BACHELOR OF ARTS EXAMINATION, 2023

(2nd Year, 1st Semester)

ECONOMICS

[ADVANCED STATISTICS]

Time : Two Hours   Full Marks : 30

Answer any *three* questions.

1. (i) Suppose X and Y be two independent N(0,1) random variables.

    If $Z = 1 + X + XY^2$

    and

    $W = 1 + X$

    Find $Cov(Z,W)$

   (ii) Suppose joint pdf of X and Y is $f(x,y) = \frac{1}{8}(c - x - y)$

    $0 \le x \le 2; 2 \le y \le 4$. What should be the value of c for f(x,y) to be considered as joint pdf? After substituting the value of c, find E(X). Are X and Y independent?   4+6

2. (i) Prove that if number of trials(n) is very large and probability of success(p) in each trial is very low, but, the product of n and p is a finite quantity then Binomial probability law converges to Poisson probability law.

   (ii) It is known that 1 in 1000 transistors are defective on average.

(a) Find the probability that there are 4 defective transistors in a batch of 2000.

(b) What is the largest number, N, of transistors that can be put in a box so that the probability of no defectives is at least 1/2?   4+3+3

3. (i) The results of a completely randomized experiment (where a one way classification, fixed effect, linear model is appropriate) to compare the effects of 3 different teaching methods on students' performances are given below. The following table gives the marks (out of 100) obtained by the students who are exposed to different teaching methodologies.

Teaching method 1	Teaching method 2	Teaching method 3
77	99	46
70	66	70
63	37	71
84	79	65
95	54	61

Test if the teaching methods are equally effective at 5% level.

(ii) State the classical definition of probability and discuss its limitations.   6+4

4. (i) According to a genetic model, the probabilities of guinea pigs being red, black and white are

respectively 9/16, 3/16 and 1/4. If the genetic model is correct, out of 10 randomly chosen offsprings of a certain cross variety of guinea pigs, what is the probability that 5 are red, 3 are black and 2 are white?

(ii) Suppose x_1, x_2,...x_n be independently and indentically distributed with pdf $f(X | \phi) = \phi x^{\phi-1}$, $0 \le x \le 1, 0 < \theta < \infty$. Find the estimate of ϕ using maximum likelihood method and method of moments.   4+3+3

5. (i) An investigator knows that the population of light bulb lifetimes is normally distributed and that the standard deviation is 36 hours. A simple random sample of 49 bulbs discloses a sample mean lifetime of 510 hours. Determine a 95% confidence interval for the mean lifetime of all such bulbs.

(ii) Show that the Moment Generating Function of Normal distribution is $M(t) = \exp\left(t\mu + \dfrac{t^2\sigma^2}{2} \right)$.   4+6