# MASTER OF MECHANICAL ENGINEERING EXAMINATION, 2017

($1^{st}$ Year, $2^{nd}$ Semester)

## MECHANICAL SYSTEMS AND VIBRATION CONTROL

Time: **Three hours** Full Marks: **100**

Different parts of a question must be answered together.
Provide sketches wherever applicable.
Answer any **Four (4)** questions

1.(*a*). Explain what do you understand by the following terms: Modal Orthogonality and Natural Coordinates. [02+02]

1.(*b*). A 2-DoF spring mass system is represented by the following governing differential equation (in matrix form): $[M]\{\ddot{x}\} + [K]\{x\} = \{0\}$. The eigenvalues of the system is given by $\omega_1$ and $\omega_2$, while the modal vectors are $\{c\}_1$ and $\{c\}_2$. Demonstrate that the modal vectors orthogonal with respect to mass matrix as well as stiffness matrix. Also demonstrate how the orthogonality property can be used to decouple the equations of motion. [08+05]

1.(*c*). An idealized model of an automobile consists of a rigid beam of mass $m$ representing the body of the car and two springs (spring stiffness = $k_1$ and $k_2$) at the two ends to simulate the suspension. Point $C$ is the center of mass of the beam. A vertical force, $F(t)$, is applied at point $O$, which is at a distance of a and b from the two springs, respectively. Assuming small translation and rotation, derive the equations of motion of the system in matrix form. What are the conditions in which the governing equations are decoupled? [08]

2.(*a*). Describe the following types of vibration absorbers with neat sketches: Vibration Neutralizer. Auxiliary Mass Damper and Tuned Mass Damper. [03]

2.(*b*). Prove that for a vibration neutralizer the response amplitude of the main mass can be reduced to zero at one particular excitation frequency. Draw a schematic representation of the response amplitude – excitation frequency plot showing the location where response amplitude is zero. What is the physical reasoning behind this zero response – Explain. [10]

2.(*c*). Derive an expression for the normalized amplitude of steady state response for a damped dynamic vibration absorber (Tuned mass damper). [12]

3.(*a*). With a neat schematic diagram explain the different philosophies/approaches of vibration control. [08]

3.(*b*). What is meant by complex stiffness – Explain. Mention two real life situations where springs show complex behaviour. Write down the complex stiffness expression for viscoelastic damping model and subsequently derive the expression for absolute transmissibility. [02+01+01+03]

3.(*c*). Define: Absolute Transmissibility and Relative Transmissibility. In case of a SDoF system with complex spring, prove that the expression for absolute harmonic transmissibility is same for both displacement and force excitations. [04+06]

4.(*a*). Free vibration equation of a SDoF system with a nonlinear restoring force is given by, $\ddot{x} + \omega^2 f(x) = 0$. Derive a general integral expression for the time period of vibration following an exact method when $f(x) = x^{2n-1}$. Consider that the system has a displacement $x_0$ and velocity 0 at the initial moment, $t = 0$. [10]

4.(*b*). The characteristic equation of a non-conservative SDoF system is given by, $\lambda^2 - p\lambda + q = 0$, where, $p$ is the trace of coefficient matrix $[A]$ and $q$ is the determinant of coefficient matrix $[A]$. Point out the different regions of stability and instability in the $p$-$q$ plane. Also mention the type of physical motion indicated by each of the zones. [05]

4.(*c*). A system is described by the following nonlinear differential equation –

$$\ddot{x} - (0.1 - 3\dot{x}^2)\dot{x} + x + x^2 = 0$$

(*i*). Transform the $2^{nd}$ order differential equation into two equivalent $1^{st}$ order differential equations.

(*ii*). Find the equilibrium positions of the system

(*iii*). Determine the type of stability about any one of the equilibrium points. [10]

5.(*a*). Mention at least two distinct attributes where a nonlinear system differs from a linear system. Write a short note on: Sources of nonlinearity. What do you understand by hardening and softening type nonlinearity? [02+05+02]

5.(*b*). The governing equation of motion of an undamped and unforced spring-mass system is given by $\ddot{x}+\omega_0^2 x+\varepsilon\omega_0^2 x^3=0$. The system is subjected to the prescribed initial conditions, $x(0)=A_0$ and $\dot{x}(0)=0$. Assume that the displacement of the system is finite but not very large. Solve for a time-displacement solution (upto 1st approximation) of the above system following straight-forward expansion perturbation method. Is the obtained solution feasible for the system under consideration? Provide explanation for the answer. [16]

6.(*a*). Draw qualitative plots representing the frequency response of a nonlinear undamped system in the excitation frequency vs. response amplitude plane, while the nonlinearity is of hardening and softening type, respectively. Also show the backbone curve in the corresponding figures. Discuss the nonlinear features exhibited in these plots. [08]

6.(*b*). Write down the standard form of the following equations: (*i*). Mathieu equation
(*ii*). Rayleigh equation [03]

Derive the standard form of Van der Pol equation from the Rayleigh's equation. Give a few examples of systems that are governed by Van der Pol equation. Explain the significance of variable damping in Van der Pol equation. [03+02+03]

6.(*c*). Write a short note explaining the concept of limit cycle with appropriate sketches. [06]

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