b) Find the number of partitions of a set with six elements into two cells.   5+5=10

OR

6. a) Define a bijective function.

   b) Prove following Lipschutz, that a function $f : A \to B$ is invertible if and only if f is bijective.   2+8=10

7. Let R be an equivalence relation on a set S. Then, prove that the quotient set S/R is a partition on S.   5

OR

8. Consider functions $f : A \to B$ and $g : B \to C$. Prove after Lipschutz the following:

   a) If $g \circ f$ is one-to-one, then f is one-to-one.

   b) If $g \circ f$ is onto, then g is onto.   2.5+2.5=5

MASTER OF ARTS EXAMINATION, 2023

(1st Year, 1st Semester)

PHILOSOPHY

[Logic (Western)]

Time : Two Hours   Full Marks : 30

Use separate answer scripts for each group.

Answer *either* in English *or* in Bengali.

All questions carry equal marks.

Group – A

1. a) Symbolize the following sentences in each case using the indicated symbols: 2.5×2=5

   i) A teacher who helps a student who has not passed in any subject, has a job for which everyone will reward him. (Tx: x is a teacher, Sx: x is a student, Ax: x is a subject, Pxy: x has passed in y, Jx: x is a job, Px: x is a person, Rxyz: x rewards y for z)

   ii) No sane witness would lie if his lying would implicate him in a crime. (Sx: x is sane, Wx: x is a witness, Lx: x lies, Ix: x implicates himself in a crime)   2

   b) Construct a formal proof of validity for the following

   $(\exists x)Px \lor (\exists y)Qy$ 3

   $(x)(Px \supset Qx) / \therefore (\exists y)Qy$

[Turn over

c) Identify and explain the mistakes found in the following erroneous proof: 2

1. $(\exists x)(Fx.Gx)$
2. $(\exists x)(\sim Fx.Gx)/ \therefore (\exists x)(Fx.\sim Fx)$
3. Fa. Ga   1, EI
4. Fa   3, Simp
5. ~Fa . Ga   2, EI
6. ~Fa   5, Simp
7. Fa. ~Fa   4, 6 Conj
8. $(\exists x)(Fx.\sim Fx)$ 7, EG

OR

2. a) Construct a formal proof of validity for the following argument:

If something is lost, then if everyone values his possessions, it will be missed. If anyone values his possessions, so does everyone. Therefore, if something is lost, then if someone values his possessions, then something will be missed. (Lx: x is lost, Px: x is a person, Vx: x values his possessions, Mx: x is missed) 5

b) Prove the invalidity of the following argument: 3

i) $(x)Nx \supset (\exists y)Oy$
$(y)Oy \supset (\exists z)Pz$
$/ \therefore (\exists x)Nx \supset (z)Pz$

ii) Prove the following: 2

$(\exists x)(Nx \cdot Px) \supset [(\exists x)Nx \cdot (\exists x)Px]$

3. a) Prove the validity of the following argument with the help of Indirect Proof: 3

$(P \lor Q) \supset (R \supset S)$
$(\sim S \lor T) \supset (P \cdot R)/ \therefore S$

b) Use the method of CP to verify that the following is a tautology: 2

$(A \supset B) \supset [(\sim A \supset B) \supset B]$

OR

4. a) Use the strengthened method of Conditional Proof to prove the validity or invalidity of the following argument: 3

$A \lor (B \supset C)$
$[B \supset (B \cdot C)] \supset (D \lor E)$
$(D \supset A) \cdot (E \supset F)/ \therefore A \lor F$

b) Use the method of I.P to prove that the following is a tautology. 2

$(P \supset Q) \lor (\sim P \supset Q)$

Group – B

1. a) Let f(n, k) represent the number of partitions of a set S with n elements into k cells (for k=1, 2, ..., n). Find a recursion formula for f(n, k).

[Turn over