ANALYTICAL STUDIES ON ANNULAR FIN PERFORMANCE AND ITS OPTIMIZATION UNDER INTERNAL HEAT GENERATIONS

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Abstract

In the present work, a new analysis model based on the **Frobenius** power series is developed for the thermal analysis of an annular disc fin (ADF). A linear variation of temperature dependent internal volumetric heat generation inside the fin has been taken into account. The temperature distribution in fins has been determined by solving the homogeneous nonlinear governing differential equation with the help of infinite **Frobenius** power series. The thermal performance has been evaluated over a wide range of thermo-geometric parameters. Results of thermal analysis has been validated by finite difference method. Heat transfer analysis has been carried out for both the convected tip and insulated tip fin. From the result, it has been observed that the maximum fin performance has been achieved at a particular value of thermo-geometric parameter under internal heat generation which can be the practical design condition to operate fins for enhancing heat transfer rate. Also a convected tip of ADF is giving more advantages over insulated tip for the thermal performance with the range of Biot number value. Maximum fin heat transfer rate has been optimized using Lagrange multiplier technique. Optimized curves are obtained for various thermo-geometric parameters. These optimized curves are generally converging.

Research Publication from this Thesis

- ## Conferences

1. **T. Majhi,** B. Kundu,“New approach for determining fin performances of an annular disc fin with internal heat generation”International Conference on Recent Innovations and Developments in Mechanical Engineering (ICRIDME),2018,NIT Meghalaya.

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NOMENCLATURE

Abbreviation

ADF	Annular Disk Fin

Symbols

A_n	coefficient shown in equ. (14) (dimensionless)
a_n	coefficient of expansion series
Bi	Biot number (dimensionless)
Bi_t	fin tip Biot number (dimensionless)
$C_1 \& C_2$	constants define in equ. (15)
h	heat transfer coefficient (W/m^{2}k)
h_t	tip heat transfer coefficient (W/m^{2}k)
k	thermal conductivity (W/mk)
Q_0	heat transfer rate, (dimensionless), $q_0 r_2^2 / k\left(T_b - T_\infty\right)$
q'''	volumetric heat generation rate as a function of temperature(W/m^3)
q_0	volumetric heat generation rate at temperature T_∞ (W/m^3)
R	radius ratio, r/r_2 (dimensionless)
R_1	inner radius ratio of fin, r_1/r_2 (dimensionless)
R_2	outer radius ratio of fin, $r_2/r_2 = 1$(dimensionless)
r	radius of the annular fin (m)
r_1	inner radius of fin (m)
r_2	outer radius of fin (m)

T	local fin temperature (k)
t	half thickness of fin (m)
T_b	fin base temperature (k)
T_∞	temperature of surrounding (k)
U	volume ratio (dimensionless)
V	volume of annular disk fin (m^3)
Z_0	fin geometry parameter, $\sqrt{Bi/\psi^2}$ (dimensionless)
Z_1	dimensionless fin geometry parameter

Greek symbols

α	coefficient of internal heat generation (/k)
β	coefficient of heat generation (dimensionless)
ε	effectiveness
φ	temperature in fin (dimensionless)
ϕ_0	temperature at R_1 (dimensionless)
η	efficiency
θ	non-dimensional temperature
ψ	half thickness to outer radius ratio

Superscripts

n	positive real number used in equ. (12)
s	roots shown in equ. (12)

Subscript

t	for tip condition, used in equ. (2b)

Chapter-1

1. INTRODUCTION

Avoiding overheating and increasing the life span of components of various thermal application fins is used. It is an extended surface equipped on the component to enhancing the heat transfer rate from the thermal system to the surrounding environment. In the design and construction of various types of heat-transfer equipment and components such as air conditioner, refrigerator, superheaters, automobile, power plants, heat exchangers, convectional furnaces, economizers, gas turbines, chemical processing equipment, oil carrying pipelines, computer processors, electrical chips, etc., fins are used to implement the flow of heat between a source (primary surface) and sink. Kern and Kraus[1] give's the three basic geometries of fins. They are longitudinal fin, annular or radial fin and pin fin or spines respectively. Based on the primary surface of the heat transfer component these fins are used. Longitudinal fins are used when the fluid flowing parallel to the axis on the thermal system. Pin fins or spines are the rod protruding from a plane surface used to increase the surface area (when the heat transfer coefficient of the fluid is relatively low) and consequently to increase the total heat transfer rate. On the cylindrical surface annular or radial fins are suitable the most.Apart from various types of fins with different geometries used, annular disk fin is widely applied in heat transfer equipment's due to its ease of design and fabrication. A more realistic design of fin need a close assumptions to be perfectly efficient based on working condition. Internal heat generation can be considered temperature dependent which is very realistic for the fins as applied on electric current carrying conductor, nuclear rods exposing to gamma rays or any other heat generating components of thermal systems.

Chapter -2

2. LITERATURE REVIEW

Initially through a measurement, thermal conductivities of the long metallic rods of iron and copper experimentally determined by Stewart[2] using the temperature distribution in the rod. For any of the above fin constant thickness or straight profile is very common to use as its design and casting process is easier. Parsons and Harper[3] derived an equation for the efficiency of straight fins of constant thickness in the course of a paper on airplane-engine radiators. Harper and Brown[4] in connection with air-cooled aircraft engines investigated the equation of the efficiency of straight fins of constant thickness, wedge-shaped straight fins, and annular fins of constant thickness. They also evaluated the errors involved in the assumptions for the investigations. Schmidt[5], first covered the same three types of fin from the standpoint of material economy. He stated that the least metal is required for given conditions if the temperature gradient is linear, and showed how the thickness of each type of fin must vary to produce this result. Finding, in general, that the calculated shapes were impractical to manufacture, he proceeded to show the optimum dimensions for straight and annular fins of constant thickness and for wedge-shaped straight fins under given operating conditions. Murray[6] presented equations for the temperature gradient and the effectiveness of annular fins of constant thickness with a symmetrical temperature distribution around the base of the fin.Carrier and Anderson[7] discussed straight fins of constant thickness, annular fins of constant thickness, and annular fins of constant cross-sectional area, presenting equations for the fin efficiency of each. In the latter two cases the solutions are in the form of infinite series.

However the performance of a fin is reduces towards the fin tip due to the reduction of temperature from base to tip. It is very necessary to save the material from both the economical background as the performance and the cost are the basic criteria for design a fin. This point of view had let various exercises to optimization a fin dimension. Two approaches have been developed to analyse the optimum dimension of the fin. First criterion gives for the particular amount of heat transfer, the fin shape has to optimize such that the fin volume will be minimum. In contrast to the criterion the fin profile becomes a curve. Second one for a given shape of fin profile, satisfying the required condition of heat transfer, the fin dimension has to optimize such that the volume will be minimum. Using the calculus of variation, Duffin[8] exhibits a rigorous proof of the criteria of Schmidt[5]. Both Schmidt[5] and Duffin[8] estimated the surface area neglecting the curvature fin profile. Liu[9] extended the variational principle to find out the optimum profile of fins with internal heat generation. Liu[10] and Wilkins[11] addressed for the optimization of radiating fins. Solov"ev[12] determined the optimum radiator fin profile. From the above literature works, it can be indicated that the above works were formulated based on the "length of arc idealization (LAI)". LAI was used for optimizing fin shapes under convecting, radiating, convective-radiating condition, for fins with heat generation and for variable thermal conductivity. The correct formulation for the optimization of longitudinal fin with the elimination of LAI and a different profile rather than Duffin[8] has been proposed using numerical integration by Maday[13]. It is interesting to note that an optimum convecting fin neither has a linear temperature profile nor possesses a concave parabolic shape suggested by Maday[13]. The profile shape contains a number of ripples denoted as wavy fin. Further this analysis has been extended for radial fin by Guceri and Maday[14].Later Razelos and Imre[15] using Pontryagin's minimum principle to evaluate the minimum mass of convective fins with variable heat transfer coefficient.

From the above literature review it has been conclude that the resulting fin profile achieved, was difficult to manufacture from the first approach, so the second approach has been considered for the analysing the fin dimension. Over the years many researches has been carried out to enhance the fin performance based on second approach as it is very popular. Following the second approach analysis has been carried out on straight or constant thickness as well as triangular profile to design the fin with least volume of material. For different fin profiles, optimizational steps have been thoroughly demonstrated by Aziz[16]. Annular fins are important for the fin-tube heat exchangers. And most of the heat exchangers are depend on the performance of the annular fin. Chambers and Somers[17] determined the performance of an annular fin with a rectangular profile for boundary conditions consisting of a constant temperature at the fin base and insulation at the fin tip. Sparrow and Niewerth[18] developed a numerical linearized solution for the efficiency of a radiative-convective fins. Smith and Sauce[19] provide the analytical solution for the efficiency of the annular fin with triangular profile by Frobenius method. Whereas, Sikka and Iqbal[20] adopted a finite-difference procedure to analyze the effectiveness of radiative convective fins. A general analysis has been carried out for arbitrary fin profile with coordinate dependent internal heat generation, thermal conductivity and heat transfer coefficient by Melese and Willkins[21]. Aziz et al.[22] studied a uniformly thick radial fin with convective heating at the base and convective-radiative cooling at the tip for homogeneous and functionally graded materials, with internal heat generation.

In most of the application the optimum fin shape is very important from both the economical as well as performance criterion. The optimum dimensions of circular fins with different profiles and temperature dependent thermal conductivity, has been determined by

Zubair et al.[23]. An increasing in heat transfer rate through the optimum profile fin has been found almost 20% as compare to the constant thickness fin. As the annular fin with constant thickness is easy to design and fabrication, it is used widely in various applications. Ahmadi and Razani[24] derive the expressions of optimum profiles for straight and circular fins, with variable thermal conductivity and arbitrary heat generation per unit width of the fin. Optimization studies for radial fins of specific profiles have been studied byLaor and Kalman[25], Yu and Chen[26], Heggs and Ooi[27] andLai et al.[28]. Kundu and Das[29] described a generalized methodology to determine the optimum design of thin fins with uniform volumetric heat generation. A genetic algorithm for fin profile optimization was proposed by Fabbri[30]. Kundu and Lee[31] demonstrated a novel analysis which is based on the calculus of variation to determine the smallest envelope fin shape for wet fins with a nonlinear mode of surface transport. Hanin and Campo[32] described an analysis on optimum shape of straight fin by minimizing the volume for a given amount of heat transfer per unit width. Kundu and Barman[33] established an analysis based on a Frobenius series expansion to determine the performance and optimum dimensions of annular disc fins under dehumidifying conditions based on linear relationships between the temperature and humidity. Peng and Chen[34]used a hybrid numerical technique based on the differential transform method and finite difference to analyse an annular disc fin for temperature dependent thermal conductivity. The temperature distribution and fin efficiency of annular fins with different cross sectional area subjected to heat and mass transfer were analysed by Sharqawy et al.[35]. Moinuddin et al.[36]extended this analysis to determine the optimum dimensions. Campo and Morrone[37]presented a thermal analysis using simple computational procedure of annular fins with tapered cross section. Minkler and Rouleau[38]developed analytical solutions for the temperature distribution and optimum fin parameters, and examined a convective fin with uniform internal heat generation. As economy is the consideration of design so Kundu and Das[39]analysed the temperature distribution of a concentric annular fin with a step change in thickness for a constant heat generation per unit volume. Bessel functions were used to determine the solution.

All the above analysis was concerned with annular fin which is constant or variable thickness. It is very easy to fabricate the constant thickness compared to a variable one. The research has been made on annular fins for various method and assuming different thermal properties. Some of the literature assumed constant internal heat generation and few of them considered it for temperature dependent. The assumption of temperature dependent heat generation is very closer to an actual case to study. In addition, no exact analysis has been presented to determine heat transfer in annular fins based on the above design condition.

Both economic consideration and ease in fabrication of annular disc fin have been considered to study the thermal performance in the present work. Convective heat transfer process is considered here. The internal heat generation is assumed to be linearly dependent on temperature. The Frobenius power series expansion approach has been used to solve the governing for the temperature in fins analytically. The present analysis is validated with the numerical values.

Chapter-3

3. Mathematical Formulation

The schematic diagram of annular fin with rectangular profilecircumscribing a cylindrical tube has been depicted in Fig. 1. The fin dimensions are taken such that the thickness is $2t$, and inner and outer radius are r_1 and r_2 respectively. Before analyzing the heat transfer analysis of an annular disk fin (ADF), a few assumptions has been considered here. The following assumptions are

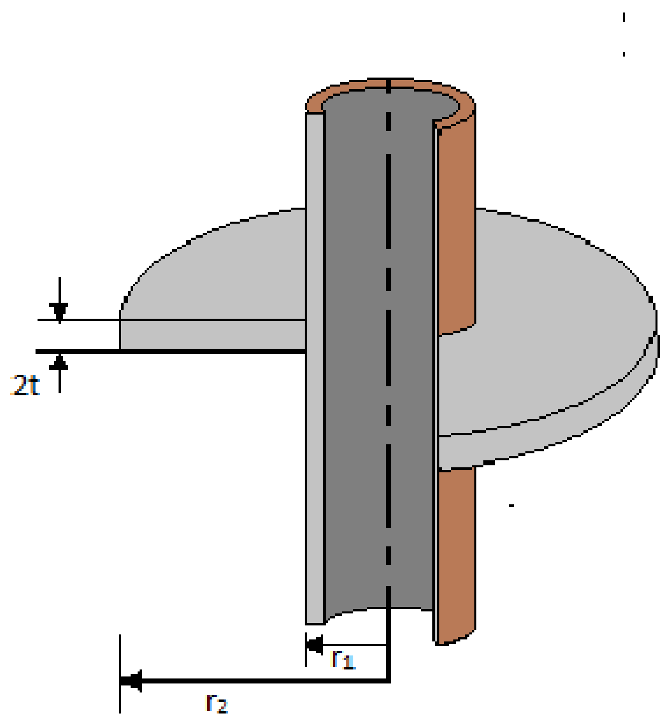


Figure 1: Schematic diagram of annular fin

1. Heat transfer is taking place at steady state condition.
2. Base temperature of the ADF is assumed to be constant.
3. Surrounding temperature is constant.
4. Only convective and conductive modes of heat transfer take into consideration.

5. Heat conduction is taking place only radial direction only, as the thickness is very small, heat transfer doesn't taken place in vertical direction.
6. Volumetric internal heat generation $q^{'''}$ is linearly temperature dependent.
7. Thermal conductivity 'k' and convective heat transfer coefficient 'h' are assumed constant.

Following the assumptions for the steady state heat transfer analysis, the governing equation of the ADF can be written by energy balance for a differential control volume of the fin in polar coordinate system as follows

$$\frac{\mathrm{d}}{\mathrm{dr}}\left(r\frac{dT}{dR}\right)+\frac{q^{'''}r}{k}=\frac{hr}{kt}\left(T-T_{\infty}\right) \tag{1}$$

The problem is considered for the annular fin has constant base temperature with convected tip fin. According to this consideration, the boundary conditions for the heat transfer analysis can be written as following

$$\text{at}\, r=r_1;\quad T=T_b \tag{2a}$$

$$\text{at}\, r=r_2;\quad -k\,dT/dr=h_t(T-T_{\infty}) \tag{2b}$$

where T_b is defined as base temperature of the ADF. Equation (1) has the term volumetric heat generation which is considered linearly temperature dependent.

$$q^{'''}=q_0\left[1+\alpha\left(T-T_{\infty}\right)\right] \tag{3}$$

Substituting the equation (3) in equation (1) and choosing appropriate dimensionless parameter equation (1) can be expressed in dimensionless form as follow

$$\frac{d}{dR}\left(R\frac{d\theta}{dR}\right)+Q_0R(1+\beta\theta)=Z_0^2R\theta \tag{4}$$

where

$$R=\frac{r}{r_2};\ \theta=\frac{T-T_\infty}{T_b-T_\infty};\ Q_0=\frac{q_0r_2^2}{k(T_b-T_\infty)};\ \beta=\alpha(T_b-T_\infty);\ Z_0=\sqrt{\frac{Bi}{\psi^2}};\ \psi=\frac{t}{r_2} \tag{5}$$

Converting the convenient boundary conditions into non-dimensional form using the non-dimensional parameter from equation (5), equation (2a) and (2b) are formed as following

$$\text{at } R=R_1,\ \theta=1 \tag{6a}$$

$$\text{at } R=1,\quad \frac{d\theta}{dR}=-Bi_t\theta \tag{6b}$$

The non-dimensionalised governing differential equation (4) is transform to a homogeneous equation. It has been solved using a new approach is called “Frobenius expansion series”.

$$R\frac{d^2\phi}{dR^2}+\frac{d\phi}{dR}-Z_1^2R\phi=0 \tag{7}$$

where,

$$Z_1^2=Z_0^2\left(1-\frac{Q_0\beta}{Z_0^2}\right);\quad \phi=\theta-\frac{Q_0/Z_0^2}{1-Q_0\beta/Z_0^2} \tag{8}$$

The newly transformed homogeneous governing equation (6) boundary conditions changes to

$$at\ R=R_1,\ \phi=\phi_0 \tag{9a}$$

$$at\ R=1;\quad \frac{d\phi}{dR}=-Bi_t\phi-Bi_t(1-\phi_0) \tag{9b}$$

where, $\phi_0 = 1 - \dfrac{Q_0 / Z_0^2}{1 - Q_0\beta / Z_0^2}$ (10)

so $\theta = \phi + 1 - \phi_0$ (11)

The transformed governing linear homogeneous differential equation (7) solution has been approximated semi-analytically by the approach ofFrobenius series expansion. According to the explanation of Frobenius series expansion, temperature can be expressed in power series as shown

$$\phi = \sum_{n=0}^{\infty} a_n R^{n+s} \tag{12}$$

Replacing the equation (12) in equation (7), the following is derived

$$\sum_{n=0}^{\infty} a_n (n+s)(n+s-1) R^{n+s-1} + \sum_{n=0}^{\infty} a_n (n+s) R^{n+s-1} - Z_1{}^2 \sum_{n=2}^{\infty} a_{n-2} R^{n+s-1} = 0 \tag{13}$$

Where the coefficient of the series a_n can be expressed as a function of a_0 & a_1 , for $n \geq 2$. Solving the equation (13) for $n = 0$, the value of the roots comes as $s = 0,0$. Considering these values of s all the coefficients has been calculated. For $n = 0$ and 1, a_0 and a_1 have been evaluated from the above equation (13), whereas $a_0 \neq 0$ and a_1=0. For $n \geq 2$ the value of a_n is calculated in dimensionless as

$$A_n = \frac{Z_1^2}{(n+s)^2} A_{n-2} \tag{14}$$

As the two roots are equal $(s = 0,0)$ the solution of the equation (13) in Frobenius method gives

$$\phi = C_1 \sum_{n=0}^{\infty} a_n R^n + C_2 \frac{d}{ds}\left[\sum_{n=0}^{\infty} A_n R^{n+s}\right]_{s=0} \tag{15}$$

where $C_1 \& C_2$ are two constants to be determined by using boundary conditions. Progressing further calculation it arrives to

$$\phi = C_1 \sum_{n=0}^{\infty} A_n R^n + C_2 \left[\sum_{n=0}^{\infty} C_n R^n + \sum_{n=0}^{\infty} A_n R^n \ln R\right] \tag{16}$$

where,

$$C_n = -\frac{2Z_1^2}{n^3} A_{n-2} + \frac{Z_1^2}{n^2} C_{n-2} \tag{17}$$

So the values of the C_n can be calculated as the values of A_n evaluated earlier from the equation (14).

Now using equations (9a) and (9b) the values of $C_1 \& C_2$ is evaluated from equation (16). Replacing the value of $C_1 \& C_2$ in the equation (16), the final mathematical expression of non-dimensional temperaturedistribution has been found out as following

$$\phi/\phi_0 = \frac{Lp + KMp - KPm - Nm}{PL - NM} \tag{18}$$

where,

$$K = \frac{B_{it}(1-\phi_0)}{\phi_0}; \qquad L = \left(\sum_{n=0}^{\infty} nC_n + \sum_{n=0}^{\infty} A_n + B_{it} \sum_{n=0}^{\infty} C_n\right);$$

$$m = \left(\sum_{n=0}^{\infty} C_n R^n + \sum_{n=0}^{\infty} A_n R^n \ln R\right); \quad M = \left(\sum_{n=0}^{\infty} C_n R_1{}^n + \sum_{n=0}^{\infty} A_n R_1{}^n \ln R_1\right);$$

$$N = \left(\sum_{n=0}^{\infty} nA_n + B_{it} \sum_{n=0}^{\infty} A_n\right); \qquad p = \sum_{n=0}^{\infty} A_n R^n; \quad P = \sum_{n=0}^{\infty} A_n R_1{}^n \tag{19}$$

Here the equation (18) gives the non-dimensional temperature distribution. Now the dissipated heat transfer rate from the fin can be define by the formulation as below

$$
\begin{aligned}
Q &= \frac{q}{4\pi r_2^2 h\left(T_b - T_\infty\right)} \\
&= \int_{R=R_1}^{1} R\theta dR + Bi_t R_1 \psi \left.\theta\right|_{R=1}
\end{aligned}
\tag{20}
$$

Integrating the equation (20) using the equation of (11), yields the heat transfer equation

$$
\begin{aligned}
Q = f\left(R_1, \psi\right) &= \frac{1}{2}\left(1-\phi_0\right)\left(1-R_1^2\right) + C_1 \sum_{n=0}^{\infty} \frac{A_n}{n+2}\left(1-R_1^{n+2}\right) + C_2 \sum_{n=0}^{\infty} \frac{C_n}{n+2}\left(1-R_1^{n+2}\right) \\
&- C_2 \sum_{n=0}^{\infty} \frac{A_n}{n+2}\left(1-R_1^{n+2}\right)\ln R_1 - C_2 \sum_{n=0}^{\infty} \frac{A_n}{\left(n+2\right)^2}\left(1-R_1^{n+2}\right) + Bi_t \psi R_1\left(1-\phi_0\right) \\
&+ Bi_t \psi R_1\left(C_1 \sum_{n=0}^{\infty} A_n + C_2 \sum_{n=0}^{\infty} C_n\right)
\end{aligned}
\tag{21}
$$

Where these value of constants C_1 and C_2 are evaluated earlier. And the second term is considered for convected tip otherwise the it is zero for insulated tip. To determine the fin efficiency$\left(\eta\right)$ and fin effectiveness$\left(\varepsilon\right)$ ideal heat transfer rate and heat transfer rate through the same base area of for the no-fin condition is determined from the following expressions, respectively.

$$
Q_i = \frac{q_i}{4\pi r_2^2 h\left(T_b - T_\infty\right)} = \theta_{\max}\left[\frac{\left(1-R_1^2\right)}{2} + \frac{Bi_t}{Bi}\psi\right]
\tag{22}
$$

$$
Q_b = \frac{q_b}{4\pi r_2^2 h\left(T_b - T_\infty\right)} = \psi R_1
\tag{23}
$$

where q_i defines ideal heat transfer rate and Q_i is the non-dimensional heat transfer rate, similarly q_b is the heat transfer rate from the base and Q_b is the non-dimensional heat transfer rate from the base. The maximum temperature $\theta_{\max}$ for cooling fins always occurs at the fin base for cases without heat generation in the fin. However, this may not be true with heat generation, depending on the rate. In such cases, a search technique may be required to determine the maximum temperature and calculate the fin efficiency.

Therefore the calculation of efficiency can be written as follows

$$\eta = \frac{Q}{Q_i} \tag{24}$$

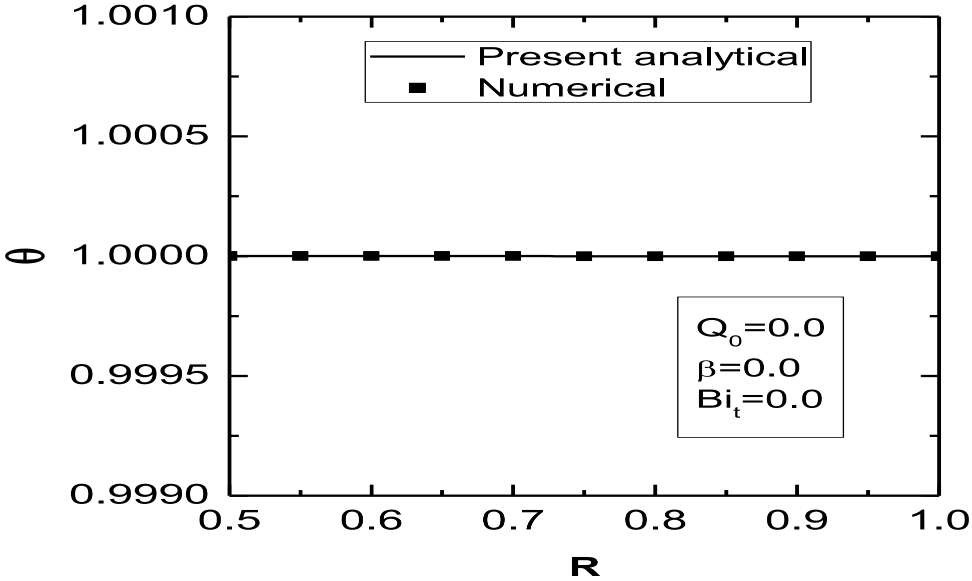
And from the definition of effectiveness it is calculated as

$$\varepsilon = \frac{Q}{Q_b} \tag{25}$$

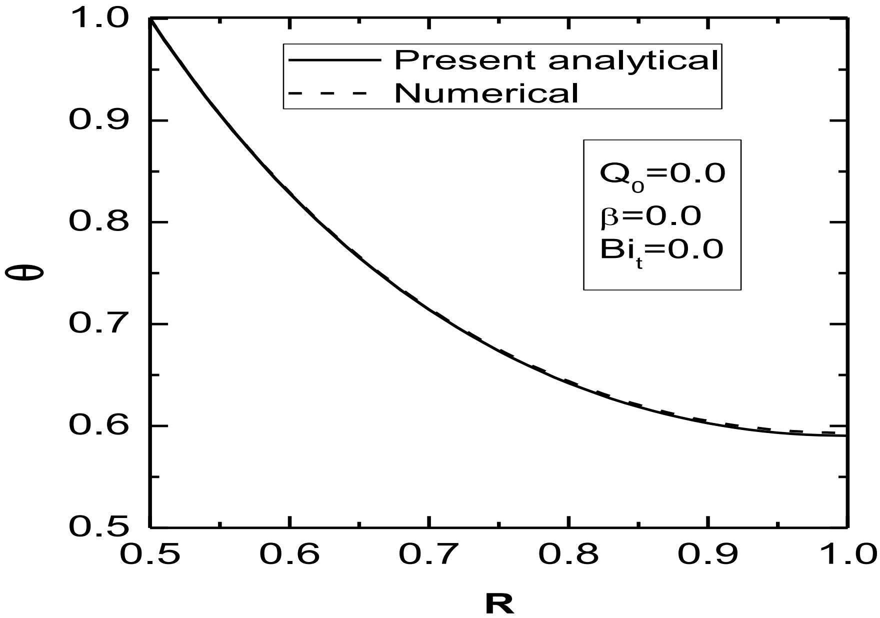
Chapter- 4

4. RESULTS AND DISCUSSION

Using above analysis the thermal performances of annular disc fin (ADF) with constant thickness has been studied for wide variation of thermo-geometric parameter. The fin temperature distribution, efficiency and effectiveness are analysed with various parameter for both insulated and convective tip fin. As well as the optimization has been carried out to evaluate its applicability based on the cost and materialistic demand. For the validation, the present analysis has been compared with the numerical values. The figures shows here for the performances are well realistic to have interest for examine. Optimizational figures are much practical to suit in applications effectively.



(a) $Z_0 = 0.001$



(b) $Z_0 = 2.0$

Fig. 2 Validation of the proposed analysis with a numerical analysis based on the finite difference method.

Considering no internal heat generation for the insulated tip fin, the non-dimensional temperature distributional profile along the radial direction for a given fin parameter Z_0 has been depicted in Fig. 2. The value of Z_0 is considered very less as shown in fig 2a when the fin temperature remain constant throughout the radius. It is an ideal case. The Frobenius analysis gives the same result as depicted by line in the Fig. 2a. Non-dimensional temperature distribution for a practical case shows the decreasing of radial temperature with a designable value $Z_0 = 2$ as shown in Fig.2b. The analytical present approach gives a very closer result with the numerical one.

Fig. 3 has been plotted when the internal heat generation has been considered for finding out the non-dimensional temperature distribution. This analysis has been carried out using the value of non-dimensional internal heat generation coefficient $\beta = 0.01$. It is very much practical from a designable value of a parameter $Z_0 = 2$. From this figure, for the insulated tip, the temperature at any radius of the annular fin is always higher than the convected tip fin. The heat dissipation from a fin with the convected tip is at a higher rate due to an extra heat transferring through the tip surface instead of the insulated tip. The temperature at the base of the fin is same for both the cases and the difference of non-dimensional temperature increases gradually along the radial direction. This result has been predicted by the finite difference method also and a good agreement of results has been found.

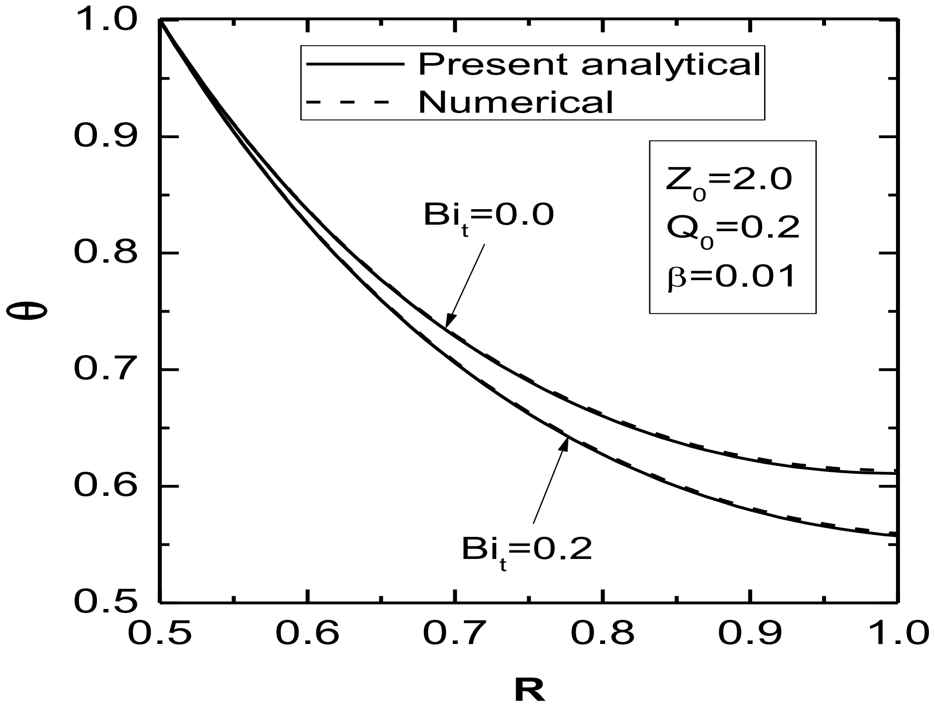


Fig. 3 Temperature distribution in an annular disc fin with internal heat generation determined by proposed analysis and numerical method

Earlier we discussed the temperature profile for both the convected tip and insulated tip fin under a particular condition. Now the temperature profile dependency of convected tip fin on the radius ratio has been provided on figure 4 so do we have the clear idea of an ADF with convective tip condition as the analysis done based on a practical case (Z_0=2). Keeping internal heat generation and outer radius constant, the inner radius is increasing. At R_1=0.2, the temperature profile has been shown. As the radius ratio increased the fin surface area reduced to only pipe surface when the radius ratio becomes 1. As the surface area reduced the heat transfer rate from the fin surface reduced respectively. So the temperature distribution inside the fin increases due to this decrease inheat dissipation rate.

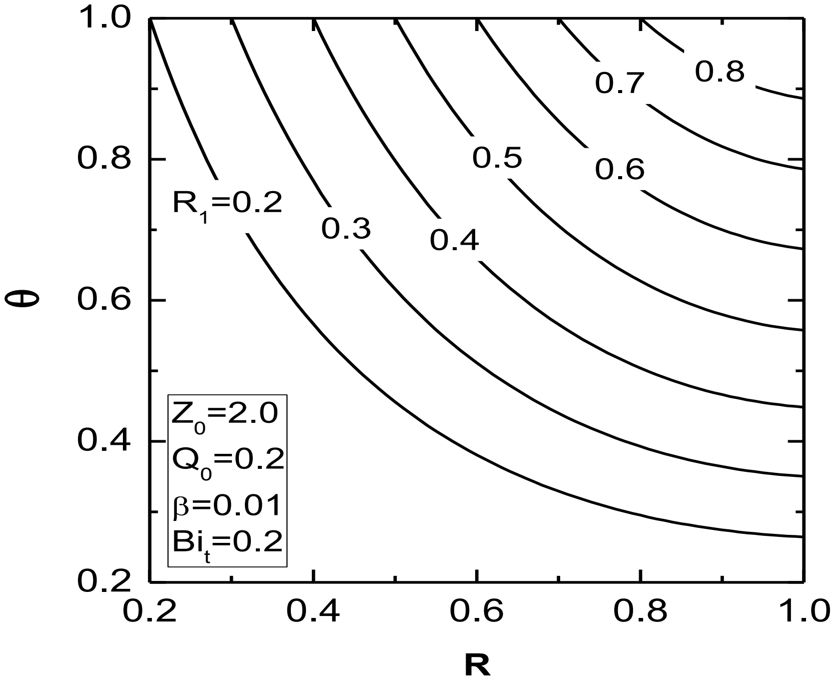


Fig. 4 Temperature distribution in an annular disc fin as a function of dimensionless inner radius

Later in the figure 5 temperature distributions in an ADF with insulated tip has been depicted. Here the internal heat generation is increasing gradually for the insulated tip fin which is depicted in Fig. 5. This case may be a practical case where Z_0 has taken 2 and the non-dimensional internal heat generation coefficient is considered $\beta = 0.01$. It is obvious that the ADF's temperature will decrease towards the tip for low value of internal heat generation. As the value of internal heat generation has been increased gradually, keeping other parameter same, it has been seen from analysis the temperature of the fin increases towards tip compare to a low value of heat generation. At a particular amount of heat generation ($Q_0 = 4$) the temperature remain constant with the base temperature along the fin. As the value of non-dimensional internal generation increases from the value of 4, the non-dimensional temperature increases from base to tip. The trend may be due to the rate of internal heat generation getting higher than the rate of heat dissipation from the fin surface.

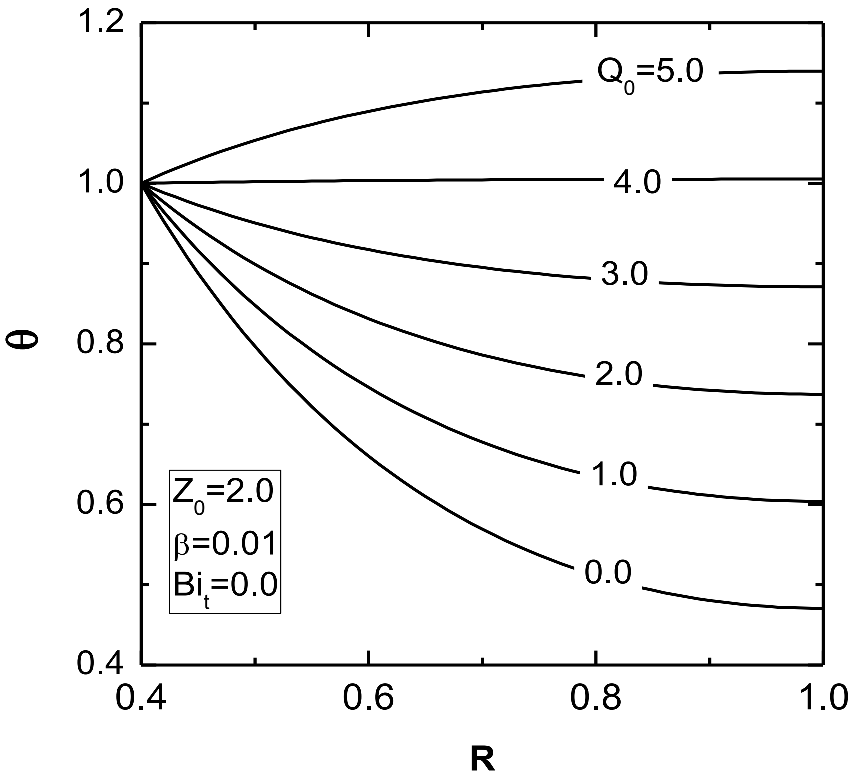
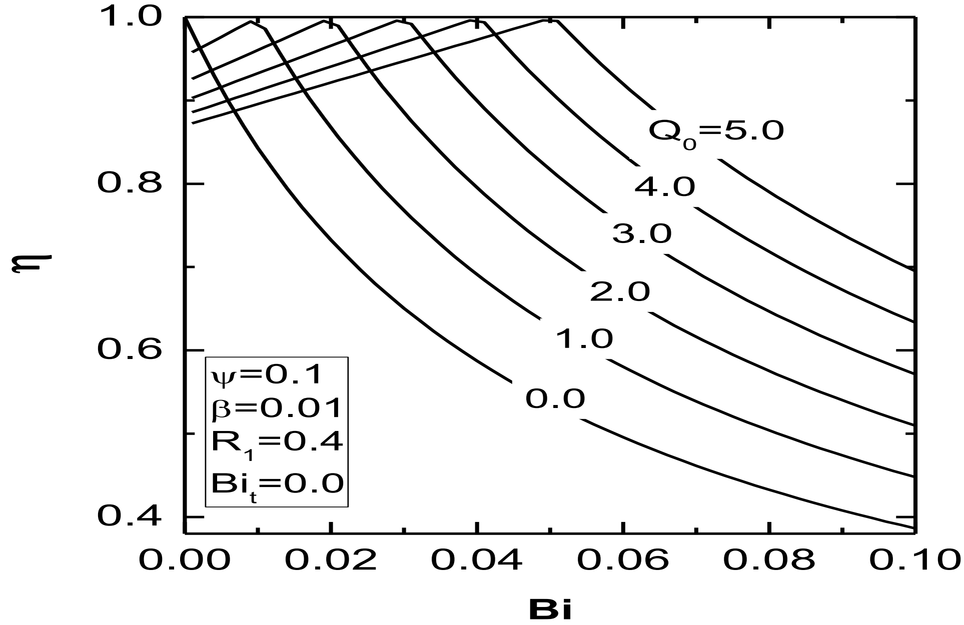


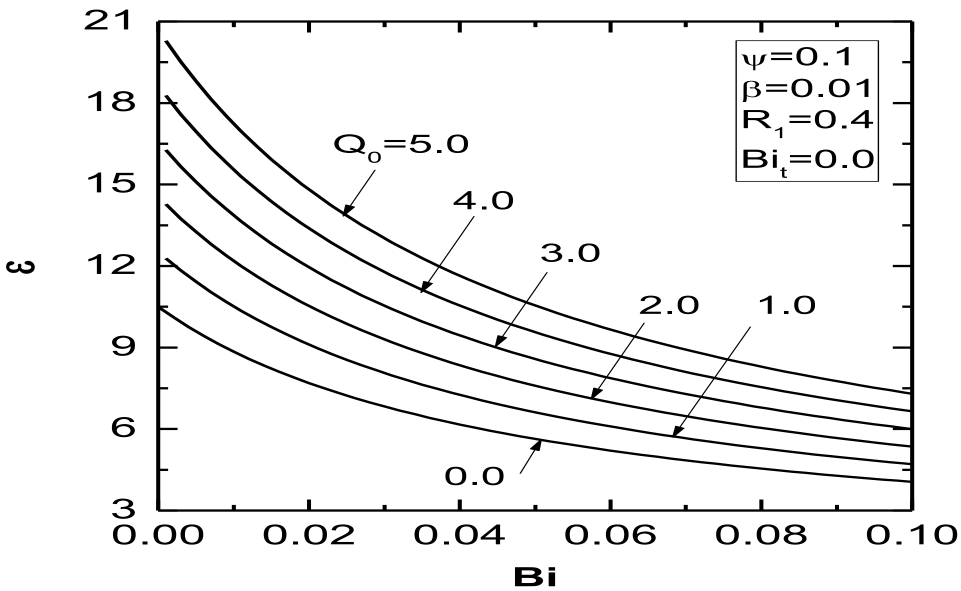
Fig. 5 Effect of internal heat generation on temperature distribution

As the temperature distribution in the ADF with both the insulated tip and convected tip has been provided earlier, analysis of efficiency and effectiveness of the fin can be understood easily. The efficiency & effectiveness of the ADF with insulated tip, have been depicted in the Fig 6. Both the efficiency & effectiveness curves have been drawn with varying Biot number considering a specific thermo-geometric parameter. At the condition of Biot no. is zero, for no internal heat generation the efficiency is maximum due to low conduction resistance for heat flow. This condition allows the highest fin efficiency with 100%. With a definite amount of heat generation, efficiency increases initially. Then the efficiency reaches at its maximum value, close to 90 percent or more for an optimum Biot no. as depicted in Fig. 6a. Efficiency drops as the Biot no. increase further from this optimum value for a certain amount of heat generation. With the increase in heat generation, initially the efficiency drops accordingly but as the Biot no. moves rightward efficiency increases. It is very good in nature for the practical application to design accordingly. Effectiveness curves have been drawn for different heat generation, keeping the other parameter same as shown in Fig. 6b. Each effectiveness curves have downwards slope with the increase of Biot no. At an ideal condition of thermo-geometric parameter ADF gives the maximum effectiveness. But at the same condition of thermo-geometric parameter of the fin effectiveness rises as the internal heat generation increases. It is shown an opposite behaviour of efficiency curve in Fig. 6b. Effectiveness increases with increase of internal heat generation and decrease as the Biot number increases.

Here both the efficiency & effectiveness has been plotted for convected tip of ADF as displayed in fig.7. Keeping other parameter same as the earlier case, the value of non-dimensional internal heat generation is taken from 0 to 5. At the no internal heat generation and zero Biot no., efficiency is found to be around 83 %. But when heat generation is considered in the ADF with Biot no is zero, the efficiency maximum but less than 100%. This id due to the temperature distribution in the ADF along the radius is more in case of heat generation condition compare to no heat generation condition. In convected tip fin the temperature distribution is shown in Fig.3. As the temperature variation in convected tip fin along the radial direction is more compare to the insulated tip fin, the efficiency drops. Unlike insulated tip fin, Biot number range for maximum efficiency gradually increase with the internal heat generation. This is an interesting observation found. Also the effectiveness curves has been show for the ADF with convected tip in fig. 7b. It is found from the analysis that the effectiveness decreases with the rightward direction of Biot no. Effectiveness value enhances for convected tip significantly for internal heat generation as depicted in fig. 7b. Unlike insulated tip ADF, effectiveness is very much higher in convected tip fin. For an example from the observation it is found that for an internal heat generation 0 to 5, at $Bi = 0.06$effectiveness is belonging from 4.5 to 9.2 approximately for insulated tip ADF, but in case of convected tip condition effectiveness is belonging from 7 to 16 for the same condition. Therefore it is suggested to analyse fin heat transfer based on the convected tip if internal heat generation is present.

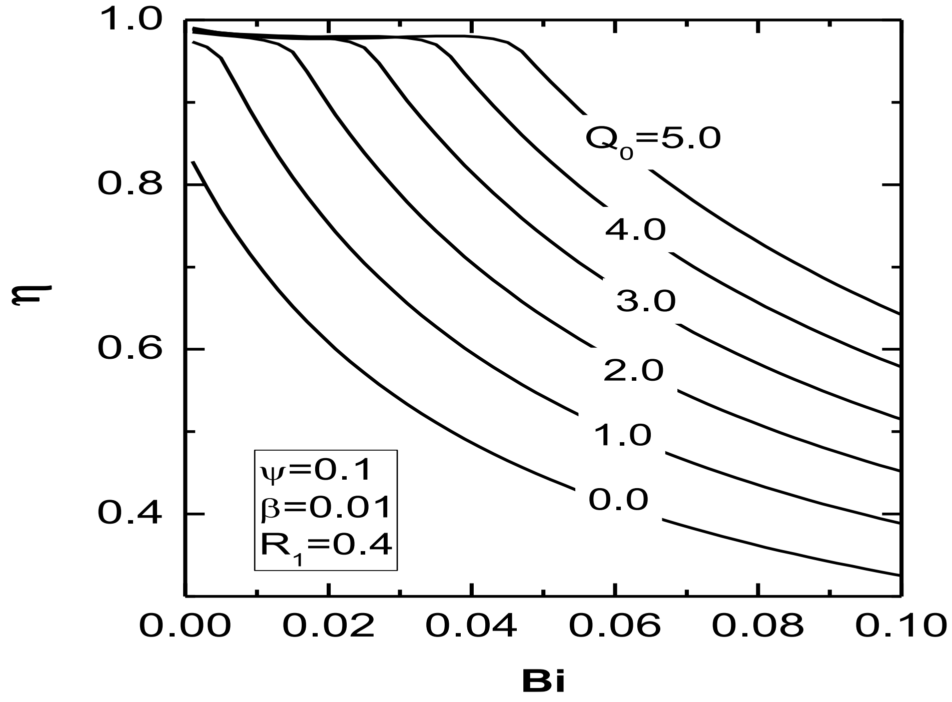


(a) Fin efficiency

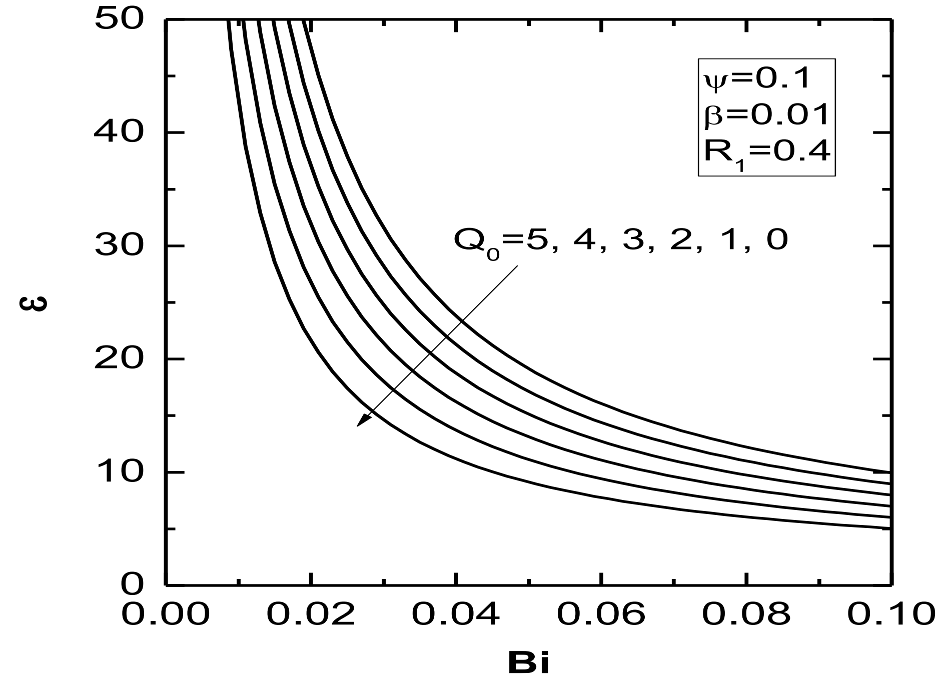


(b) Fin effectiveness

Fig. 6 Fin performance as a function of Biot number and internal heat generation for insulated tip condition



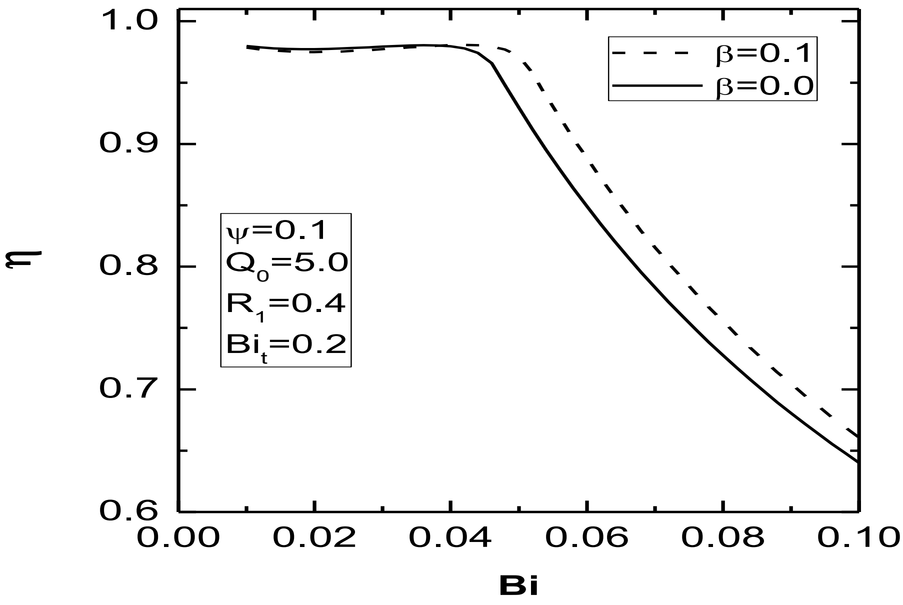
(a) Fin efficiency



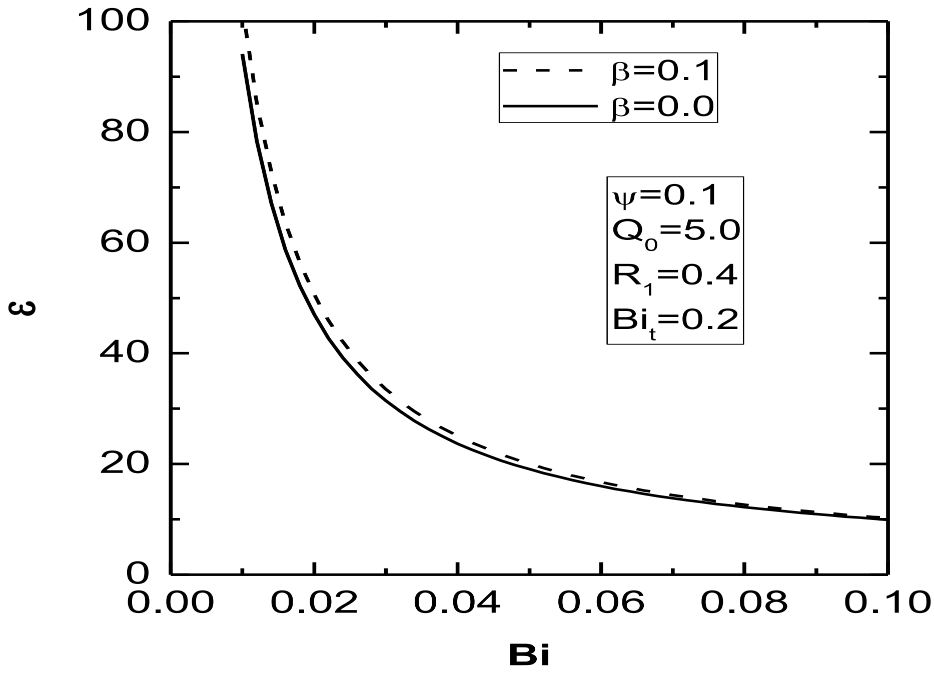
(b) Fin effectiveness

Fig. 7 Fin performance as a function of Biot number and internal heat generation for convected tip condition, $Bi_t = 0.2$

Earlier in the discussion of results of efficiency and effectiveness of the ADF with both insulated tip and convected tip has been checked out with the variation of Biot no. using different heat generation rate. Whereas ADF with convected tip give advantages instead the insulated tip. So here the depicted fig.8 has been provided the information about the convected tip of the ADF's efficiency and effectiveness with the change in Biot no. but with the effect of heat generation parameter. The non-dimensional coefficient of the internal heat generation "β" is considered 0 and 0.1 for the study. For the value$\beta = 0$, the fin has constant internal heat generation. It doesn't change upon the variation of temperature. But in some case internal heat generation does depend on temperature. Whereas $\beta = 0.1$ considered for this case to study the performance. Here in the fig.8a for a certain radius ratio of the convected tip fin with constant heat generation gives less efficiency compare to variable heat generation. The efficiency remains constant as close to 98% for the Biot no. range0.01 to 0.04 for the constant heat generation. But due to increase in internal heat generation from the ADF, when $\beta = 0.1$ considered, the efficiency increases with the Biot no. range and close to 0.05. After such value of Biot no. the efficiency decreases with the increasing value of Biot no. for the cases. So the important thing has been got from this study is the effect of internal heat generation parameter with the value of $\beta = 0.1$ gives the efficiency same as$\beta = 0$, but with a greater value Biot no, which helps to simulate the analysis over practical approach. Similarly the effectiveness has been studied for the same condition of thermo-geometric parameter for the ADF with convected tip. It is found to be decreases with the rise of Biot no. as we have seen earlier in the fig.7b. But the effect of heat generation parameter with nonzero value have improved performance compare to the value$\beta = 0$. So this advantage of improved performance for the heat generation parameter has greater influence over this study more.



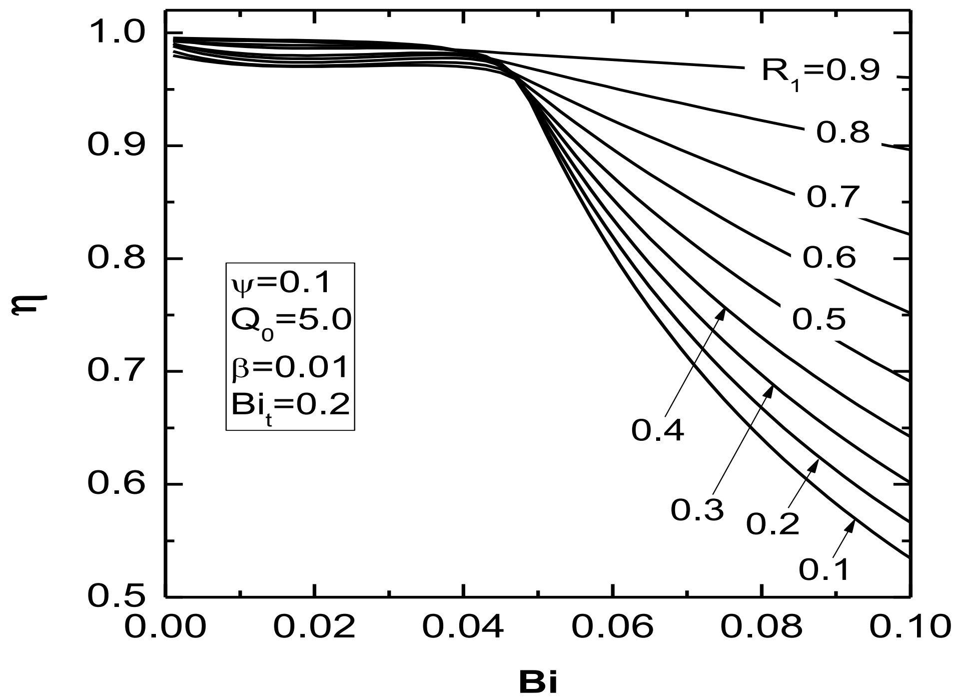
(a) Fin efficiency



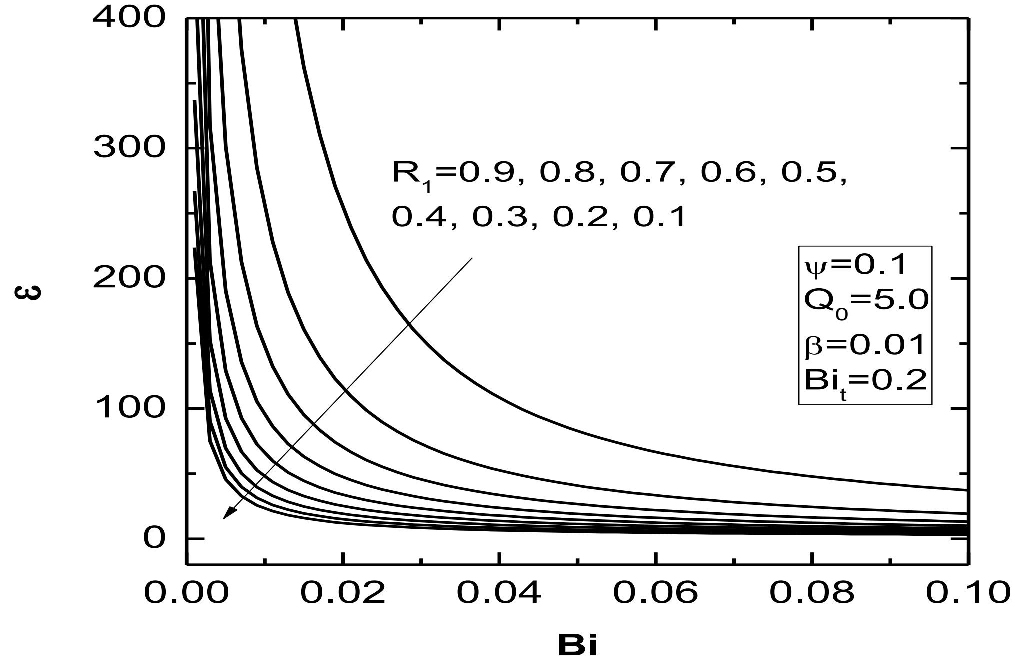
(b) Fin effectiveness

Fig. 8 Influence of variable heat generation parameter on fin performances

Earlier in the fig.7a we have seen the efficiency is increasing as the value of internal heat generation increased under a specified condition on convected tip fin. Along with that we also have discussed the effect of ADF performance under the heat generation parameter. Next we would like to study this ADF performance for another geometric parameter is radius ratio. Fig. 8a totally describe the efficiency curve under above mentioned condition, where we can found out the condition of getting the maximum efficiency for a certain radius ratio. Using those condition of Fig. 7a and the value of dimensionless internal heat generation keeping 5, we have got the curve for $R = 0.4$. In the fig. 9a efficiency is close to 95% for a range up to of Biot no. 0.04 for any radius ratio. It is very good character of observed in the study. As we know reducing the radius ratio increases surface area. Temperature inside the fin also decreases accordingly due to enhance in heat transfer rate. As the temperature variation increases with decrease in radius ratio so the efficiency drops. In case of radius ratio increase from 0.4 to 0.9 the surface area of fin reduced. So the temperature variation inside the fin is lessening towards the increasing radius ratio, which causing in enhancing the efficiency. This result is very interesting to have risen in efficiency with the low volume of material of the ADF as the Biot no. is increases too. As well the effectiveness has been found out for the ADF with convected tip fin. As effectiveness is an opposite character to efficiency, it drops with increase in radius ratio.



(a) Fin efficiency



(b) Fin effectiveness

Fig. 9 Effect of dimensionless inner radius on fin performances for an annular disc fin having internal heat generation

Chapter-5

5. Optimization Analysis

5.1 Optimization Procedure

The optimization study of any fin can be done either by maximizing the rate of heat transfer for a given fin volume or by minimizing fin volume for a given heat transfer rate but the results obtained from both the optimization schemes yield the same value. However, selection of the objective function and constraint equations depends upon the requirement of a design. In the present study, an optimization model is proposed in a generalized way with satisfying the above fact. The volume V of an ASF can be expressed in dimensionless form as

$$U = \frac{V}{2\pi r_1^3} = \left(1 - R_1^2\right)\frac{\psi}{R_1^3} \tag{26}$$

Also the equation (21) is dependent on parameter $R_1 \,\&\, \psi$. From the equations (27) and (26) expressions of heat transfer rate and volume it can be demonstrated that the optimum design of the fin assembly depends upon the geometric parameters R_1 and ψ for a design condition. The optimality criterion is then derived from Euler equations after eliminating the Lagrange multiplier

$$\left(\partial Q/\partial \psi\right)\left(\partial U/\partial R_1\right) - \left(\partial Q/\partial R_1\right)\left(\partial U/\partial \psi\right) = 0 \tag{27}$$

In order to expand the equation (27), equation (21) and (26) is replaced. It gives

$$g\left(\psi, R_1\right) = \left(\frac{\partial f}{\partial \psi}\right)\left(-\frac{2\psi}{R_1^2} - 3\left(\frac{1-R_1^2}{R_1^4}\right)\right) - \left(\frac{\partial f}{\partial R_1}\right)\left(\frac{1-R_1^2}{R_1^3}\right) = 0 \tag{28}$$

In order to determine the optimum parameters, Eq. (28) can be solved along with the constraint either heat transfer rate (Eq. (21)) or fin volume (Eq. (26)) depending upon the requirement ofa design. Thus the constraint equation can be formed by combiningEqs. (21) and (26) as follows:

$$O\left(\psi, R_1\right) = 0 = \begin{cases} f\left(\psi, R_1\right) - Q \\ \left(1 - R_1^2\right)\dfrac{\psi}{R_1^3} - U \end{cases} \tag{29}$$

A numerical scheme, namely Newton-Raphson iterative method is adopted for solving eqs. (28) & (29). For finding the multiple roots by Newton-Raphson method, it is worthy to mention that the initial value for the roots have been taken cautiously so that the convergence criteria for each iteration has been specified (29).For the present problem, a brief outline of the generalized Newton–Raphson method and the convergence criteria for each step of iterations are described in the following paragraph.The optimum values of design variables such as R_1 and ψ can be approximated from the Newton–Raphson formula by using justlyprevious iterative or initial guess values of these variables.

$$\begin{bmatrix} R_{1j+1} \\ \psi_{j+1} \end{bmatrix} = \begin{bmatrix} R_{1j} \\ \psi_j \end{bmatrix} - \left[J\left(R_1, \psi\right)\right]^{-1} \begin{bmatrix} g\left(R_{1j}, \psi_j\right) \\ O\left(R_{1j}, \psi_j\right) \end{bmatrix} \tag{30}$$

where J defines jacobian matrix which can be expressed as

$$\left[J\left(R_1,\psi\right)\right]=\begin{bmatrix}\left(\partial f/\partial R_1\right)_j & \left(\partial f/\partial \psi_1\right)_j \\ \left(\partial O/\partial R_1\right)_j & \left(\partial O/\partial \psi_1\right)_j\end{bmatrix} \tag{31}$$

The subscript ‘‘j” denotes the value of jth iteration. The convergence criteria given below at each step of iteration must be satisfied.

$$\text{Max}\left\{\Delta_1,\Delta_2\right\}<1 \tag{32}$$

where the expression of Δ_1,Δ_2 can be written as

$$\begin{bmatrix}\Delta_1 \\ \Delta_2\end{bmatrix}=\begin{bmatrix}\left|\partial\Gamma_1/\partial R_1\right|_j+\left|\partial\Gamma_2/\partial R_1\right|_j \\ \left|\partial\Gamma_1/\partial\psi\right|_j+\left|\partial\Gamma_2/\partial\psi\right|_j\end{bmatrix} \tag{33}$$

$$\begin{bmatrix}\Gamma_1 \\ \Gamma_2\end{bmatrix}=\begin{bmatrix}R_1-Det\Omega_1/DetJ \\ \psi-Det\Omega_2/DetJ\end{bmatrix} \tag{34}$$

$$\left[\Omega_1\right]=\begin{bmatrix}g & \partial g/\partial\psi \\ O & \partial O/\partial\psi\end{bmatrix} \tag{35}$$

And

$$\left[\Omega_2\right]=\begin{bmatrix}\partial g/\partial R_1 & g \\ \partial O/\partial R_1 & O\end{bmatrix} \tag{36}$$

The above procedure are repeated till the geometric roots R_1 and ψ are obtained to a desire accuracy.

5.2 Results and Discussion

Earlier we have discussed the results of the fin performance based on various thermo-geometric parameter. It shows that ADF with convected tip has got more priority to perform over insulated tip. So here we have evaluated the optimum dimension of the ADF with convected tip for the maximum thermal performance. Below fig.10 provided the results for maximum amount of heat transfer rate for a certain volume of fin material. For a particular amount of heat generation in the convected tip within the ADF gives the curves for maximum heat transfer rate with the varying radius ratio. Different curves have been plot for different non-dimensional volume of fin. These curve having a low value of heat transfer rate but as the radius ratio increasing rightwards the heat transfer rate has become maximum at a certain radius ratio. It is decreasing after the pick value of the curve. Each curve has a pick point which has been shown by drawing a locus passing through these pick points. As the dimensionless volume of the fin is decreasing, the pick point is forwarding right. So does the value of radius ratio increasing means the fin surface areas reduced too.

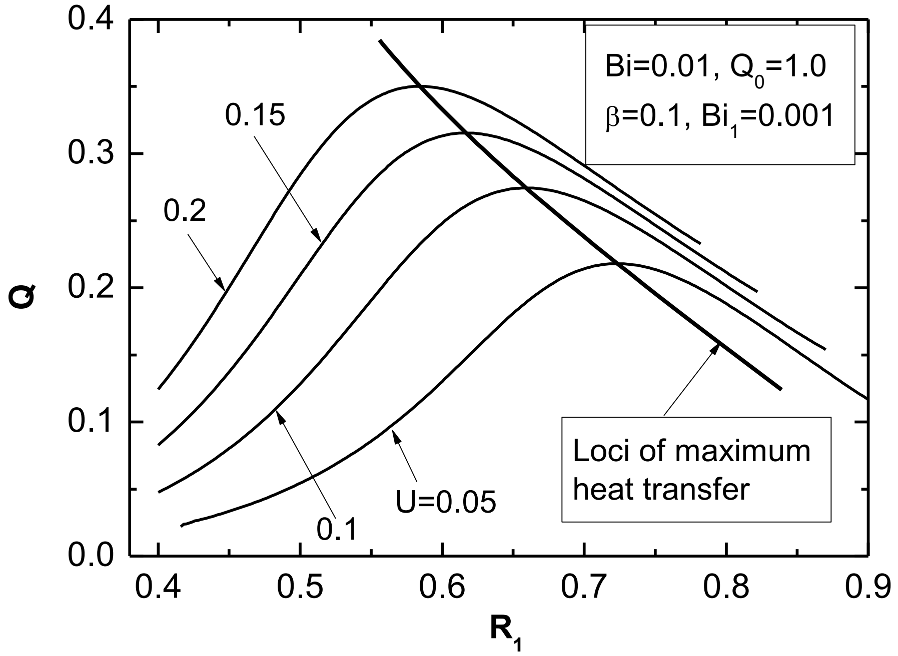


Fig. 10 Maximum heat transfer rate in an annular fin as a function of fin volume

Here the depicted fig. 11, we have optimize the Biot no. for maximum heat transfer rate for constant volume of ADF with convected tip. For a constant value of heat generation coefficient and tip Biot no. this analysis has been carried out considering the dimensionless heat generation rate at surrounding temperature is 1.0. This graph has been shown that maximum heat transfer rate plotted against the radius ratio for different Biot no. The curves are similar to the previous fig. 10. Here also the pick points of each curve have located by a locus, named as loci of maximum heat transfer. It can be noticed for each curve that the maximum heat transfer rate is decreases as the Biot no increases. For a value of $Bi = 0.003$ gives the highest amount of heat transfer rate close to the value of 0.35.

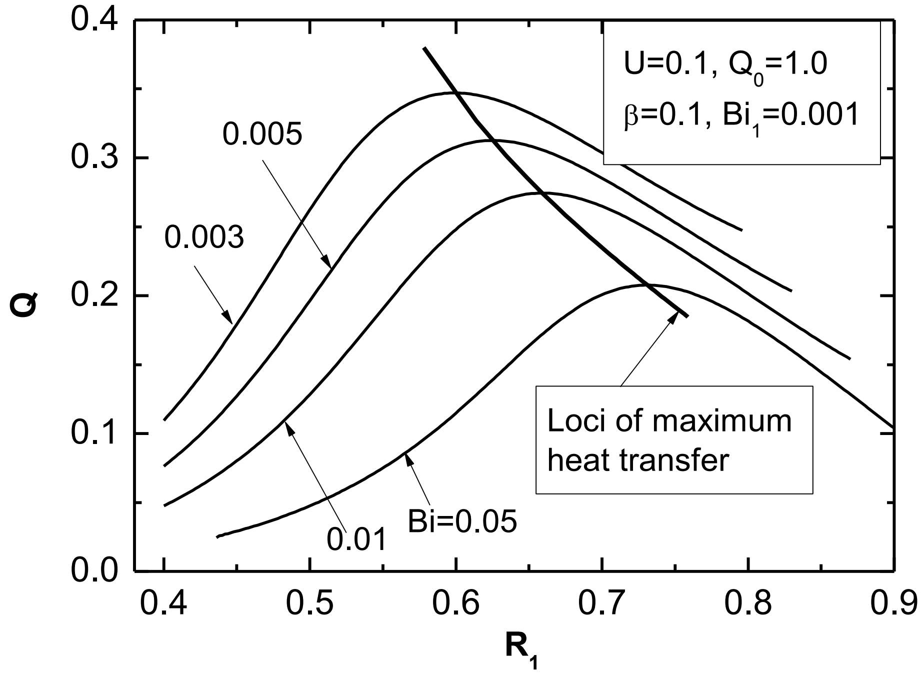


Fig. 11 Maximum heat transfer rate as a function of Bi for a constraint fin volume

The below fig. 12 has shown to optimize the ADF with convected tip for maximum heat transfer like above fig. shown. But this fig. 12 has described the analysed curve like valley having a pick, where the ADF dimensionless volume, Biot no. and tip Biot no. are considered constant. Each curve have been plotted for different value of dimensionless heat generation parameter with the consideration of heat generation coefficient value of 0.1. Optimum value of radius ratio can be evaluated form the diagram by drawing a vertical line to the radius ratio axis from the pick points of the curve. These entire pick points are joined by a locus as defined loci of maximum heat transfer. We can see the figure gives a curve with no internal heat generation gives the very less amount of heat transfer rate. But as the heat generation is having inside the fin, the maximum heat transfer rate increasing like exponentially. So the change of maximum heat transfer rate is so more with the change in radius ratio gives when the internal heat generation present. As the value of dimensionless internal heat generation is increasing, it is giving the maximum amount of heat transfer rate. Drawing the vertical line to the radius ratio axis from the pick point from each curve, gives the optimum value of radius ratio. So the study of the effect of internal heat generation gives the very likely optimum result.

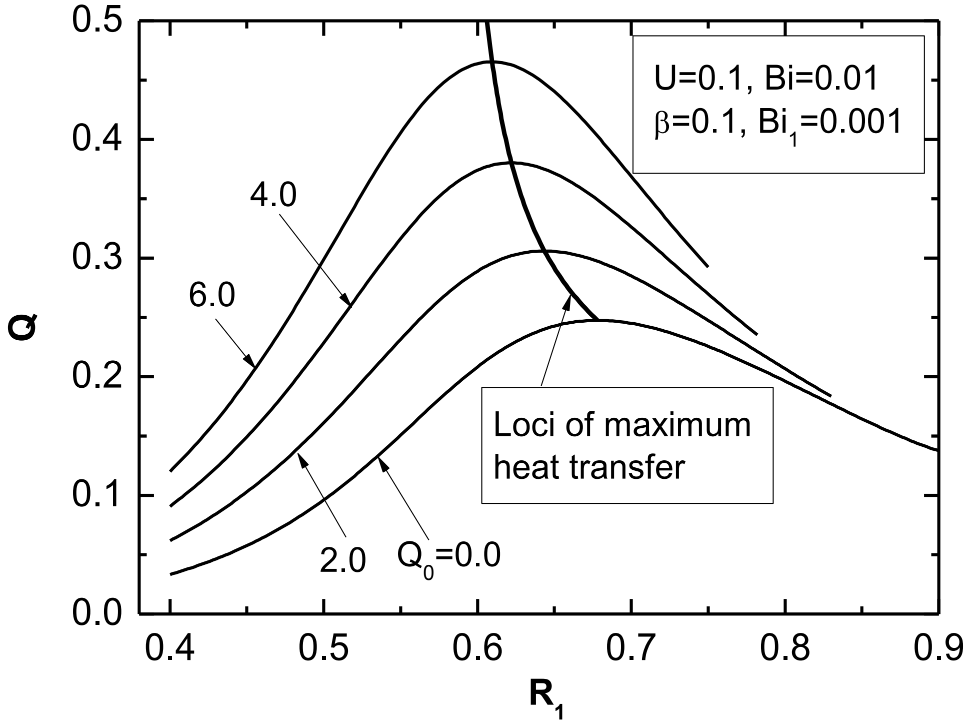


Fig. 12 Maximum heat transfer rate as a function of heat generation parameter Q_0 for a constraint fin volume under a design condition

Above all those figures give the optimum dimension of the geometry of ADF based on the various thermo-geometric parameters. The figure 12 described the optimum dimension based on the effect of internal heat generation parameter. No we will have the optimization based on the effect of variable heat generation coefficient as described below fig. 13. Earlier in fig.8, we have the results with improved effect on the thermal performance of the ADF. Both the effectiveness and efficiency increases with the nonzero value of the coefficient of internal heat generation. Here this analysis for a convected tip ADF with constant volume and constant Biot no. has been considered. A constant heat generation parameter $Q_0$with the variable coefficient of heat generation gives the below curves with a pick value for each coefficient. We can see the negative value of the coefficient having the curve of heat transfer rate below the greater value of the coefficient. All this pick points are shown by plotting a locus through it. The maximum value of heat transfer rate increasing very fast with the increase of the coefficient of the heat generation.

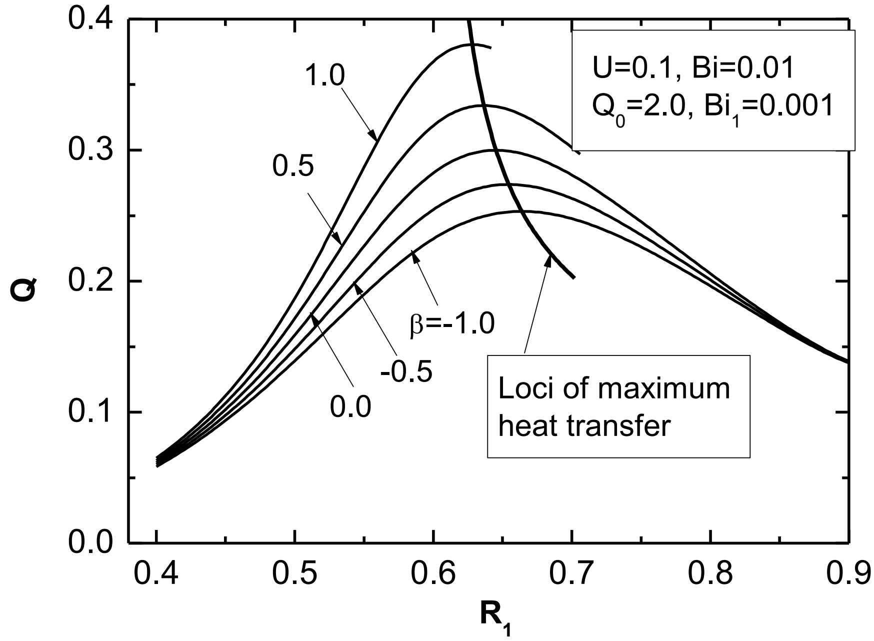


Fig. 13 Influence of variable heat generation parameter β on maximum heat transfer rate for a constraint fin volume

We have figure out the maximum possible heat transfer can take place through the fin for an optimum dimension of the ADF through various effective parameters. The last one we are considering for maximum heat transfer rate is tip Biot no. Here the tip loss can be changeable due to the variable convective heat transfer coefficient. Constant volume of the fin with the constant heat generation parameter and constant coefficient of heat generation parameter has been considered to plotting the curve. Tip loss is more when theBi_thaving higher value. So here in the fig. 14 shown the curves for increasing order of Bi_t having the heat transfer rate is more at any R_1except below the value$R_1 = 0.5$. Maximum value of the heat transfer rate has been pointed out by drawing a locus through the pick points of the curve. So a tip loss on the optimization has great influence as the value of the Bi_t increasing towards upward direction gives a flatting curve. The maximum heat transfer rate remain almost same for higher value of Bi_t as the curve shown for$Bi_t = 0.008$. So the condition helps to choose a

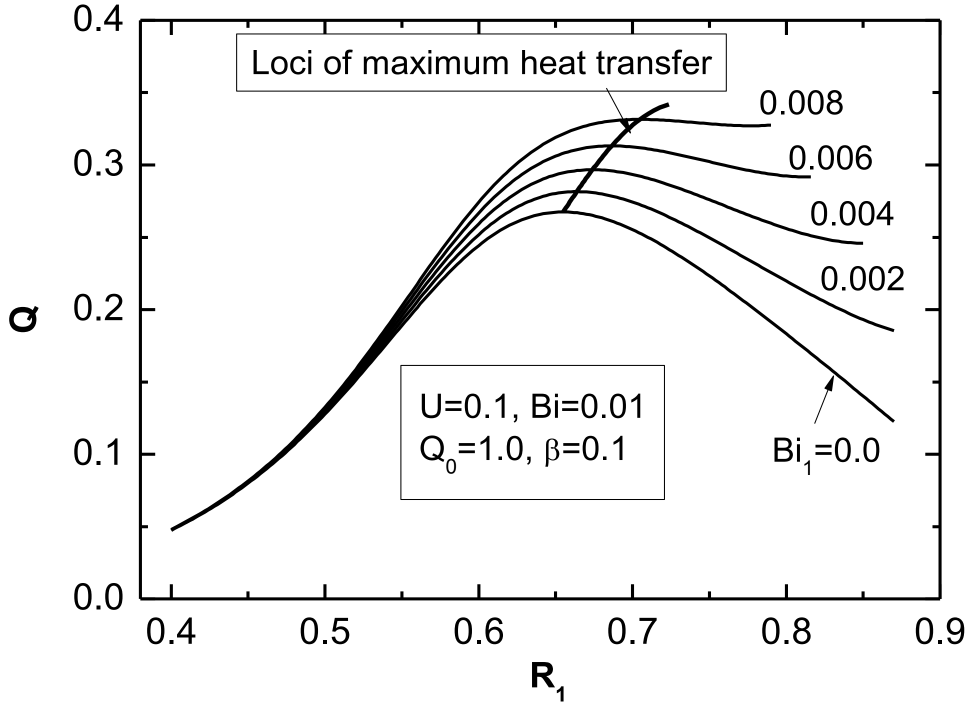


Fig. 14 Enhancement of maximum heat transfer rate with tip heat loss parameter

Chapter-6

6. Conclusion

An analytical methodology for the thermal analysis of the annular disk fin (ADF) with constant thickness has been developed. The approach adopted here for the thermal analysis is a generalized one. It is assumed a case of variable internal heat generation witha constant base temperature. Using the Frobenius expanding series, the generalized governing differential equation has been solved. The fin temperature distribution has been evaluated analytically. The thermal performance has been studied over radius ratio, Biot number, internal heat generation parameters. The internal heat generation has shown greater influence on the fin performance. For a particular Biot no. performances has maximum value for a convected tip fin, instead an insulated one. The present analysis has demonstrated an optimum design condition for the maximum efficiency of a fin subject to internal heat generation, both fin efficiency and fin effectiveness have been increased substantially which may be favourable condition in a practical design. So the interesting observation on convected tip ADF has been studied further for optimization.

Later on the results of ADF’s dimension has been optimized for maximum heat transfer rate. These study on optimization has been carried out using elimination of Euler lagrangian multiplier. Optimum results has been obtained for various thermo-geometric parameters. These are fin volume, Biot no., heat generation, tip condition, heat generation parameter, etc. In practical case, the value provided on the study of optimization of ADF, is very useful for design on the basis of internal heat generation.

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