Master of Arts Examination, 2023

(1st Year, 1st Semester)

ECONOMICS

[Microeconomics - I]

Time : Two Hours Full Marks : 30

Part – I

Answer any of the ***four*** of the following. 4×2.5=10

1. Suppose the indirect utility function of a utility maximizing consumer is

$$v(p_1, p_2, u) = \max\left(\frac{y}{\sqrt{p_1 p_2}}, \frac{2y}{p_1 + p_2}\right).$$

The consumer's income is 10 while prices are $p_1 = 4$ and $p_2 = 1$. Now suppose p_1 falls to 1. What is the welfare effect of this price fall in terms of EV (Equivalent variation)?

2. Consider a decision maker with utility function

$$u(w) = \alpha - \beta e^{-rw},\ \alpha, \beta, r > 0$$

where w denotes the total wealth of the decision maker. Suppose she can accept or reject a lottery. If she rejects the lottery she keeps a certain wealth level w_0. If she accepts the lottery she gain or lose a random amout $\in$ which is distributed according to the distribution function

$f(\in)$ and her total (random) wealth is $w_0 + \in$. Does the decision to accept or reject the lottery depends on her initial wealth w_0?

3. Suppose a firm produces output y with two inputs z_1 and z_2 using the production technology

$$y = \sqrt{\min\left\{2\sqrt{z_1 z_2}, z_1 + z_2\right\}}.$$

The output and input prices are given by p and (w_1, w_2) respectively. Derive the net supply function of input z_i, $i = 1,2$.

4. Suppose C = {1, 2, 3) and two compound lotteries are defined as follows $\boldsymbol{L_1} = \left\{l_1, l_2, l_3, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}$ and $\boldsymbol{L_2} = \left\{l_4, l_5, \frac{1}{2}, \frac{1}{2}\right\}$ with $l_1 = (1,0,0)$, $l_2 = (0,1,0)$ and $l_3 = (0,0,1)$, $l_4 = (0, p, 1-p)$ and $l_5 = (q, 1-q, 0)$. Find the value of p and q such that $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ generate same simple lottery.

5. Derive the cost function $c(w, q)$ where $w = (w_1, w_2)$ for the following single-output constant returns technologies with production functions given by

$f(z) = \max\{az_1, az_2\} + \min\{z_1, z_2\}$ for all $0 < a < 1$.

these restrictions, $u(w)$ displays increasing absolute risk aversion. 4

10. Consider a Perfectly competitive market where the cost function of the representative firm is $C(y)$ for the production of y unit of output. Suppose the market price p is random variable with mean $\bar{p}$. Further, the firm's utility of profit is $u(\pi)$ with $u' > 0$, $u'' < 0$. Show that in case of output decisions under price uncertainty the optimum output increases with expected price if the firm exhibits decreasing absolute risk aversion. 10

8. a) State and prove the Shepard's Lemma for Cost Minimization.

b) Consider a profit maximizing firm with the production function $y = f(z_1 + sz_2) + f(z_2 + sz_1)$, facing output price $p = 1$ and factor prices w_1 and w_2.

i) Give an interpretation of the parameter s. 1

ii) Show that the second order condition of profit maximization always holds. 2

iii) If z_i^* denote optimum input choice then find the expression of $\frac{\partial z_i^*}{\partial w_i}$ and $\frac{\partial z_j^*}{\partial w_j}$ for all $i, j = 1,2$ and $i \neq j$. Comment on their sign. 3

9. a) Let u and v be utility function (not necessary VNM) representing $\succsim$ on the lottery space $\mathcal{L}$. Show that v is a positive affine transformation of u if and only if for all lottery $L_1, L_2, L_3 \in \mathcal{L}$, with no two indifferent, we have $\frac{u(L_1) - u(L_2)}{u(L_2) - u(L_3)} = \frac{v(L_1) - v(L_2)}{v(L_2) - v(L_3)}$. 6

b) Let $u(w) = -(b - w)^c$. What restrictions on w, b and c are required to ensure that $u(w)$ is strictly increasing and strictly concave? Show that under

6. Suppose there are three identical firms in the industry. The demand is $1 - Q,\ Q = q_1 + q_2 + q_3$. There is no cost of the production.

i) Compute the Cournot equilibrium.

ii) Show that if two of the three firms merge, the profit of these firms decreases. 1+1.5

Part – II

Answer any ***two*** of the following. 10×2=20

7. a) Show that if a choice function satisfies WARP and budget balanced-ness, then it must satisfy two properties implied by the utility maximization, namely, homogeneity of degree zero and negative semi definiteness of the Slutsky matrix.

b) Let $(-\infty, \infty) \times \mathbb{R}^{L-1}$ denote the consumption set, assume that preferences are strictly convex and quasilinear. Normalize p_1=1.

i) Show that the Walrasian demand functions for good 2, ..., L are independent of the wealth. What does this imply about the wealth effect of the demand for good 1. 2.5

ii) Argue that the indirect utility function can be written in the form $v(p, w) = w + \phi(p)$ for some function $\phi(\cdot)$. 2.5