

M.E. Mechanical Engineering 1<sup>st</sup> Year 2<sup>nd</sup> Semester Examination, 2017

Subject: Control of Mechatronic Systems

Time : Three hours

Full Marks: 100

Answer any **FOUR** questions.

1. a) What is the basis of evolving a fuzzy control and what kind of uncertainty is dealt by it?  
b) Show typical variations of linear and singleton membership functions.  
c) What are meant by the reaching phase and sliding phase in the context of SMC?  
d) What are meant by an autonomous system and an LTI system?  
e) What are meant by positive definite function and radially unbound function? 5×5
2. a) Explain the difference between crisp and fuzzy sets. 5  
b) The Gaussian membership functions of the sum of the nondimensional error  $e$  and its rate  $de$  involve five sets, namely 1, 2, 3, 4 and 5, which have membership value of 1.0 respectively at -4.0, -2.0, 0, 2.0 and 4.0 and acquire membership value of 0.5 respectively at -3.0, -3.0, 1.0, 1.0 and 3.0. For the output variable  $u$ , the fuzzy sets are singleton membership value of 1.0 at -4, -2, 0, 2 and 4 for the subsets LN, N, Z, P and LP respectively. The rule base is: if  $x$  is  $i$  then  $u$  is  $i$  for  $i=1$  to 5. Find the expressions for the membership functions of all the fuzzy sets. 20
3. a) What is meant by chattering in the context of sliding mode control? 5  
b) Consider a first order dynamic system  $\dot{x}=u+d$ ,  $u=\alpha \text{sgn}(x)$ ,  $|d|<C$  and  $\alpha>C$  where  $x$ ,  $u$  and  $d$  are the output error, input signal and the disturbance respectively. For an initial point on the positive side of the  $x$ -axis in the  $x-\dot{x}$  phase-plane, draw the system trajectory with proper explanations. Also determine the time to reach the sliding surface. 20
4. Consider a second order dynamic system  $\ddot{x}=-\lambda\dot{x}+u+d$  and  $|d|<C$ , where  $x$ ,  $u$  and  $d$  are the output error, input signal and the disturbance respectively and the initial state  $x_o>0$ ,  $\dot{x}_o=0$ ,  $\ddot{x}_o=0$ . With proper explanations, draw the system trajectory in the  $x-\dot{x}$  phase plane with  $\alpha>C$  for the input signal  $u = \alpha \text{sgn}(\dot{x} + \beta \text{sgn}(x)\sqrt{|x|})$ . 25
5. State and provide proofs for Lyapunov stability theorem and Lyapunov asymptotic stability theorem. 25
6. Using  $V(x_1, x_2) = x_1^2 + x_2^2$  for each of the following system, show that the origin is asymptotically stable:  
(a)  $\dot{x}_1 = -x_1 + x_2^2$ ;  $\dot{x}_2 = -x_2$ , 10  
(b)  $\dot{x}_1 = (x_1 - x_2)(x_1^2 + x_2^2 - 1)$ ;  $\dot{x}_2 = (x_1 + x_2)(x_1^2 + x_2^2 - 1)$ . 10  
Can the second system be globally asymptotically stable? 5