.360	36.812	0.0295
710	321.06	81.20	30.706	30.466	0.0307
736	199.90	52.31	40.257	20.915	0.0337
735	297.50	74.99	32.674	28.498	0.0310
737	349.36	86.75	26.208	34.964	0.0324
738	335.02	82.68	27.832	33.340	0.0322
711	310.99	76.49	29.821	31.351	0.0326
740	290.16	71.21	31.657	29.515	0.0329
741	280.37	68.89	31.921	29.251	0.0338
775	229.05	64.85	43.687	17.484	0.0245

Table 4 illustrates optimal DG with unbalanced kVAR support, loss saving with DG support and loss saving per unit DG (kVA) support, and shows that the bus no.741has the best loss saving per unit kVA support.

DC	Optima	l size of D	G & Reactive	e Support	Loss soving with	Loss
placed at	$P_a = P_b = P_c$	$Q_a$	$Q_b$	$Q_c$	DG & WAP	saving per
bus no	(1.11)				$\frac{DU}{dK} \sqrt{AK}$	unit kVA
ous no.	(KW)		(KVAK)		support (KW)	suppport
799		No	support			
701	855.23	526.74	384.67	339.17	27.318	0.010
702	762.94	448.13	349.68	308.53	37.160	0.015
705	437.75	274.19	184.81	173.00	22.918	0.016
712	348.55	225.29	144.19	133.36	18.661	0.016
742	308.02	187.68	127.53	129.86	17.581	0.017
713	597.55	352.01	263.85	249.47	31.915	0.016
704	468.82	270.32	201.95	206.72	27.660	0.018
714	420.61	239.08	183.77	186.91	25.813	0.018
718	277.86	155.26	127.01	120.18	18.176	0.020
720	358.22	206.62	143.41	169.65	23.480	0.020
707	240.86	135.07	85.35	129.63	17.987	0.022
724	172.10	92.48	62.34	97.10	13.929	0.024
722	224.03	122.69	80.53	123.70	17.455	0.023
706	293.21	169.54	116.31	140.26	19.414	0.020
725	252.46	146.67	98.64	121.48	16.872	0.020
703	625.79	359.31	303.36	236.27	39.384	0.019
727	491.28	288.26	239.00	178.14	32.433	0.020
744	436.73	255.01	213.92	158.53	29.462	0.020
729	341.27	199.43	167.32	122.80	23.975	0.021
728	376.71	222.14	183.38	135.30	25.893	0.021
730	493.40	280.15	242.67	185.61	39.942	0.024
709	471.25	266.38	233.51	176.35	40.075	0.026
731	377.88	213.07	183.54	145.67	32.553	0.026
708	469.27	269.97	239.69	163.56	40.067	0.026
732	380.27	223.94	192.15	128.07	32.808	0.026
733	426.89	242.06	220.42	149.21	39.940	0.028
734	399.77	228.39	211.98	132.40	39.482	0.030
710	319.35	187.38	165.30	103.26	32.625	0.031
736	198.48	114.70	97.77	70.07	21.971	0.033
735	296.06	175.69	152.39	94.14	30.537	0.031
737	346.19	193.91	186.39	115.03	37.579	0.033
738	333.22	187.84	182.98	106.06	35.971	0.032
711	309.59	176.66	168.81	97.50	33.865	0.033
740	289.02	166.65	156.94	89.40	31.899	0.033
741	278.26	158.81	148.58	90.67	31.510	0.034
775	229.02	152.40	117.78	73.01	18.443	0.024

**Table 4** optimal reduction in losses by balanced 3-phase DG in co-ordination withunbalanced KVar support at different buses

The second type of study is done to investigate the effect of considering the constraint on partial overloading of DGs on the solution of the problem of optimal DG placement for loss minimization in distribution network. In this study, unlike the studies of the first type, the modified load flow proposed in chapter 3 is used when no partial overloading constraint is considered, and the load flow described in chapter 4 is applied when the problem is solved with partial overloading constraint considered. No additional kVAR support has been considered. IEEE 37 node network has been used for the case studies.

The results of the study without taking into account any constraint on partial overloading is shown in Table 5 for the significant buses only while a list of optimal solution for all the buses is given in Table 7. The positive sequence terminal voltage of the DG was specified at 1.0 p.u.. From Table 7 it is observed that bus no. 733 is the optimal DG location, and 1310kW is the optimal size. The corresponding value of loss is 20.5kW.From Table 5 per phase loading of the DG is found to be  $P_a$ =457.7kW,  $P_b$ = 378.0kW,  $P_c$ = 474.3kW. It is obvious that partial overloading has taken place in phase-a and phase-c of the DG as their loading has exceeded the allowable limit of per phase loading (1310/3  $\approx$  436.67 kW)

#### Table 5 Optimal size and location of DG without partial overloading

#### a) The voltage profile

	Bus No.	Voltag	ge Magnitude (	p.u.)	Voltage Phase Angle (Degree)			
		$V_{a}$	$V_b$	V <sub>c</sub>	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
	799	1.000	1.000	1.000	-30.0	-150.0	90.0	
	733	0.996	1.001	1.003	-30.2	-149.9	89.7	
	INT	1.015	1.015	1.015	-28.6	-148.6	91.4	

	Active Power (kW)				Reactive Power (kVAR)			
	Pa	$P_b$	Pc	Total P	$Q_a$	$Q_b$	Qc	Total Q
Grid Power	404.3	293	458.1	1155.4	280.6	152.3	121.6	554.5
DG Power	457.7	378	474.3	1310	269.9	208.7	170.3	648.9
Total load	855.1	665.7	924.1	2444.9	547.7	359.7	287.7	1195.1
Loss	6.9	5.3	8.3	20.5	2.8	1.3	4.3	8.4

#### b) Power balance

Table 6 shows the corresponding results for the significant buses with partial overloading constraint considered in the study. A list of optimal solution for all the buses is given in Table 7 which reveals that bus no. 733 is the optimal DG location and 1430kW is the optimal size. The corresponding loss is 20.5kW. However, from Table 6 it is found that the DG can be loaded only up to 1317.2 kW (total 3-phase) because at this loading, the maximum loaded phase (phase-c) reaches the per phase limit of (1430/3) = 476.7 kW.

Table 6 Optimal size and location of DG with partial overloading is imposed

Bue No	Volta	ge Magnitude	(p.u.)	Voltage Phase Angle (Degree)			
Bus No.	$\mathbf{V}_{\mathrm{a}}$	$V_b$	V <sub>c</sub>	$\theta_{\mathrm{a}}$	$\theta_{b}$	$\theta_{\rm c}$	
799	1	1	1	-30	-150	90	
733	0.996	1.001	1.003	-30.2	-149.9	89.7	
INT	1.015	1.015	1.015	-28.5	-148.5	91.5	

		Active P	ower (kW	)	Reactive Power (kVAR)			
	Pa	P <sub>b</sub>	Pc	Total P	Qa	$Q_{b}$	Qc	Total Q
Grid Power	402.1	290.5	455.7	1148.2	283.7	155.4	124.4	563.5
DG Power	460	380.6	476.7	1317.2	266.8	205.6	167.5	639.9
Total load	855.1	665.7	924.1	2444.9	547.7	359.7	287.7	1195.1
Loss	7	5.3	8.2	20.5	2.8	1.3	4.2	83

### b) Power balance

Table 7 shows the results of optimal DG sizes at all the buses of IEEE 37 node network with and without partial over loading constraint imposed.

DG bus	Without parts	ial over loading	With partial over loading		
20045	DG size (kW)	Total loss(kW)	DG size(kW)	Total loss(kW)	
799	-	-	-	-	
701	2500	34.91	2500	34.96	
702	2240	24.58	2340	24.58	
705	1390	38.91	1500	38.91	
712	1140	43.29	1260	43.29	
742	1080	44.45	1180	44.45	
713	1750	29.53	1850	29.53	
704	1430	34.03	1510	34.03	
714	1340	35.92	1420	35.92	
718	950	43.91	1030	43.91	
720	1110	38.36	1180	38.36	
707	780	44.15	820	44.15	
724	600	48.24	650	48.23	
722	750	44.64	790	44.64	
706	910	42.53	980	42.53	
725	800	45.17	870	45.17	
703	1830	21.68	1940	21.68	
727	1490	28.69	1600	28.69	
744	1330	31.73	1430	31.73	
729	1110	37.43	1200	37.44	
728	1180	35.44	1270	35.44	
730	1560	20.81	1660	20.85	
709	1490	20.61	1600	20.62	
731	1210	28.47	1310	28.47	
708	1390	20.55	1510	20.55	
732	1160	28.02	1280	28.02	
733	1310	20.5	1430	20.50	
734	1190	20.8	1300	20.80	
710	980	27.93	1080	27.93	
736	680	39.27	740	39.28	
735	920	30.08	1020	30.08	
737	1070	22.48	1170	22.48	
738	1000	24.12	1110	24.12	
711	930	26.27	1030	26.27	
740	880	28.3	980	28.30	
741	870	28.64	970	28.64	
775	710	49.78	780	49.78	

**Table 7** Optimal sizes of DG at all the buses with and without partial over loading

Now a load flow is solved with a DG of 1310 kW rating installed at bus no. 733. The specified positive sequence terminal voltage of the DG is 1.0p.u..With partial overloading limit imposed, the DG can now be loaded upto 1194.71 kW only as, at this

loading, the maximum loaded phase (phase-c) reaches the per phase limit of 436,67 kW. Table 8 shows the corresponding results. The resulting loss is also much higher in this case.

Table 8 Load flow results with DG of 1310 kW capacity installed at Bus no.733.

Bus No.	Volta	age Magnitud	le (p.u.)	Voltage Phase Angle (Degree)			
	Va	V <sub>b</sub>	$V_{c}$	$\theta_{a}$	$\theta_{b}$	$\theta_{c}$	
799	1	1	1	30.00	-90.00	150.00	
733	0.997	1.004	0.999	29.62	-90.45	149.26	
INT	1.021	1.021	1.021	30.91	-89.09	150.91	

a)	The	voltage	profile
u)	1110	vonuge	prome

		Active p	ower (kW	Reactive power (kVAR)			
	Pa	P <sub>b</sub>	P <sub>c</sub>	Total P	Qa	Q <sub>b</sub>	Qc
Swing bus	448.45	336.93	499.53	1284.91	185.40	61.17	30.95
DG	418.92	339.12	436.67	1194.71	366.77	301.23	265.52
Total Gen	867.37	676.05	936.20	2479.62	552.18	362.40	296.47
Total Load	855.06	665.74	924.11	2444.90	547.70	359.70	287.70
Loss	12.31	10.32	12.09	34.72	4.48	2.70	8.77

#### b) Power balance

## 6.5 Chapter Summary

The important problem of optimal sizing and sitting of DG units to minimize the network power loss in distribution network has been addressed in this chapter. The problem has been solved taking into account the partial overloading constraint to demonstrate its effect on the optimal DG placement and the resulting loss. The results of the study have shown that with partial overloading limit considered, higher DG size is required to achieve same amount of loss reduction. If DG of same size is used then with the DG operated with partial over loading limit imposed can be loaded up to a much lower value than its full capacity and loss reduction is less compared to that achieved with the DG operated without the constraint, but a safer operation of the DG ensured.

# CHAPTER 7

# CONCLUSIONS AND FURTHER SCOPE OF STUDIES

Outline of the chapter

7.1 Conclusions

7.2 Future scope of studies

## 7.1 CONCLUSIONS

- 1. The present research work has focused on some issues on optimal planning and operation of DGs in unbalanced distribution networks. Three-phase distribution load flow analysis plays an extremely important role in the studies related to such problems. The existing methods for considering models in 3phase distribution load flow programs suffer from several limitations. In chapter 2 and chapter 3 of this thesis, two methods have been proposed to overcome those limitations. Efficacy of the two methods have been shown by necessary results on some benchmark distribution systems.
- 2. Distribution systems usually operate at relative high degree of unbalanced condition, and are, therefore, prone to the problem of partial over loading of its DGs. According to the existing literature, no attention has been given so far to this problem. In this thesis the term 'partial overloading' has been defined and the problem has been explored in detail in chapter 4 and chapter 5. The methodologies are based on load flow calculation with appropriate modifications introduced in the formulations to impose limits on the output of the maximum loaded phase of DG units. In chapter 4, a N-R load flow based methodology is presented, while in chapter 5, a method more suitable for distribution system has been proposed which is based on the load flow formulation presented in chapter 3. The outcome of these studies enables the power system planners and operators to determine the maximum loading limits of the DGs units in the system without causing partial overloading of those units. Thus, these studies will be helpful for planning and operation by the distribution network operators for safe operation of DGs in the system.
- 3. The important problem of optimal sizing and sitting of DG units to minimize the network power loss in distribution network has been addressed in chapter 6. This problem was previously addressed by a number of researchers. But in this thesis, the problem has been solved taking into account the partial overloading constraint to demonstrate its effect on the optimal DG placement and the resulting loss. The results of the study have shown that with partial overloading constraint considered, higher DG size is required to achieve the

same benefit in terms of loss reduction. Thus the same benefit in loss reduction is achieved at the cost of higher capital expenditure. If DG of same size is used with and without imposition of partial over loading limit, then in case of operation with partial over loading limit, the DG can be loaded upto a lower value . In that case higher loss takes place but it provides the benefit of safer operation of the DG.

## **7.2 FUTURE SCOPE OF STUDIES**

- 1. In the present studies related to partial overloading, only the active power loading of the generators (DGs) has been considered. However, more appropriate variable to be considered will be the armature current in each phase of the DGs. Further study is therefore required to modify the formulation of the problem to consider current over loading instead of active power over loading.
- 2. In this thesis, the studies done on partial overloading generate solutions that give the safe loading limits of the DGs for a given loading condition only. Accordingly, the optimal DG planning considering partial overloading has been carried out for the given loading condition. The problem can be extended for energy minimization (instead of loss minimization) over some planning period.

# Appendices

Outline of the chapter

Appendix-I	A Brief Introduction to Direct Approach load flow
Appendix-II	DG Active Power Correction
Appendix-III	A Brief Introduction to PSO Algorithm
Appendix-IV	Line diagrams of the three phase Networks

#### Appendix-I

#### A Brief Introduction to Direct Approach load flow

The direct approach load flow uses two constant matrices, [*BIBC*] and [*BCBV*] to update the node voltages in each iteration [38]. The relation between branch currents and bus injected currents can be established as follows

$$[B] = [BIBC][I] \tag{A-1}$$

where, [B] and [I] are, respectively, the vectors of branch currents and bus injected currents.

The relation between bus voltages and branch currents can be given by

$$[V] = [V_0] - [BIBC][B]$$
(A-2)

where,

[V] is the vector of bus voltages.

 $[V_0]$  is a vector where all entries are equal to the slack bus voltage.

From equation (A-1) & (A-2)

$$[V] = [V_0] - [BCBV][BIBC][I]$$
  
=  $[V_0] - [DLF][I]$  (A-3)

where,

[DLF] = [BCBV][BIBC]

The node voltage vector can be updated in each iteration with the help of equation (A-3) as follows

$$\begin{bmatrix} V^{k+1} \end{bmatrix} = \begin{bmatrix} V_0 \end{bmatrix} - \begin{bmatrix} DLF \end{bmatrix} \begin{bmatrix} I^k \end{bmatrix}$$
(A-4)

where  $[I^k]$  is obtained by calculating the node injected current for each of the three phases.

For the  $i_{th}$  node, the current is given by

$$I_{i,p}^{k} = \left(\frac{P_{i,p} + jQ_{i,p}}{V_{i,p}^{k}}\right)^{*}$$
(A-5)

Here, k indicates  $k^{th}$  iteration.

p=a, b, c indicates the phases.

P and Q, respectively, indicates active and reactive power.

After each iteration, convergence is checked using the following relation

 $Max \left| V^{k+1} - V^k \right| \le \varepsilon$ 

where,

 $\epsilon$  is the specified tolerence.

#### Appendix-II

#### DG Active Power Correction

The active power and reactive power flowing in any of the phases of a 3-phase DG from its internal node to its terminal node are given by equations (A-6) and (A-7) as follows

$$P = EV[G\cos(\delta - \theta) + B\sin(\delta - \theta)]$$
(A-6)

$$Q = EV[G\sin(\delta - \theta) - B\cos(\delta - \theta)]$$
(A-7)

where,

*P* and *Q* indicate active power and reactive power respectively.

*E* indicates the internal voltage magnitude of the concerned phase.

V indicates the terminal voltage magnitude of the concerned phase.

 $\delta$  indicates the angle of the internal voltage of the concerned phase.

- $\boldsymbol{\theta}$  indicates the angle of the terminal voltage of the concerned phase.
- *G* is the conductance per phase of the DG
- *B* is the susceptance per phase of the DG

The resistance of a generator is significantly less than its reactance, hence, neglecting G in equations (A-6) and (A-7) these two can be converted into the following form.

$$P = EV[B\sin(\delta - \theta)] \tag{A-8}$$

$$Q = EV[-B\cos(\delta - \theta)] \tag{A-9}$$

Now,

$$\frac{\partial P}{\partial \delta} = EVB\cos\left(\delta - \theta\right)$$

$$\frac{\partial P}{\partial \delta} = \overline{P} \tag{A-10}$$

where,

 $\overline{P}$  is called the synchronizing power co-efficient.

For any small changes in  $\boldsymbol{\delta},$  the corresponding change in P can be obtained as

$$\Delta P = \overline{P} \Delta \delta \tag{A-11}$$

#### Appendix-III

#### A Brief Introduction to PSO Algorithm

PSO is an evolutionary optimization technique developed by Kennedy and Eberhart in 1995 [56]. It is inspired by social behavior of fish schooling and bird flocking and has gained popularity owing to its easy implementation and a faster convergence rate than the traditional evolutionary algorithms [57].

The PSO algorithm starts with a swarm of particles, where position 'y' and velocity 'v' of each particle are randomly initialized for a size of population 'pop' in the search space. The search for optimal position is carried out by updating the velocities and positions of the particles iteratively using the following expressions:

$$v_{i}^{k+1} = w * v_{i}^{k} + c_{1} * rand_{1} * (pbest_{i}^{k} - y_{i}^{k}) + c_{2} * rand_{2} * (gbest_{i}^{k} - y_{i}^{k})$$
(A-12)  
$$y_{i}^{k+1} = y_{i}^{k} + v_{i}^{k+1}$$
(A-13)

where,  $v_i$  = velocity of the  $i^{th}$  particle

 $y_i$  = position of the  $i^{\text{th}}$  particle i=1,2,...,pop

 $c_1 = \text{cognitive constant}$ 

 $c_2 =$ social constant

w = inertial weight of particle

k = current iteration number.

The search of swarm is focused towards the particle best positions '*pbest*' (also called local best positions), which generates the minimum value of the objective function (also called fitness function) attained by each individual particle. The search continues for the entire population size. Finally, the global best position '*gbest*' is obtained from the best value of '*pbest*' positions that gives the global minimum of the objective function.
## Appendix-IV

Line diagrams of the three phase Networks

a) IEEE 13 Node Test Feeder [58].



**Fig. A-1** Line diagram of IEEE 13 Node Test Feeder Network and load data are available in reference **[58]**.



Fig. A-2 Line diagram of 25 Node URDN Feeder

Network and load data are available in reference [59].



Fig. A-3 Line diagram of IEEE 37 Node Test Feeder

Network and load data are available in reference [60].

## **BIBLIOGRAPHY:**

- [1] T. Ackermann, G. Andersson and L. Soder, "Distributed generation: a definition," Electric Power Systems Research, vol. 57, pp. 195-204, 2001.
- [2] W. El-Khattam and M.M.A. Salama, "Distributed generation technologies, definitions and benefits," Electric Power Systems Research, vol. 71, pp. 119-128, 2004.
- [3] Pepermans G., Driesen J., Haeseldonckx D., et al. "Distributed Generation: Definition, Benefits and Issues," Centre for Economic Studies, Energy Transport & environment, Energy Policy,. 36(5): 787-798, Aug, 2003.
- [4] P. A. Daly and J. Morrison, "Understanding the potential benefits of distributed generation on power delivery systems," IEEE Rural Electric Power Conference, pp. A2/1-13, May, 2001.
- [5] J.A. Lopes, N. Hatziargyriou, J. Mutale, P. Djapic and N. Jenkins, "Integrating distributed generation into electric power systems: A review of drivers, challenges and opportunities," Electric Power System Research, vol. 77, pp-1189-1203, Oct, 2007.
- [6] A. Losiand and M. Russo, "Dispersed generation modeling for object-oriented distribution load flow," IEEE Transactions on Power Delivery, vol. 20, no. 2, pp. 1532-1540.
- [7] L.A.Gallego, E.Carreno and A.P.Feltrin, "Distributed generation modeling for unbalanced three-phase power flow calculations in smart grids," IEEE/PES Transmission and Distribution Conference and Exposition, vol. 1, pp. 323-328, 2010.
- [8] J.-H. Teng, "Modelling distributed generations in three-phase distribution load flow," IET Generation Transmission Distribution, vol. 2, no. 3. pp. 330-340, 2008.
- [9] T.H. Chen, et.al, "Three phase co-generator and transformer models for distribution system analysis," IEEE Transactions on Power Delivery, vol. 6, no. 4, pp. 1671-1681, Oct, 1991.

- [10] D. N. Hussein, M. A. H. El-Sayed, and H. A. Attia, "Modeling and simulation of distributed generation (DG) for distribution systems load flow analysis," Power Systems Conference MEPCON vol. 1, pp. 285-291, Dec, 2006.
- [11] Wasley R.G. and Shlash M.A., "Newton-Raphson algorithm for 3-phase load flow," IEE Proc, 121, (7), pp 630-638., Jul 1974.
- [12] Birt K.A., Graffy J.J., McDonald J.D. and El-Abiad A.H., "Three phase load flow program," IEEE Transactions on Power Apparatus and Systems, vol.95, no.1, pp. 59-65, 1976.
- [13] J. Arrillagaa and B.J. Harker, "Fast-decoupled three-phase load flow," Proc. of IEE, vol. 125, no. 8, pp. 734-740, Aug, 1978.
- [14] Smith B.C. and Arrillaga J., "Improved three-phase load flow using phase and sequence components," IEE Proc. Generation Transmission & Distribution, vol. 145, no.3, pp. 245-250, May I998.
- [15] Akher M.A., Nor K.M. and Rashid A.H.A., "Improved three-phase power-flow methods using sequence components," IEEE Transactions on power Systems, 20, (3), pp. 1389-1387, Aug, 2005.
- [16] Zhang X.P. and Chen H., "Asymmetrical three-phase load-flow study based on symmetrical component theory," IEE Proc. Generation Transmission & Distribution, 141, (3), pp. 248-252, May I994.
- [17] Lo K.L. and Zhang C., "Decomposed three-phase power flow solution using the sequence component frame," IEE Proc., 140, (3), pp. 181-188, May 1993.
- [18] D. Rajicic and A. Bose, "A modification to the fast decoupled power flow for networks with high R/X ratio," IEEE Transactions on Power systems, vol. 3, no. 2, pp 743-746, May 1988.
- [19] R.D. Zimmerman and H.D. Chiang, "Fast decoupled power flow for unbalanced radial distribution systems," IEEE Transactions on Power systems, vol. 110, no. 4, pp 2045-2052, Nov, 1995.

- [20] A.V. Gracia and M.G. Zogo, "Three-phase fast decoupled power flow for distribution networks," IEE Proc. Generation, Transmission and Distribution, vol. 143(2), pp. 188-192, 1996.
- [21] F. Zhang and C.S. Chang, "A modified Newton method for radial distribution system power flow analysis," IEEE Transactions on Power systems, vol. 12, no. 1, pp 389-397, Feb 1997.
- [22] H.L. Nguyen, "Newton-Raphson method in complex form," IEEE Transactions on Power systems, vol. 12, no. 3, pp 1355-1359, Aug 1997.
- [23] P.A.N. Garcia, J.L.R. Pereira, S. Carneiro, V.M.D. Costa and N. Martins, "Three-phase power flow calculations using the current injection method," IEEE Transactions on Power systems, vol. 15, no.2, pp. 508-514, May, 2000.
- [24] W.M. Lin and J.H. Teng, "Phase-decoupled load flow method for radial and weakly-meshed distribution networks," IEE Proc. Generation, Transmission and Distribution, vol. 143, no. 1, pp. 39-43, Jan, 1996.
- [25 T.H. Chen, et.al, "Distribution system power flow analysis- A rigid approach," IEEE Transactions on Power Delivery, vol. 6, no. 3, pp. 1146-1152, July, 1991.
- [26] N.C. Yang, "Three-phase power flow calculations using direct ZBUS method for large-scale unbalanced distribution networks," IET Generation, Transmission and Distribution, vol. 10, no. 4, pp. 1048-1056, 2015.
- [27] J.C.M. Viera Jr, W.Freitas and A. Morelato, "Phase decoupled method for three-phase power flow analysis of unbalanced distribution system," IEE Proc. Generation, Transmission and Distribution, vol. 151, no. 5, Sept, 2004.
- [28] W.H. Kersting and D.L. Mendive, "An application of ladder Network theory to the solution of three-phase Radial load flow problems," IEEE-PES 1976 winter meeting, New York, A76044-8, Jan, 1976.
- [29] W.H. Kersting and W.H. Philips, "A Radial three-phase power flow program for the PC," Conference paper, Frontiers power conference, Stillwater, OK(USA), Oct, 1987.

- [30] D. Rajicic, R. Ackovaki and R. Taleski, "Voltage correction power flow," IEEE-PES 1993 Summer meeting, Vancouver, B.C., Canada, 93 SM 570-2, Jul, 1993.
- [31] H.D. Chiang, "A Decoupled load flow method for Distribution power Networks: Algorithm, Analysis and convergence study," Electrical power and Energy System, vol. 13, no. 3, pp. 130-138, Jun, 1991.
- [32] [T. Alinjak, I. Pavic and M. Stojkov, "Improvement of backward/forward sweep power flow method by using modified breadth-first search strategy," IET Generation, Transmission and Distribution, vol. 11, no. 1, pp. 102-109, 2017.
- [33] D. Shirmohammadi, H.W. Hong, A. Semlyen and G.X. Luo, "A compensation based power flow method for weakly meshed Distribution and Transmission Networks," IEEE Transactions on Power systems, vol. 3, no. 2, pp. 753-762, May,1988.
- [34] G.X. Luo and A. Semlyen, "Efficient load flow for large weakly meshed Networks," IEEE Transactions on Power systems, vol. 5, no. 4, pp. 1309-1316, Nov, 1990.
- [35] C.S. Cheng and D. Shirmohammadi, "A three phase power flow method for real-time distribution system analysis," IEEE Transactions on Power systems, vol. 10, no. 2, pp. 671-679, May, 1995.
- [36] S. Khushalini and N.N. Schutz, "Unbalanced distribution power flow with distributed generation," Proceedings of IEEE Transmission and Distribution conference, Dallas, TX, May, 2006.
- [37] S. Kushalini, J.M. Solanki and N.N. Schulz, "Development of three-phase unbalanced power flow using PV and PQ models for distributed generation and study of the impact of DG models," IEEE Transactions on Power systems, vol. 22, no. 3, pp. 1019-1025, Aug, 2007.
- [38] Teng J.H.: "A direct approach for distribution system load flow solutions," IEEE Transactions on Power Delivery, 18, (3), pp-882-887, Jul 2003.

- [39] T.H. Chen and N.C. Yang, "Three-phase power-flow by direct ZBR method for unbalanced radial distribution systems," IET Generation, Transmission and Distribution, vol. 3, no. 10, pp. 903-910, Mar, 2009.
- [40] J.H. Teng, "A modified Gauss-Seidel algorithm of three phase power flow analysis in distribution networks," Electrical Power and Energy Systems, vol. 24, pp. 97-102, 2002.
- [41] D. Thukaram, H.M.W. Banda and J. Jerome, "A robust three phase power flow algorithm for radial distribution systems," Electric Power Systems Research, vol. 50, pp. 227-236, Oct, 1999.
- [42] H.E. Farag, E.F. El Saadany, R. El Shatshat and A. Zidan, "A generalized power flow analysis for distribution systems with high penetration of distributed generation," Electric Power Systems Research, vol. 81, pp. 1499-1506, 2011.
- [43] U. Ghatak and V. Mukherjee, "A fast and efficient load flow technique for unbalanced distribution system," Electrical Power and Energy Systems, vol. 84, pp. 99-110, 2017.
- [44] W.M. Lin, Y.S. Su, H.C. Chin and J.H. Teng, "Three-phase Unbalanced Distribution Power Flow Solutions with Minimum Data Preparation," IEEE Transactions on Power systems, vol. 14, no. 3, pp. 1178-1183, Aug, 1999.
- [45] M.Z. Kamh and R. Iravani, "Unbalanced model and power flow analysis of microgrids and Active distribution systems," IEEE Transactions on Power Delivery, vol. 25, no. 4, Oct, 2010.
- [46] H. Ahmadi, J.R. Marti and Meier, "A Linear power flow formulation for threephase distribution systems," IEEE Transactions on Power systems, vol. 31, no. 6, Nov, 2016.
- [47] M.A.Kashem, A.D.T.Le, M.Negnevitsky, G.Ledwich, "Distributed Generation for Minimization of Power Losses in Distribution Systems," IEEE PES general meeting, pp-1-8, 2006.

- [48] S.R.A.Rahim et al, "Implementation of DG for Loss Minimization and Voltage Profile in Distribution System," IEEE, pp-490-494, 2010.
- [49] A.T.Dada, M.D.Desai, B.R.Parekh, "Integration of Renewable Distributed Generation in Distribution system for loss Reduction: A case study," International journal of computer and Electrical engg., vol.3, No. 3, pp. 413-416, jun, 2011.
- [50] S.Dahal and H.Salehfar, "Impact of distributed generators in the power loss and voltage profile of three phase unbalanced distribution network," Electrical Power and Energy Systems, vol. 77, pp. 256-262, 2016.
- [51] A.A.A.Esmin, G.Lambert-Torres, "Loss Power Minimization Using Particle Swarm Optimization," IEEE Conference on Neural Networks vol.27, no.2, pp-1988-1994, 2006.
- [52] D.Q.Hung, N.Mithulananthan and R.C.Bansal, "Analytical strategies for renewable distributed generation integration considering energy loss minimization," Applied Energy, Volume 105, Pages 75-85, , May 2013.
- [53] K. Mahmoud, N. Yorino and A. Ahmed, "Optimal distributed generation allocation in distribution systems for loss minimization," IEEE Transactions on Power systems, vol. 31, no. 2, March, 2016.
- [54] D. Singh, D. Singh, and K. S. Verma, "GA based Optimal Sizing & Placement of Distributed Generation for Loss Minimization," International Journal of Electrical and Computer Engineering, pp. 557-563, 2007.
- [55] <u>M. Imran A; Kowsalya M</u>, "Optimal distributed generation and capacitor placement in power distribution networks for power loss minimization," IEEE Conference, ICAEE 2014, pp. 1-6, Jan. 2014.
- [56] J.Kennedy, R.Heberhart, "Particle Swarm Optimization," IEEE International conference, NN, vol.4, pp.1942-1948, 1995.
- [57] T.Niknam, M.R.Narimani, J.Aghaei, R.A.Abarghooee, "Improved particle swarm optimization for multi-objective optimal power flow considering the

cost, loss, emission and voltage stability index," IET Generation, Transmission & Distribution, Vol. 6, No.6, pp.515-527, Sep.2011.

- [58] IEEE/PES Distribution System Analysis Subcommittee, "IEEE-13 Test feeder," Special Publication, 2004.
- [59] T. Ramana, V. Ganesh and S. Sivanagaraju, "Distributed Generator Placement And Sizing in Unbalanced Radial Distribution System," Cogeneration & Distributed Generation Journal, Taylor & Francis, vol. 25, no. 1, pp. 52-70, Jan, 2010.
- [60] IEEE/PES Distribution System Analysis Subcommittee, "IEEE-37 Test feeder," Special Publication, 2004.

-820/1/ 27/3/22

SAMA SUDHAKAR REDDY

Sumla Halder 25/3/22

Professor Electrical Engineering Department JADAVPUR UNIVERSITY Kolkata - 700 032

Sondarit Paul 25/3/22

Professor Electrical Engineering Department JADAVPUR UNIVERSITY Kolkata - 700 032