current prices, W, and a general price index, P, for the manufacturing sector for a certain industrialized country for the period 1973 to 2002. L is measured as the average number of workers employed. Y and P are measured as index numbers equal to 100 in the year 2002. He fits the following regression (standard errors in parentheses):

$$\underset{}{\log\hat{L}} = \underset{(0.13)}{-3.12} + \underset{(0.09)}{0.42}\log Y - \underset{(0.10)}{0.34}\log W - \underset{(0.06)}{0.11}\log P \qquad (1)$$

The RSS of regression (1) turns out to be 1.99. He next regresses L on real output, Y/P, and real wages, W/P:

$$\log\hat{L} = \underset{(0.13)}{-2.56} + \underset{(0.07)}{0.46}\log\frac{Y}{P} - \underset{(0.07)}{0.32}\log\frac{W}{P} \qquad (2)$$

The RSS of regression (2) is 2.03. Finally he fits the following specification:

$$\log\hat{L} = \underset{(0.13)}{-3.12} + \underset{(0.09)}{0.42}\log\frac{Y}{P} - \underset{(0.10)}{0.34}\log\frac{W}{P} - \underset{(0.04)}{0.03}\log P \qquad (3)$$

The RSS of specification (3) is 1.99. Explain why specification (2) is a restricted version of specification (1), stating the restriction. Perform a t test of the restriction. Note: $t^{crit}_{5\%} = 2.056$.

Master of Arts Examination, 2023

(1st Year, 1st Semester)

ECONOMICS

[Econometrics - I]

Time : Two Hours Full Marks : 30

Answer any ***five*** of the following questions: 6×5=30

1. Let the loglikelihood function be $\ell(y,\theta)=\sum_{t=1}^{N}\ell_t(y^t,\theta)$, where y^t is the vector $(y_1, y_2, ..., y_1)'$, $t=1,2,...,N$. Prove that

$$E_\theta\left[g(y,\theta)g'(y,\theta)\right]=\sum_{t=1}^{N}E_\theta\left[G_t'(y^t,\theta)G_t(y^t,\theta)\right],$$

where E_θ represents expectation with respect to DGP characterized by θ, a typical element of $g(y,\theta)$ is

$$g_i(y,\theta)=\frac{\partial\ell(y,\theta)}{\partial\theta_i}=\sum_{t=1}^{N}\frac{\partial\ell_t(y^t,\theta)}{\partial\theta_i}, \quad i=1,2,...,K$$ and

$G_t(y^t,\theta)$ is the t-th row of the matrix $G(y,\theta)$ with typical element $G_{ti}(y^t,\theta)=\dfrac{\partial\ell_t(y^t,\theta)}{\partial\theta_i}$.

2. Consider the following model: $y_1 = z_1\delta_1 + \alpha_1 y_2 + u_1$, where z_1 and z_2 are (row) vectors of regressors, δ_1 is

(column) vector of coefficients corresponding to z_1, $z = (z_1, z_2)$, $E(z'u_1) = 0$ and y_2 is potentially endogenous. Regress y_2 on z_2 and obtain the fitted values as $\tilde{y}_2$. Regress y_1 on z_1, $\tilde{y}_2$ to obtain $\tilde{\delta}_1$ and $\tilde{\alpha}_1$. Show that $\tilde{\delta}_1$ and $\tilde{\alpha}_1$ are generally inconsistent and explain the reason.

3. Consider the least square estimates of the model: $y_{N\times 1} = X_{N\times K}\beta_{K\times 1} + u_{N\times 1}$, where $E(u \mid X) = 0$ and $E(uu' \mid X) = \Sigma = \sigma^2(I + AA')$, where A is an $N \times m$ matrix with $K < m < N$. Assume, for simplicity, that σ^2 and A are known.

   a) Obtain the variance of the OLS estimator of β.

   b) Compare your answer in (a) with default OLS variance $\sigma^2(X'X)^{-1}$. Are the default OLS standard errors biased/inconsistent in any particular direction?

   c) Determine the variance of the GLS estimator of β, using the result $(I + AA')^{-1} = I_N - A(I_m + A'A)A'$.

   d) Compare the default variance $\sigma^2(X'X)^{-1}$ of OLS with the true variance of GLS. Does your finding violate that fact that GLS must be BLUE when disturbances are non-spherical?

4. Let the model be $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + u$, where x_1, x_2 are non-stochastic and u satisfies all standard classical linear regression model properties. Run the regression of x_1 on a constant and x_2 and obtain the residuals $\hat{v}_1$. Regress y on $\hat{v}_1$ alone. Prove that resulting estimate of the slope coefficient is $\hat{\beta}_1$.

5. Take a single draw of $Y \sim N(0, 1)$ and define a sequence of random variables, $\{X_N\}$, as follows:

$$X_N = \begin{Bmatrix} -Y & \text{if N is even} \\ Y & \text{if N is odd} \end{Bmatrix}$$

   Show that $X_N \xrightarrow{d} Y$ but not $X_N \xrightarrow{p} Y$.

6. Consider the following regression

$$y = X_1\beta_1 + X_1\beta_2 + u$$

   where X_1 and X_2 are partitions of data matrix containing n observations each and K_1 and K_2 regressors, respectively. Prove that, $\hat{\beta}_2$ is equal to the set of coefficients obtained from OLS regression of $\hat{\varepsilon}_1$ on $\hat{\varepsilon}_2$, where ε_1 and ε_1 are defined as follows:

$$y = X_1\theta_1 + \varepsilon_1,\ X_2 = X_1\theta_2 + \varepsilon_2$$

7. A researcher has annual data on demand for labor, L, aggregate output in current prices, Y, average wages in