Ref. No. EX/M.SC./PHY/E/I/103/23/2017

M.SC. 1ST YEAR 1ST SEMESTER (EVE.) 2017

Subject: QUANTUM MECHANICS I **Time: 2 hours** **Full Marks: 40**

Answer any four questions.

1. a) Write down the Schrodinger equation in three dimensions. b) Write down the complete set of commutating observables for hydrogen atom problem and complete set of quantum numbers. c) Which corrections contribute to the fine structure of hydrogen spectrum? d) Calculate the lowest order relativistic correction to the Hamiltonian.

2+3+2+3=10

2. Define projector P_Ψ onto a ket $|\Psi>$. Then, show that $P_\Psi^2 = P_\Psi$. Prove that P_Ψ is Hermitian operator. Find out the matrix representation of $|\Psi'>= A\,|\Psi>$ in a basis set $\{|u_i>\}$. 2+3+5=10

3. Prove that a) the eigenvalues of Hermitian operators are real, b) two eigenvectors of a Hermitian operator corresponding to two different eigenvalues are orthogonal, and c) if two operators A and B commute, one can construct an orthonormal basis of state space with eigenvectors common to A and B. 3+3+4=10

4. a) Consider an arbitrary ket $|\Psi>$ corresponding to the wave function $\Psi(r)$. Then, show that the value of $\Psi(r_0)$ at r_0 is the component of ket $|\Psi>$ on the basis vector $|r_0>$ of $\{|r_0>\}$ representation. b) Define Clebsch-Gordon coefficient for addition of two angular momenta.

5+5=10

5. a) Find the transformation relation of the matrix elements of an operator A from the basis set $\{|u_i>\}$ to $\{|t_i>\}$. b) Write down all postulates in quantum Mechanics.

5+5=10

6. Two operators are constructed for a one dimensional harmonic oscillator: $a = \frac{1}{\sqrt{2}}(X + iP)$, and $a^\dagger = \frac{1}{\sqrt{2}}(X - iP)$, where $X = \sqrt{\frac{m\omega}{\hbar}}x$ and $P = \frac{1}{\sqrt{m\hbar\omega}}p$.

   a) Express the Hamiltonian H in terms of these two operators.

   b) $N = aa^\dagger$ satisfies the eigenvalue equation $N|\phi_v> = v|\phi_v>$. Then, i) prove that $v \geq 0$ and ii) if $v = 0$, then prove that $a|\phi_{v=0}>= 0$.

   c) Show that a and $a^\dagger$ act as ladder operator. 3+(2+2)+3=10

7. a) Find the first order corrections to energy and the wave function due to the time independent perturbation H_I to the unperturbed HamiltonianH_0.

   b) Prove the Schwarz inequality $|<\phi_1|\phi_2>|^2 \leq <\phi_1|\phi_1><\phi_2|\phi_2>$

6+4=10