Ref. No. *Ex/PG/EE/T/127E/2017*
Ex/PG/CTLE/T/127B/2017

Master of Electrical Engg. & Control System Engg. 2nd Semester Examination, 2017

Optimal and Robust Control

Time: Three Hours; Full Marks: 100

Answer any ***four*** questions
All questions carry equal marks

1. a) Distinguish between structured and unstructured uncertainty. With the help of diagrams, explain the terms additive and multiplicative unstructured uncertainty. 8+2+15

   b) State one limitation of Kharitonov's theorem.

   c) For a system whose characteristic polynomial is given by
   $$s^3 + (a-1)s^2 + 2as + a^2 = 0,$$
   where a=3, check for the robust stability of the system for $\pm 10\%$ variation in each of the coefficients.

2. a) Explain the terms 'supremum' and 'infimum' with the help of appropriate examples. 5+8+12

   b) Find the 2-norm and ∞-norm of the following signal:
   $$u(t) = \begin{cases} 0, & if \quad t \le 0 \\ 3t^2, & if \quad 0 < t \le 1 \\ 0, & if \quad t > 1 \end{cases}$$

   c) For the system with transfer function $G(s) = \dfrac{as+1}{bs+1}$, find $\|G\|_\infty$ when (i) $a > b$, (ii) $a = b$, (iii) $a < b$.

3. For a unity feedback control system, the forward path transfer function is given by $P(s) = \dfrac{5000}{s(1+50s)}$. 10+5+10

   Find

   (i) the largest value of the complementary sensitivity M_t and the corresponding frequency ω_{mt}.

   (ii) the allowable size of the process uncertainty ΔP

   (iii) permissible values of gain and phase variations when a controller is designed to provide the largest value of the complementary sensitivity M_t.

4. a) Define *sensitivity function*. Justify that the sensitivity function of a system is also a measure of its robustness. 5+(6+6)+8

b) Correct or justify the following statements w.r.t. the system shown in Fig. 1:

(i) Disturbance rejection is high for frequencies where the sensitivity function S is small.

(ii) Tracking performance is good when the complementary sensitivity function T is close to 1.

c) Consider the system shown in Fig. 1. Assume that the feedback system is internally stable and $n = d = 0$. If r is a step, then prove that $e(t)\rightarrow 0$ as $t\rightarrow\infty$ iff the sensitivity function S has at least one zero at the origin.

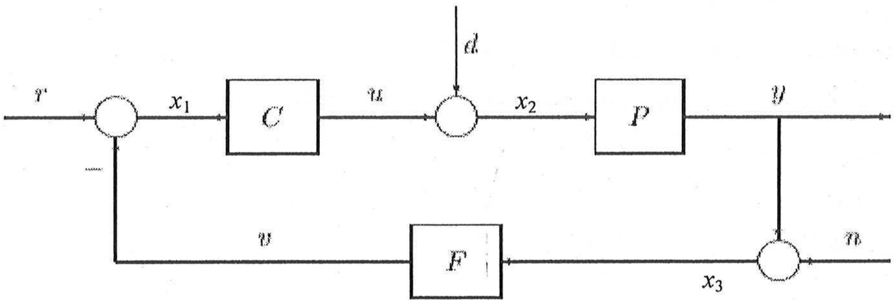

Fig. 1

5. The regulator shown in Fig.2 contains a plant described by 10+8+7

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \quad ; \quad y=\begin{bmatrix}1 & 0\end{bmatrix}x$$

and has the performance index

$$J=\int_0^\infty\left\{x^T\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}x+u^2\right\}dt.$$

Determine

(a) the Riccati matrix P

(b) the state feedback matrix K

(c) the closed loop eigenvalues.

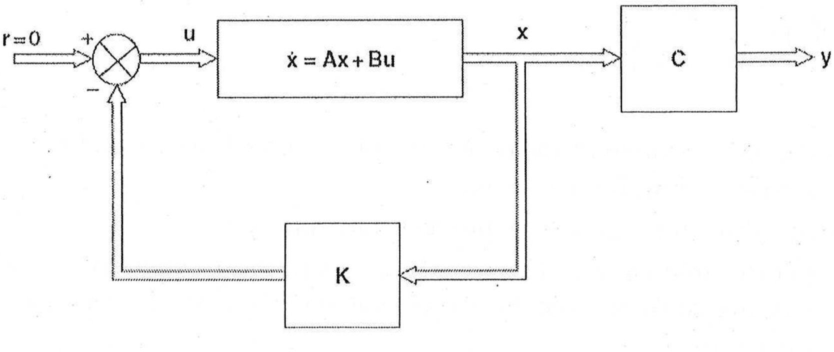

Fig.2

6 a) Given a functional $J = \int_a^b F\left(x; y; y'\right) dx$. For the fixed end-point problem, prove that the necessary condition to achieve an extremum of the functional is $F_y - \frac{d}{dx} F_{y'} = 0.$ 5+5+15

b) Determine the shortest curve y connecting two points (0, 0) and (1, 1) from the calculus of variations.

c) A plane curve, $y = y(x)$ passes through the points $(0, b)$ and (a, c), $(b \neq c)$ in the $x - y$ plane (a, b, c are positive constants) and the curve lies above the x-axis as shown in Fig. 3.
Determine the curve $y(x)$ which, when rotated about the x-axis, yields a surface of revolution with minimum surface area.

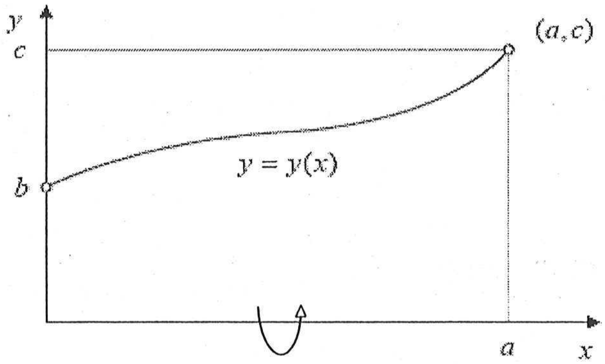

Fig. 3

7 a) State the Brachistochrone problem and derive an expression for the time of descent of a particle in a Brachistochrone problem. 12+13

b) Find the curve which is the solution to the above Brachistochrone problem, given initial position is A(x_1, y_1) and final position is B (x_2, y_2).