MASTER OF MECHANICAL ENGINEERING EXAMINATION, 2017

(1st Semester)

THEORY OF ELASTICITY

Time: Three Hours **Full Marks: 100**

Parts of the same question must be answered together
Any missing data may be assumed with suitable justification

ANSWER ANY FOUR QUESTIONS

Q1. [8+12+5]

(a) Derive the governing equations for finding principal stresses and the corresponding principal directions.

(b) Given the six stress components $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}$ and τ_{xz} with respect to the orthogonal reference frame *x-y-z* at any point *O* (**Fig. 1b**), derive the stress components for any rotated orthogonal reference frame *X-Y-Z*, for which the direction cosines are given as per **Table 1b**.

(c) Show that the state of stress at any given point can be decomposed into hydrostatic and pure states of stress.

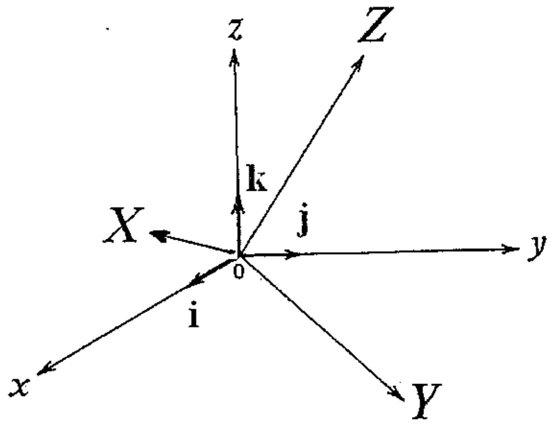

Fig. 1b

Table 1b: Direction Cosines

	x	y	z
X	l_1	m_1	n_1
Y	l_2	m_2	n_2
Z	l_3	m_3	n_3

Q2. [8+14+3]

(a) Show that the stress surface of Cauchy completely defines the state of stress at a point.

(b) Show that the normal strain (ε) at any point in any given direction (direction cosines: *l, m, n*) is given by,

$$\varepsilon = \varepsilon_{xx} l^2 + \varepsilon_{yy} m^2 + \varepsilon_{zz} n^2 + \gamma_{xy} lm + \gamma_{yz} mn + \gamma_{xz} ln .$$

(c) Write down the equations of equilibrium in cylindrical coordinates.

Q3. **[15+10]**

(a) For plane elastic problems, show that the compatibility equation in terms of stress in polar coordinates is given by, $\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right](\sigma_r+\sigma_\theta)=0$.

(b) Considering a third degree polynomial function for Airy's stress function, determine the stress fields. Explain the case of pure bending of a thin rectangular plate and show it on the boundary using a neat sketch.

Q4. **[4+17+4]**

(a) For torsion problem of straight prismatic bars, write down the governing equation and the associated boundary conditions, in terms of Prandtl's torsion stress function.

(b) Show that, for torsion of an elliptic cross-section bar, the maximum shear stress occurs at the ends of the minor axis of the cross-section.

(c) Write down the expressions of Lame's constants in terms of Young's modulus and Poisson's ratio. Express the general stress-strain relations in terms of Lame's constants.

Q5. **[20+5]**

(a) For plane stress condition, determine the radial and circumferential components of stresses of a thick cylinder (having internal and external radii, r_i and r_o respectively) under uniform internal pressure p_i and external pressure p_o.

(b) Briefly discuss the following: "The elasticity problem of a plate with a small hole under uniaxial tension can be formulated by considering two different problems of a circular disk, one under uniform external pressure and the other under varying external pressure".

Q6. Write short notes on the following: **[5 × 5]**

(a) Field equations of elasticity

(b) Volumetric strain

(c) Components of strain in polar coordinates

(d) Plane stress problems

(e) Axisymmetric problems in elasticity

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