Ref. No. : Ex/PG/PE/T/113A/31/2017

Master of Power Engineering, 1st Semester Examination, 2017

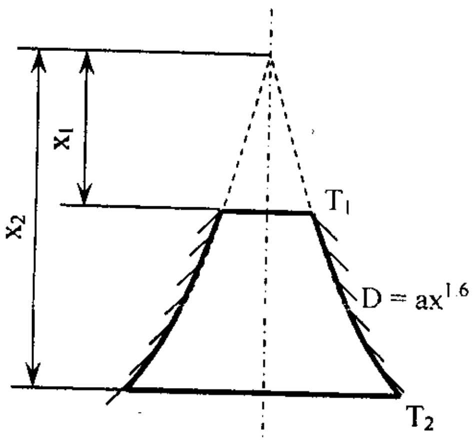
Heat and Mass Transfer **Time : 3 hour** **Full Marks: 100**

Answer any **five** questions taking at least two from each group

Group - A

1.a) A truncated solid cone is of circular cross section and its diameter is related to the axial coordinate by an expression of the form $D=ax^{1.6}$, where $a=10.0\ m^{-0.6}$ and x is in m. The sides are well insulated, while the top and bottom surfaces of the cone are maintained at constant temperatures T_1 and T_2 respectively. Obtain an expression for the temperature distribution along the cone if the thermal conductivity of the cone material is expressed as $k=60(1+2.5\times10^{-5}T^2)$ W/m-K, where T is in Celsius. What is the rate of heat transfer across the cone if the end temperatures are $T_1=200^0$ C and $T_2=60^0$ C? **(12)**

$x_1 = 0.01$ m
$x_2 = 0.025$ m



b) A spherical lead bullet of 5 mm diameter is moving at a Mach number of approximately 3. The resulting shock wave heats the air around the bullet to 750 K and the average convection coefficient for heat transfer is 525 W/m^2-K. If the bullet leaves the barrel at 300 K and the time of flight is 0.6 s, what is the surface temperature on impact? (for lead ρ = 11340 kg/m^3, c_p = 129 J/kg-K, k = 35.3 W/m-K). **(8)**

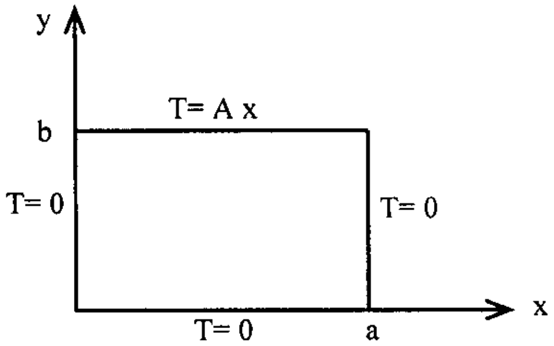
2. Consider an annular fin on a cylindrical pipe. The outer radius of the pipe is r_1, the outer radius of the fin is r_2 and the fin thickness is t. The fin material has a thermal conductivity k, while the convective heat transfer coefficient at the surface of the fin is h. The pipe outer surface is at a constant temperature T_w and the heat transfer from the fin tip may be neglected. From the energy balance across a small element in the fin, develop the governing equation for heat transfer through the fin and write down its boundary conditions.

If a modified Bessel equation in the form $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}-y=0$ has a general solution as, $y=C_1I_0(x)+C_2K_0(x)$, where $I_0(x)$ and $K_0(x)$ are the modified zero order Bessel functions of the first and second kind, respectively, show that the temperature distribution across the annular fin can be expressed as,

$$\frac{T - T_\infty}{T_w - T_\infty} = \frac{K_1(mr_2)I_0(mr) + K_0(mr)I_1(mr_2)}{K_1(mr_2)I_0(mr_1) + K_0(mr_1)I_1(mr_2)}$$

$I_l(x)$ and $K_l(x)$ are the modified first order Bessel functions of first and second kind, given as $I_1(x) = \frac{d}{dx}(I_0(x))$, and $K_1(x) = \frac{d}{dx}(K_0(x))$ **(20)**

3. A two-dimensional rectangular plate is subjected to the boundary conditions as shown in the figure below. Derive an expression for the steady state temperature distribution T(x,y) across the plate. **(20)**



4.a) Consider an one-dimensional steady state heat conduction problem with the following governing equation and boundary conditions:

$$\frac{d^2T}{dx^2} + \frac{1}{k}\dot{q}_o''' \alpha e^{-\alpha x} = 0 \qquad \text{for } 0 < x < L,\ \alpha \text{ is a constant}$$

$$\frac{dT}{dx} = 0 \qquad \text{at } x = 0$$

$$k\frac{dT}{dx} + h(T - T_\infty) = 0 \qquad \text{at } x = L$$

Write down the finite difference formulation of this problem through discretization of the above equations considering uniform grids in the domain and discuss a methodology for the solution. **(10)**

b) Using Taylor series, show that the accuracy of central differencing is more than either forward or backward differencing. **(6)**

c) Define Biot number and Fourier number and state their physical significance. **(4)**

Group - B

5.a) In a two dimensional flow of Newtonian fluid in Cartesian co-ordinate system, the surface stresses on a fluid element are expressed as,

$$\sigma_{xx} = -p + 2\mu\frac{\partial u}{\partial x}, \quad \sigma_{yy} = -p + 2\mu\frac{\partial v}{\partial y}, \quad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

Find out an expression of the rate of work transfer due to surface force on the fluid element per unit mass of the fluid.

(8)

b) What is the basic philosophy of "Order of Magnitude" analysis? For a steady, two dimensional, incompressible flow over a rectangular flat plate, the conservation of mass and momentum equations are expressed as follows:

Mass :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

x- Momentum :

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

y- Momentum :

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

where, all the symbols have their usual nomenclature.

Apply the order of magnitude analysis to simplify the equations for the application within the boundary layer above the plate.

(12)

6. The conservation of energy in a fluid element in absence of internal energy generation in two-dimensional Cartesian coordinate is expressed as

$$\frac{\partial}{\partial t}\left[\rho\left\{e+\frac{1}{2}\left(u^2+v^2\right)\right\}\right]+\frac{\partial}{\partial x}\left[\rho u\left\{e+\frac{1}{2}\left(u^2+v^2\right)\right\}\right]+\frac{\partial}{\partial y}\left[\rho v\left\{e+\frac{1}{2}\left(u^2+v^2\right)\right\}\right]=$$
$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\left(u\,f_{bx}+v\,f_{by}\right)-\left[\frac{\partial}{\partial x}(up)+\frac{\partial}{\partial y}(vp)\right]+\mu\phi$$

where, ϕ is the dissipation function. The momentum equations are

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+f_{bx}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

and

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+f_{by}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

Show that the energy equation can be expressed in terms of fluid temperature (T) as

$$\rho c_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\beta T\frac{Dp}{Dt}+\mu\phi$$

$\frac{Dp}{Dt}$ represents the substantial derivative of p and β is the coefficient of volumetric expansion terms have their usual nomenclature.

7. a) The temperature distribution within the thermal boundary layer over a flat plate with fluid flow parallel to it is assumed as $T = a + by + cy^2 + dy^3$.

   Show using suitable conditions (with proper justification) that the temperature distribution may b expressed as

$$\frac{T - T_\infty}{T_w - T_\infty} = 1 - \frac{3}{2} \cdot \frac{y}{\delta_T} + \frac{1}{2} \cdot \left(\frac{y}{\delta_T}\right)^3$$

   δ_T represents the thickness of the thermal boundary layer, T_w is the wall temperature consic constant and T_∞ the free stream temperature.

   b) The energy equation within the thermal boundary layer above the flat plate can be expressed a

$$\frac{\partial}{\partial x} \int_0^{\delta_T} u(T - T_\infty) dy = \frac{\dot{q}_w}{\rho\, c_p}$$

   where $\dot{q}_w$ is the local wall heat flux on the flat plate at constant temperature. Considering the P and the temperature distribution obtained in (a), find an expression of local Nusselt number fc case.

   c) From the local Nusselt number expression obtained in (b), find the expression of average Nu number for a plate length L.

8. In a hydrodynamically and thermally fully-developed, laminar, steady pipe flow with constant heat flux q_w, the axial velocity profile has a parabolic shape given by,

$$v_x = 2\bar{v}_x \left(1 - \frac{r^2}{R^2}\right)$$

   where, R is the radius of the pipe and $\bar{v}_x$ is the average velocity of flow. The energy equation m expressed, after simplification, as

$$v_x \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

   All terms in the equations have their usual nomenclature. For the above case, answer the followir
   (a) Define bulk mean temperature and show that it varies linearly along the length of the pipe.
   (b) Find an expression for the difference between the wall temperature and bulk mean temperatu any axial position of the pipe.
   (c) Show that the Nusselt number (Nu) for the forced convective heat transfer has a value 4.36.

9. (a) Define Grashof number and state its significance.
   (b) State Fick's law of mass diffusion. Show that for a binary mixture of A and B, $D_{AB} = D_{BA}$. (
   (c) Consider a liquid 'A' maintained at a fixed height in a vertical glass cylinder and a gas mixtu A and B to be flowing across the top of the cylinder. The concentration of A in the flowing g less than the concentration of A at the liquid –vapour interface. Considering steady, dimensional transport along the tube and the liquid to be impermeable to species B, deriv

expression for the mass fraction of A along the tube. From it, show that if the free stream does not contain species A then the mass transfer rate per unit area is,

$\dot{m}'' = \frac{\rho\, D_{AB}}{L} \ln\left[\frac{1}{1-C_{A,i}}\right]$, where $C_{A,i}$ is the concentration of A at the interface and ρ is the average density of the mixture. **(10)**