Bachelor of Arts Examination, 2023

(1st Year, 1st Semester)

ECONOMICS

[Mathematical Methods in B1]

Time : Two Hours — Full Marks : 30

Answer question number **1** and ***any two*** from the rest.

1. a) Check whether the following statements are true or false (Give reasons for your answer). 5

   i) The disjunct of two contingent propositions is never a contradiction.

   ii) For the function $f(x) = 4\alpha x^2 + \beta$ the condition $f'(x) = 0$ is both necessary and sufficient condition to identify the value of x for which $f(x)$ attains a maximum value.

   iii) $f(x) = 4x^3y^6 + 3x^2y^4 + 9$ is a homothetic function but not a homogeneous function.

   iv) If you maximize the function $y = f(x_1, x_2, x_3, x_4, x_5)$ and a local maximum exists and the function is twice continuously differentiable then the third principal minor of the Hessian determinant will be positive evaluated at the point of local maximum.

   v) A homogeneous function is always quasi concave.

[Turn over

b) i) Consider S=Z. $\forall x, y \in \mathrm{S}\ x\mathrm{R}y$ if $2 \mid (x+y)$. Show that R is an equivalent relation.

ii) Given the properties of a function below, identify the value of the function at relative maximum (if any), the relative minimum (if any) and the point of inflexion (if any):

$f : \mathbf{R} \to R$; $f(3)=1$, $f(-3)=-1$, $f(0)=0$, $f'(3)=0$, $f'(-3)=0$, $f''(x)>0\ \forall\ x \in (-\alpha, 0)\ f''(x)<0\ \ \forall x \in (0, \alpha)$. Give reasons for your answer. 2+3=5

2. a) Consider $f(x)=9-12x+9x^2-2x^3$ (Domain: R). Identify the subdomains over which i) The function is concave and increasing, ii) Concave & decreasing, iii) Convex & increasng, iv) Convex & decreasing. Identify the relative maximum &/or relative minimum if they exist.

b) Find out and classify the critical points of the following function $f(x,y)=x^3+y^3-3xy$. Also if there is a relative optimum check whether it is an absolute optimum.

c) Prove that convexity of a function is sufficient to establish it's quasi-convexity. 4+4+2=10

3. a) Prove that the tangent lines of the level curves of a homogenous function $f(x; y)$ have constant slopes along each ray from the origin.

b) Find the optimal value(s) of f from the following constrained optimization problem:

Optimize $f(x,y,z)=xz+yz$ subject to $y^2+z^2=1$ & $xz=3$. 4+6=0

4. a) Identify the relative minimum, relative maximum, saddle point of the following function (if they exist).

$$f(x,y)=x^3+y^3+3y^2-12x-9y$$

b) Let $z=f(x,y)$ be a function, which is twice continuously differentiable. Write down the condition for quasi-convexity in terms of the bordered determinant. Prove that if the bordered determinant satisfies the sufficient condition for quasi-convexity, the bordered Hessian then must satisfy the 2nd order sufficient condition for minimization given that the constraint function is linear. 7+3=10