proposition. If he accepts, the game is over. If he rejects, they move on to period 2 where B proposes (1–y,y), where B gets y. If A accepts game ends or moves to period 3, where A will again propose a distrib ution. Payoffs are discounted with the discount factor δ. Find the sub game perfect Nash equilibrium when this bargaining game is repeated infinitely. If $\delta = 1/4$, what are the equilibrium payoffs.

(*Note: If B is indifferent between accepting and rejecting, we assume that he always accepts.*) 5+1=6

10. Consider a firm (F) that selects the number of workers $L \geq 0$ and a union (U) that fixes wages, $W \geq 0$. Firm's are given by $\Pi(w,L) = 100L - 0.1L^2 - wL$ whereas union's and the firm observes w and then chooses labour input L.
   (a) Draw the extensive form of the game and find the sub game perfect equilibrium.
   (b) Suppose that the union is worried about reaching an employment level of X at least, so that its payoff function si now
   $U(w,L) = (L - X)w$
   Draw the extensive form of the game and find the sub game perfect equilibrium, as a function of E.

3+3+6

BACHELOR OF ARTS EXAMINATION, 2023

(3rd Year, 1st Semester)

ECONOMICS

[TOPICS IN MICROECONOMICS I]

Time : Two Hours Full Marks : 30

Answer any ***five*** questions.

1. (a) Compute the set of all rationalizable strategies in the following game.

	A2	B2	C2	D2
A1	2,9	4,7	9,2	2,3
B1	7,4	5,5	7,4	2,3
C1	9,2	4,7	2,9	2,3
D1	2,1	2,0	2,2	12,1

   (b) Now find the pure strategy Nash equilibrium as well as mixed strategy Nash equilibrium of this game. 2+1+3=6

2. Two friends, A and B, would like to share Rs. 101. They agree on the following procedure. Each friend writes (without knowing what the other writes) a number on a sheet of paper. They share the money according to the following rules :
   i. If both numbers are even, A obtains Rs. 51 and B Rs. 50;

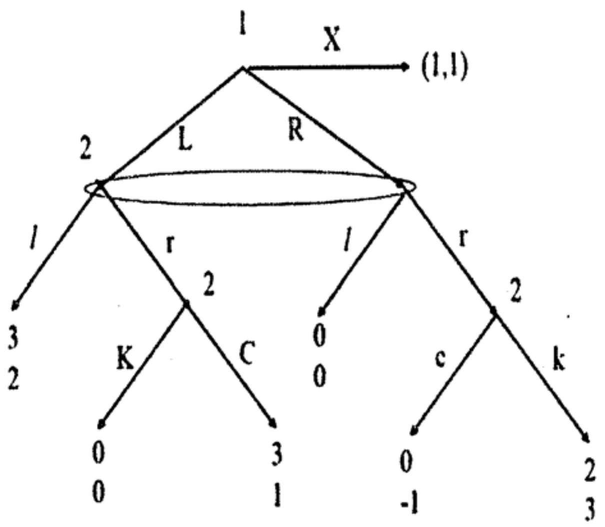
ii. if both numbers are odd, A obtains Rs. 50 B Rs. 51; and

iii. in the remaining cases, both obtain Rs. 50, one rupee is lost.

(a) Represent this game in normal form, and find the pure strategy Nash and mixed strategy Nash Equilibria.

(b) Now suppose another strategy 'do not write a number' is introduced in the game, that is each player will have three strategies. If one player chooses this strategy, while other does not, then he gets 101. However, if both choose this strategy each will get 1. Write down the normal form and find the Nash equilibrium. 4+2=6

3. Write down the normal form of the following game. How many non-trivial subgames are there? Find the sub game perfect Nash equilibrium of the game. 2+1+3=6.



(a) Write the normal form representation of this game. Find the pure strategy Nash equilibrium.

(b) Write is the Nash equilibrium result if the game is repeated for 10 time periods with a discount factor of δ.

(c) If the firms play this game repeatedly for an infinite period of time using a trigger strategy, derive the discount factor δ where the firms collude to sustain a profit Rs. 20 crores each in n every time period. 2+1+3=6

8. Suppose a parent and a child play the following game. First, the child takes an action, $A \in R$, that produces income for the child, $IC(A) = 5 - (A - 3)^2$ and income for the parent, $IP\ (A) = 5 - (A - 1)^2$. Second, the parent observes the incomes IC and IP and then chooses a bequest, B, to leave to the child. The child's payoff is U(IC+B); the parent's is V(IP–B)+U(IC+B), where the utility functions U(x)=In x and V(x) = In(4+x).

(a) Find the backwards-induction outcome of the game.

(b) Prove that, the child chooses the parent's payoff exhibits altruism. 3+3=6

9. Consider a bargaining (negotiation) game of 2 periods. In the first one, Player A offers Player B to share Rs. 1 as (x, 1–x), where x is the quantity that A would receive. Player B can then choose to accept or reject A's

6. An open access fishery for lobster is available in open waters. There are n identical fishing boats. The total harvest revenue from all n boats is $Y = AX - BX^2$ where A and B are parameters while X is total inputs from all fishing boat such that $X = \sum x_i$.
The revenue for an individual fishing boat is a proportion of total harvest revenue such that $y_i = \frac{x_i}{X} y$ where x_i is the input by one fishing boat. Total cost by one boat is Cx_i. [C=Constant]
   (a) Solve for the Nash equilibrium input level by one fishing boat. Given this individual level, solve for the aggregate level of input.
   (b) Assume that there is a single private owner of the entire fishery. Solve for their optimal level of input.
   (c) Compare the level of inputs under Nash equilibrium and private ownership and give an intuitive explanation why the input levels are such. 2+2+2=6

7. Two firms are in competition. They can both produce a large amount leading to a lower price and lower profits, i.e. Rs 10 crores each. Alternatively, they can reduce production to raise price and increase profit with each firm receiving Rs 20 crores. If one firm produces a lot but the other produces little, the firm producing a lot receives Rs 30 crores and the firm producing a low output has 0 profit.

[Turn over

4. Consider the following normal form game.

	A2	B2	C2
A1	4, 4	1, 5	0,3
B1	7,1	3, 4	0,1
C1	3, 0	2, 0	1, 1

   (a) Find the Nash equilibria in pure strategies.
   (b) Assume that the above stage game is played two times. After the first round, players observed the moves done by the other player. The total payoffs of the repeated game are the sum of the payoffs obtained in each round. Is there a sub game perfect Nash equilibrium in pure strategies in which (A1, A2) is played in the first round? 2+4=6

5. There are n profit maximizing firms in Neverland, with 0 production costs. Each firm decides their own production q_i. Total Production is thus $Q = q_1 + q_2 + ...q_n$ Inverse demand function is : $p(Q) = e^{-Q}$.
   (a) Find firm's profit function for i = 1, ...n
   (b) Consider the case n = 2. Draw reaction functions and find the Cournot equilibrium.
   (c) Consider the general case, calculate the Cournot equilibrium. What happens to price and total quantity as the number of firms increases? What would happen in the limit? 1+2+3=6

[Turn over