Masters of Chemical Engineering Examination, 2017

(1^{st} Semester)

Advanced Heat Transfer

Time: Three Hours | Full Marks: 100

Answer Question No. 1 __and__ Any Two from the rest.

[1] As a good model problem, we consider steady state heat transfer to fluid in steady flow through a tube. The fluid enters the tube at a temperature T_0 and encounters a wall temperature at T_w, which can be larger or smaller than T_0. A simple version of this problem was first analyzed by **Graetz** (1883).

a) Obtain the steady temperature distribution, $T(r,z)$ in the fluid,
b) Calculate the rate of heat transfer from the wall to the fluid.
c) Sketch (qualitatively) the behavior of the Nusselt number as a function of dimensionless axial position.
d) Explain the Lévêque approximation in connection to the Graetz problem.

The following assumptions may be adopted,

- Steady fully developed laminar flow; steady temperature field.
- Constant physical properties: ρ, μ, k, C_p; This assumption also implies incompressible Newtonian flow.
- Axisymmetric temperature field $\Rightarrow \frac{\partial T}{\partial \phi}$, where we are using the symbol ϕ for the polar angle. Use the symbol θ to represent dimensionless temperature.
- Negligible viscous dissipation.

[15+25+3+7=50]

[2] Starting with the following governing equation, develop a suitable model of heat transfer for laminar flow in a circular tube under hydro-dynamically and thermally developed conditions with *__Uniform Wall Temperature__*. At any point in the tube the boundary layer approximations may be applied. Here α is the thermal diffusivity.

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Hence show that, for a __*parabolic*__ velocity profile, $Nu_D = 3.656$

[25]

[3] Starting with the following governing equation, develop a suitable model of heat transfer for laminar flow in a circular tube under hydro-dynamically and thermally developed conditions with *__Uniform Wall Heat Flux__*. At any point in the tube the boundary layer approximations may be applied. Here α is the thermal diffusivity.

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Hence show that, for a *parabolic* velocity profile, the fully developed temperature profile is:

$$T(r) = T_W + \frac{\dot{q}_W r_0}{k}\left[\left(\frac{r}{r_0}\right)^2 - \frac{1}{4}\left(\frac{r}{r_0}\right)^4 - \frac{7}{24}\right],$$ k is the thermal conductivity.

[25]

[4]

a) Explain various regimes of Boiling with the help of a diagram. Explain the significance of *Peak Heat Flux* and *Leidenfrost point*.

b) Develop an expression for the thickness of the boundary layer developed on a vertical flat plate during free convection heat transfer.

[7+18=25]

[5] Consider film condensation on a vertical plate of length L. The plate temperature is maintained at Tw, and the vapour temperature at the edge of the film is the saturation temperature Tg. The film thickness is represented by δ. It is assumed that the viscous shear of the vapour on the film is negligible. Obtain the following Nusselt number correlation.

$$Nu_x = \left[\frac{\rho(\rho - \rho_v) g h_{fg} x^3}{4\mu_f k (T_g - T_w)}\right]^{1/4}$$

Where, h_{fg} is the latent heat of condensation and μ_f is the viscosity of vapour respectively, k is the thermal conductivity of the material of the plate wall. Choose a coordinate system with positive direction of x being measured downward. Also find out a correlation for the average heat transfer coefficient.

[25]

Continuity Equation

Cylindrical

$$\frac{\partial \rho}{\partial t}+\left\{\frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta)+\frac{\partial(\rho V_z)}{\partial z}\right\}=0$$

Navier Stokes Equations

Cartesian

$$x: \quad \rho\left\{\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}\right\}=\rho g_x-\frac{\partial p}{\partial x}+\mu\left\{\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\right\}$$

$$y: \quad \rho\left\{\frac{\partial V_y}{\partial t}+V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}+V_z\frac{\partial V_y}{\partial z}\right\}=\rho g_y-\frac{\partial p}{\partial y}+\mu\left\{\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_y}{\partial z^2}\right\}$$

$$z: \quad \rho\left\{\frac{\partial V_z}{\partial t}+V_x\frac{\partial V_z}{\partial x}+V_y\frac{\partial V_z}{\partial y}+V_z\frac{\partial V_z}{\partial z}\right\}=\rho g_z-\frac{\partial p}{\partial z}+\mu\left\{\frac{\partial^2 V_z}{\partial x^2}+\frac{\partial^2 V_z}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2}\right\}$$

Cylindrical

r:

$$\rho\left\{\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}-\frac{V_\theta^2}{r}+V_z\frac{\partial V_r}{\partial z}\right\}$$
$$=\rho g_r-\frac{\partial p}{\partial r}+\mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)\right)+\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}+\frac{\partial^2 V_r}{\partial z^2}\right\}$$

θ:

$$\rho\left\{\frac{\partial V_\theta}{\partial t}+V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_rV_\theta}{r}+V_z\frac{\partial V_\theta}{\partial z}\right\}$$
$$=\rho g_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}+\frac{\partial^2 V_\theta}{\partial z^2}\right\}$$

z:

$$\rho\left\{\frac{\partial V_z}{\partial t}+V_r\frac{\partial V_z}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}+V_z\frac{\partial V_z}{\partial z}\right\}=\rho g_z-\frac{\partial p}{\partial z}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}+\frac{\partial^2 V_z}{\partial z^2}\right\}$$