EX/PG/ETCE/T/111A/16/2017

# Master of E. & Tel. E. Examination, 2017

## (1st Semester)

## STATISTICAL COMMUNICATION THEORY

Time: Three hours Full Marks: **100**

Answer Question **number 1** and any **four from the rest**

Answer must be written at one place for each attempted question

Q1. a. Define the following terms (any 5) 3x5=15

- i) Random variable and random process with examples
- ii) Orthogonal and un-correlated random process
- iii) Gaussian process and binomial process
- iv) Wide Sense Stationary and Ergodic Random process
- v) Sample mean and ensemble average of random process
- vi) Autocorrelation and Power spectral density of random process

b) Given the random process $X(t)=10\cos(100t+\Theta)$ where $\Theta$ is uniformly distributed random variable in the interval $(0,\pi/2)$. What type of process it is? 05

Q2. (a) A random variable **y** is linearly estimated from the observation of another random variable **x**. Find the mean square estimation error in terms of correlation coefficients. What will happen is x and y are uncorrelated? If correlation coefficient is equal to 1 what inference do you draw? 10

(b) A discrete-time random process x(n) is generated as follows:

$$x(n) = \sum_{k=1}^{p} a(k)\, x(n-k) + w(n)$$

Where w(n) is a white noise process with variance $\sigma_w^2$. Another process z(n) is formed by adding noise to x(n),

$$z(n) = x(n) + v(n)$$

Where v(n) is a white noise with variance $\sigma_v^2$ and is uncorrelated with w(n).

- (i) Find the power spectrum of x(n)
- (ii) Find the power spectrum of z(n) 10

Q3. (a) Find the power spectrum given the autocorrelation function 03

$$r_x(k) = 2\,\delta(k) + \cos(\pi k/4)$$

(b) Find the autocorrelation function given the power spectral densities 03

$$P_x(e^{jw}) = 1/(5 + 3\cos w)$$

(c) For a WSS process x(n) the power spectrum is given by

$$P_x(e^{jw}) = (5+4\cos w) \;/\; (10 + 6\cos w)$$

Find the whitening filter that produces unit variance white noise when the input is x(n). Determine whether the filter is causal and stable. 08

(d) The input to a linear shift-invariant filter with unit sample response

$$h(n) = \delta(n) + 1/2\,\delta(n-1) + 1/4\,\delta(n-2)$$

is a zero mean WSS process with $r_x(k) = 0.5^{|k|}$, what is the variance of the output process? 06

Q4. (a) Show that for a real valued random process x(n), the spectral factorization takes the form

$$P_x(z) = \sigma_0^2\, Q(z)\, Q(z^{-1})$$ 10

(b) From the analysis in part (a) define regular process, innovation process. 05

(c) Define ARMA (p, q) process in terms of filter response and power spectral density function. How many poles and zeroes will be there for such process? 05

Q5 (a) Consider a first order AR process that is generated by the difference equation : $y(n) = a\,y(n-1) + w(n)$

Where w(n) is a zero mean while noise process with variance $\sigma_w^2$ and $|a|<1$

(i) Find the unit sample response of the filter that generates y(n) from w(n)
(ii) Find the autocorrelation of y(n)
(iii) Find the power spectrum of y(n) 10

(b) Consider the MA(q) process that is generated by the difference equation

$$y(n) = \sum_{k=0}^{q} b(k)\, w(n-k)$$

Where w(n) is a zero mean while noise process with variance $\sigma_w^2$.

(i) Find the unit sample response of the filter that generates y(n) from w(n)
(ii) Find the autocorrelation of y(n)
(iii) Find the power spectrum of y(n) 10

Q.6 (a) An FIR Wiener filter produces the minimum mean square estimate for a given process d(n) by filtering a set of observations of statistically related process x(n). Establish the Wiener-Hopf equations for the filter and find the expression for mean square error. 10

(b) We observe a signal x(n), in a noisy environment as

$$y(n) = x(n) + 0.8\, x(n-1) + v(n)$$

Where v(n) is white noise with variance $\sigma_v^2 = 1$ that is uncorrelated with x(n). The autocorrelation function for WSS process x(n) is $r_x(k)\ [4,2,1,0.5]^T$ . Find the non-causal IIR filter H(z) that produces minimum mean square estimate of x(n). 10

Q7. (a) Discuss the importance of discrete Kalman filtering process by giving the steps of this filtering. Highlight the difference with Wiener filtering process. 12

(b) What is called linear prediction?

Derive the Wiener-Hopf equation for a first order linear predictor $[W(z) = w(0) + w(1)\, z^{-1}]$ when the measurement of x(n) is noisy such that y(n) =x(n) +v(n), v(n) is white Gaussian noise with variance $\sigma_v^2$. 08

Q8.(a) What is called Hypothesis testing? Define binary and multiple Hypothesis testing? 05

(b) Let $Z_1, Z_2, Z_3 \ldots\ldots Z_n$ are random variables with zero mean and unity variance. The density function of Zi under the $H_0$ follows Gaussian $f_G(zi)$ and that for $H_1$ follows Laplacian $f_L(zi)$.

Design the Hypothesis testing rule according to maximum a posteriori probability criterion (MAP). Design the likelihood ratio test (LRT) according to Bay's rule. 05

(c ) When is Neyman-Pearson test applicable? What is the basis of this test? 03

(d) In a binary hypothesis-testing problem, the received signals under the two hypotheses are

$$H_1 : Z = X_1^2 + X_2^2$$
$$H_2 : Z = X_1^2$$

Where, $X_1$ and $X_2$ are independent, identically Gaussian distributed with zero mean and unity variance. Obtain the optimum decision rule for the Bay's criterion. Assume that $P_0 = P_1 = 1/2$, $C_{00} = C_{11} = 0$, $C_{10} = C_{01} = 1$. 07