M.C.S.E 1st Year 2017
1st Semester
THEORY OF COMPUTING

Time: Three hours | Full Marks: 100

Answer any ***five*** questions

1. (a) Prove that a boolean formula in **conjunctive normal form (CNF)** can always be constructed to represent any boolean function from its truth-table. Hence find out the CNF for $A \oplus B \oplus C$.

   (b) Show that any arbitrary boolean formula can be transformed to an equivalent formula in **CNF**, without using truth-tables. Explain the rules used for this purpose.
   Illustrate the method using the formula $(A \vee B) \rightarrow (C \rightarrow D)$.

   12+8

2. (a) Show that transforming an arbitrary boolean formula to CNF may lead to an *exponential* growth in the size of the formula.

   (b) Explain the TSEITIN scheme of transforming an arbitrary boolean formula to an equi-satisfiable CNF when the given formula may contain the operators $\vee, \wedge, NOT, \rightarrow, \leftrightarrow$.
   Explain the importance of this scheme.

   8+12

3. (a) Explain the **DAVIS-PUTNAM** algorithm for testing the satisfiability of a boolean formula using suitable examples for each step.

   (b) The *splitting rule* often increases the number of clauses but the algorithm still works - explain.
   Using a suitable example, show that the speed of the algorithm depends on the order of choosing the variables for splitting.

   15+5

4. (a) Define the *implication graph* of a boolean formula in CNF whose each clause contains exactly 2 literals. Draw the implication graphs for the following formulas :
   (i) $(A + \bar{B})(A + B)(\bar{A} + B)(\bar{A} + \bar{B})$
   (ii) $(A + B)(A + \bar{C})(\bar{A} + B)(B + C)$

   (b) Hence describe an algorithm for solving $2 - SAT$ in polynomial time and prove its correctness.
   Explain why your algorithm will work within *polynomial* amount of time.
   Using your algorithm, find out which of the above two formulas is satisfiable. Also find a satisfying assignment for the satisfiable formulas.

   6+14

5. (a) Construct a single tape Turing machine which accepts the language $\{wcw : w \in \{a, b\}^*\}$.
   Give necessary explanations and justifications.

   (b) Construct a multitape Turing Machine which accepts all strings of the form $w_1\#w_2\# \cdots \#w_n$ for any $n \geq 1$ where each w_i is a binary string and for at least one i, w_i is the binary representation of the number i.

   10+10

6. (a) A language L is accepted by a Turing Machine M with 1-way infinite tape. Let M' be a Turing Machine with 2-way infinite tape whose transitions are exactly the same as that of M.
   Explain if M' will also accept L.
   Explain how M' can be modified so that it also accepts L.

(b) Prove that a language L is accepted by a Turing Machine with 2-way infinite tape if and only if it is accepted by a Turing Machine with 1-way infinite tape.

5+15

7. (a) Explain what is a NP-complete problem.
Describe the SET-COVER problem and prove that it is NP-complete by reduction from general CNF-SAT problem.

(b) Describe the 3-COLORING problem for graphs and prove that it is NP-complete by reduction from 3-SAT problem.

7+13