

Programming Methodology

Time : 3 hours

Full Marks : 100

Answer any *five* questions
All questions carry equal marks

- 1(a) Explain the term Generator Basis and Generator Universe in connection with an abstract data type definition. We say that the Generator Functions are total and strict. What do we understand by total and strict function? Explain your answer by suitable example. 06
- 1(b) State the profile for the function '+' in connection with the abstract data type Nat representing the natural numbers. Present the semantic definitions for the above function using both case construct and the case free axiom. 06
- 1(c) What is the basic concept for Generator Induction? Assuming case free axiom for the operator + on natural number and using Generator Induction prove that 08
- $\forall x, y : \text{Nat} \bullet x + y = y + x$
2. What are the conditions for termination of a repetitive construct? Design a program that multiplies a nonnegative integer x by a positive integer y by using repetitive additions. Show that the program you design would terminate and will produce the required result. You may choose suitable post condition, integer function and invariant. 20
- 3(a) Let a guarded command set with its derived alternative construct IF and a predicate P be such that P and BB \Rightarrow wp(IF,P) holds for all states. Show that for corresponding repetitive construct DO we get P and wp(DO, T) \Rightarrow wp(DO, P and non BB) holds for all states. 10
- 3(b) Explain the theorem proved in part(a) in connection with the following program
Example program:
{0 ≤ x, 0 < y}
X:=13; y:=4; R:=x; Q:=0;
{ 0 ≤ R and 0 < y and x=y*Q+R}
do
 R ≥ y → R:=R-y; Q:=Q+1;
{0 ≤ R and 0 < y and x=y*Q+R}
od
{0 ≤ R and R < y and x=y*Q+R} 10

4(a) What are the different parameter passing mechanisms in connection with a procedure call? Illustrate your answer with examples. 08

4(b) Let a procedure be defined as

Procedure Proc(value \bar{x} ; value result \bar{y} ; result \bar{z})
 $\{P\} < \text{body} > \{Q\}$

and let a procedure call be represented by Proc($\bar{a}, \bar{b}, \bar{c}$) such that the corresponding post condition is R, then prove that the required weakest precondition PR for this call is

$$\{PR : P \frac{\bar{x}, \bar{y}}{a, b} \wedge (\forall \bar{u}, \bar{v} : Q \frac{\bar{y}, \bar{z}}{u, v} \Rightarrow R \frac{\bar{b}, \bar{c}}{u, v})\}$$

Also show how the above result may be simplified. 12

5. Design a program that finds the GCD of two positive integers. Show that your program terminates and produces correct result. You may assume the required invariant and the integer function. 20

6 Let a guarded command set with its derived alternative construct IF and a predicate R be such that 20

$(P \wedge BB) \Rightarrow t > 0$ for all states. Also for some t_0 for all states

$(P \wedge BB \wedge (t \leq t_0)) \Rightarrow wp(IF, t \leq t_0)$ then prove that

(i) $P \Rightarrow wp(DO, T)$ and (ii) $P \Rightarrow (DO, P \wedge \neg BB)$

7(a) Define wp(S,R). State and prove all the properties of wp(S,R) 12

7(b) What is a guarded command? How such commands may be used to derive repetitive construct. Explain the role of non determinacy in such construct. Also explain the term 'fairness' in this connection. 08

8(a) Let $Q \Rightarrow BB$
 and $(\forall j: 1 \leq j \leq n : (Q \wedge B_j) \Rightarrow wp(SL_j, R))$ holds for all states
 then prove that $Q \Rightarrow wp(IF, R)$ holds for all states. 10

8(b) State the expression for wp('x:=e', R)
 Hence find wp(SL, x = A \wedge y = B), where the statement list SL is given by "t:=x ; x:=y; y:=t".
 If after execution the values of x and y becomes 8 and 10 respectively what should be their initial values. 10