EX/PG/CNSE/T/113A/9/2017
EX/PG/ELE/T/113B/14/2017

MEE, & M. CONTROL SYSTEM ENGINEERING, 1ST SEMESTER EXAMINATION - 2017

DIGITAL CONTROL THEORY

Time : Three Hours **Full Marks : 100**

Answer any *FOUR* questions.
Answering a question at a place is preferred.

2x5+5+2x5

1(a) (i) If the function f(t) has the z-transform F(z) and if the final value of the function f(t) exists, then prove that

$$\lim_{k\to\infty} f(kT) = \lim_{z\to 1}(1 - z^{-1})F(z).$$

(ii) If the function f(t) has the z-transform F(z), then prove that

$$Z[f(t + nT)] = z^n[F(z) - \textstyle\sum_{k=0}^{n-1} f(kT)\, z^{-k}]$$

where **n** is a positive integer and **T** is the sampling period.

(b) With usual notations if $C(s) = F^*(s)G(s)$ show that $C^*(s) = F^*(s)G^*(s)$.

(c) Find the z-transform of the following functions:

(i) $F(s) = \dfrac{1}{s^2(s+1)}$.

(ii) $f(t) = te^{-5t}$

5+2x5+10

2(a) Define stability of an n^{th} order autonomous digital system given by

$$x(k + 1) = f[x(k)], f(0) = 0$$

in the sense of Liapunov at the origin.

(b) (i) State and explain the stability theorem (Second Method) of Liapunov

(ii) Determine the stability of the discrete – data system represented by the following state equations

$$x_1(k + 1) = 0.4x_1(k)$$
$$x_2(k + 1) = -0.6x_2(k)$$

P.T.O.

using the second method of Liapunov

(c) A linear digital system is represented by the following state equation

$$\mathbf{x(k+1) = Ax(k) + Bu(k)}$$

where

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

. Determine the optimal state feedback control $u^0(k)$ by Liapunov Method so that, given a positive definite symmetric matrix Q = I (identity matrix), the Liapunov function

$$V(x) = x^T(k)Px(k)$$

and the matrix P the solution of the matrix equation

$$A^TPA - P = -Q$$

the performance index

$$\Delta V(x) = V[x(k+1)] - V[x(k)]$$

is minimized.

5+10+5+5

3.. An open-loop digital system is described by the following state equation:

$$\mathbf{x(k+1)} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} \mathbf{x(k)} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u(k)}$$

(i) Show that the system can be transformed into a phase variable canonical form(PVCF)

(ii) Use the controllability matrix of the system to obtain a nonsingular transformation matrix M and hence transform the system into PVCF.

(iii) Find the feedback gain matrix $\mathbf{G}^*$ of the transformed system to place its poles at **0.6** and **0.4** within the unit circle in the z – plane. Hence find the feedback gain matrix **G** of the original system from $\mathbf{G}^*$

(iv) Verify the pole locations of both the systems in closed-loop with the desired pole locations in the z – plane.

12+13

4(a) State and prove the theorem of complete state controllability of an r-input, p-output n-th order digital system given by the following discrete state space model:

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k)$$

The matrices A,B,C and D are of appropriate dimensions.

(b) Determine the complete observability of the following system

$$x(k+1) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

12+13

5(a) An r-input, p-output n-th order digital system is represented by the following state space model:

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k).$$

The matrices A,B,C and D are of appropriate dimensions..Determine the state transition equation of the system by recursive method.

(b) A sampled data system is shown in Fig.5(b)

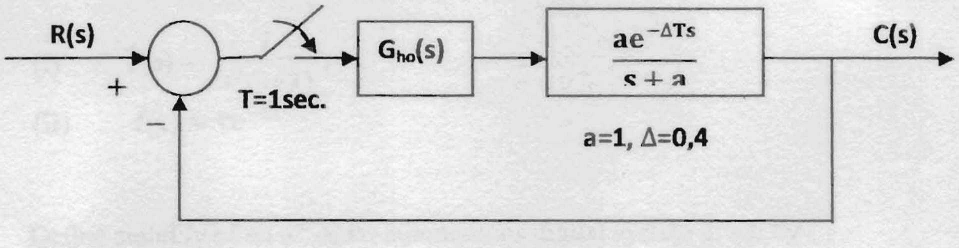

Fig. 5(b)

(i) Determine the closed-loop z-transfer function of the system - **C(z)/R(z)**

(ii) Obtain unit step input response of the system.

(12+5)+8

6(a) The OLTF of a linear digital system in z-domain is given by

$$GH(z) = \frac{0.0952Kz}{(z-1)(z-0.905)}$$

(i) Sketch the root locus of the system and indicate the salient points on it..

P.T.O.

(ii) Find the condition and frequency of sustained oscillation. Consider sampling period T = 0.1sec..

(b) The closed-loop characteristic equation of a discrete data system is as follow:

$$z^2 + (0.0952K - 1.905)z + 0.905 = 0$$

Apply extended Routh-Hurwitz criterion (in-w-plane) to determine the range of K for the stability of the system.