JADAVPUR UNIVERSITY

MASTER OF SCIENCE EXAMINATION, 2017.
(1st Year, 1st Semester, **EVENING**)

Subject: PHYSICS
Paper: MATHEMATICAL METHODS - I
PHY / TG / 102
(Ref. No. *EX/M.SC./PHY/E/I/102/23/2017*)

Time: Two Hours
Full Marks: 40

Answer any FOUR questions

1. Integrate the following functions along the specified contours.

   (a) $f(z) = \frac{1-\sin z}{\cos z}$ along the contour $|z - i| = \sqrt{5}$

   (b) $f(z) = \frac{\sin \pi z}{(z^2+1)^2}$ along the contour $4x^2 + y^2 = 4$

   Marks: 5 + 5 = 10

2. (a) Expand the function $f(z) = [z(z^2 - 3z + 2)]^{-1}$ about $z = 0$ over the following three domains:

   (i) $0 < |z| < 1$, (ii) $1 < |z| < 2$, (iii) $|z| > 2$

   (b) Let C be a contour on which the function $f(z)$ is analytic but inside the contour there is a pole at the point z_0. Show by writing a Laurent expansion about z_0 that $\int_C f(z)dz = 2\pi i R$, where R is the residue of $f(z)$ at z_0.

   Marks: (2 × 3) + 4 = 10

3. (a) Suppose that on a semi-circular contour of radius R drawn along the lower half of the complex plain, the function $g(z) \to 0$ uniformly as $R \to \infty$. Show that the integral of $e^{i\lambda z}g(z)$ goes to zero in the limit of $R \to \infty$ if $\lambda < 0$.

   (b) Evaluate the integral $\int_0^\infty \frac{x \sin ax}{x^2+b^2}dx$, where a and b are real.

   Marks: 5 + 5 = 10

4. (a) Evaluate the integral $\int_{-\infty}^{\infty} \frac{e^{ax}}{e^x+1}$ (with $0 < a < 1$) using (i) a semicircular contour and (ii) a rectangular contour.

   (b) Integrate the function $\frac{z^2+\pi^2}{1+e^{-z}}$ over the contour $|z| = 2$.

   Marks: (4 + 4) + 2 = 10

5. (a) For two arbitrary vectors $|\alpha\rangle$ and $|\beta\rangle$, show that $\langle\alpha|\alpha\rangle.\langle\beta|\beta\rangle \geq |\langle\alpha|\beta\rangle|^2$ by forming an arbitrary linear combination $|\gamma\rangle = |\alpha\rangle + \lambda|\beta\rangle$, where λ is a complex number and then minimizing the norm of $|\gamma\rangle$ with respect to λ and its complex conjugate.

   (b) Considering two Hermitean operators $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$, show that the minimum value of the product of the squares of their standard deviations is given by $(-1/4)$ times the square of the expectation value of the commutator of $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$.

   Marks: 4 + 6 = 10