REF. NO.: EX/PG/ME/T/115B/24/2017

M. MECH. ENGG. 1ST. SEM. EXAM.-2017

FULL MARKS 100

DURATION 3 HRS.

BASICS OF FINITE ELEMENT METHOD

Answer any four questions. Subsections of a question (if any) carry equal distribution of marks. Some data are presented in the Appendix-I. You may assume reasonable values to any other missi.ng data

1. Solve the DOF values at all nodes for the shown loading condition of the frame structure below.

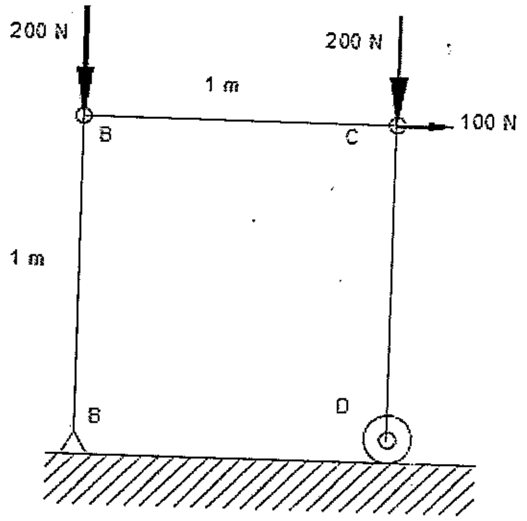

Figure (1)

2. Determine the unknown DOF values and force in each component of the truss structure below.

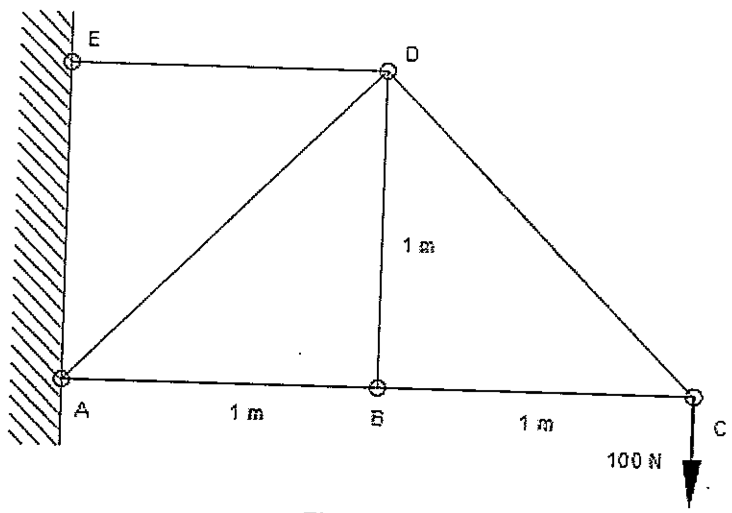

Figure (2)

3. For the simply supported beam shown below, use FE formulation of Euler-Bernoulli beam to determine the deflection at the mid-point and compare that to the analytical solution of

$\delta = \frac{5qL^4}{384EI}$, where L = Total length of the beam.

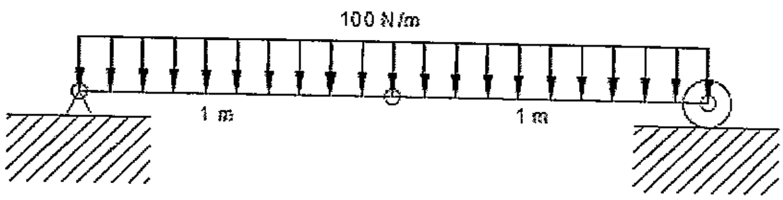

Figure (3)

4. The column shown in Figure (4) has lateral dimensions $t_y = t_z = 0.1$ m and it is subjected to a vertical traction of $t = 1E4\ N/m^2$. Using a single 4 node quadrilateral element with *Full Integration* determine the DOF values at different nodes.

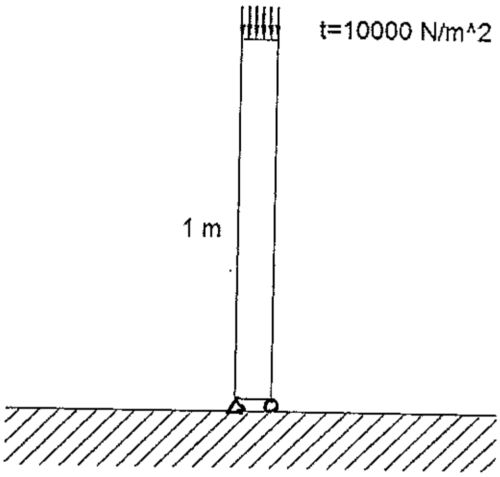

Figure (4)

5. Derive the complete FE formulation of a CST element.
6. Write short notes on the following (any 5):
   a. Galerkin's method.
   b. Rayleigh-Ritz method.
   c. Geodesic problem.
   d. p and h refinement.
   e. Equivalence of V-Statement and M-Statement.
   f. Plane stress and Plane strain conditions.

Appendix-I

i. For question number 1, 2, 3, following are the properties of each component.
   a. $A = 1.0E-3\ m^2$.
   b. $I = 1.0E-5\ m^4$.

ii. Value of Young's modulus should be universally taken as, $E = 200$ GPa.

7

iii. For 1-D elements following are the element stiffness matrices in the local (element) coordinate system.

a. $[K_e^{bar}] = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

b. $[K_e^{beam}] = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ & 4l_e^2 & -6l_e & 2l_e^2 \\ & & 12 & -6l_e \\ Sym. & & & 4l_e^2 \end{bmatrix}$

iv. The local to global transformation matrix for the frame element when the axis of the element makes an angle of θ with the x axis-

$$[Q] = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & & & \\ -\sin(\theta) & \cos(\theta) & 0 & & \mathbf{0} & \\ 0 & 0 & 1 & & & \\ & & & \cos(\theta) & \sin(\theta) & 0 \\ & \mathbf{0} & & -\sin(\theta) & \cos(\theta) & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}.$$

v. The (ξ_i, w_i) pairs for Gaussian Quadrature method with one and two integration points are $(0,2)$ and $(\pm\frac{\sqrt{3}}{2}, 1)$ respectively.

vi. The material stiffness matrix for plane stress condition-

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix}.$$

vii. Beam Shape functions:-

$$[N]_{1\times4} = \frac{1}{4}\left[(1-\xi)^2(2+\xi) \quad \frac{l_e}{2}(1-\xi)^2(1+\xi) \quad (1+\xi)^2(2-\xi) \quad -\frac{l_e}{2}(1-\xi)(1+\xi)^2\right]$$

The Element force vector $[f_e] = \int_0^{l_e} [N]^t q \, dx$.