Ref. No.: Ex/M.Sc./P/II/112/20/2017

MASTER OF SCIENCE EXAMINATION 2017

(2nd Year, 1st Semester)

PHYSICS

SOLID STATE PHYSICS AND X-RAY

PHY/TG/112

Time: Two hours **Full marks: 40**

Answer any FOUR questions.

1. (a) State and prove the Bloch's theorem. [2+2=4]

(b) Show that the Schrödinger equation of a single electron, $H(\vec{r})\,\Psi(\vec{r}) = E\,\Psi(\vec{r})$, where the Hamiltonian, $H(\vec{r}) = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})$ is invariant under the lattice translation of any Bravais vector $\vec{R}$, *i. e.*, $H(\vec{r}+\vec{R}) = H(\vec{r})$, can be expressed in reciprocal space as

$$\left(\frac{\hbar^2}{2m}\left(\vec{k}-\vec{G}\right)^2 - E\right) c_{\vec{k}-\vec{G}} + \sum_{\vec{G}'} V_{\vec{G}'-\vec{G}}\; c_{\vec{k}-\vec{G}'} = 0.$$

Symbols have their usual meaning. [6]

2. (a) Derive the expression of energy dispersion relation for the three-dimensional crystal in tight-binding approximation:

$$E(\vec{k}) = E_A - \alpha - 4\gamma \sum_m e^{i\vec{k}\cdot(\vec{R}_j - \vec{R}_m)},$$

where $\vec{R}_m$ are the nearest neighbours of $\vec{R}_j$. Other symbols have their usual meaning. [5]

(b) Derive the following expressions of dispersion relations in tight-binding approximation for fcc crystal, *i. e.*,

$$E_k = E_A - \alpha - 4\gamma\left[\cos\frac{k_x a}{2}\cos\frac{k_y a}{2} + \cos\frac{k_y a}{2}\cos\frac{k_z a}{2} + \cos\frac{k_z a}{2}\cos\frac{k_x a}{2}\right].$$

[3]

(c) Show that E_k can be approximated as $E_k \approx E_{\text{Min}} + \frac{\hbar^2 k^2}{2m^*}$, when k is very small. Find the expression of m^*. [1+1=2]

3. (a) By using the Sommerfeld expansion,

$$\int_0^\infty f(E)\,\frac{\partial F}{\partial E}\,dE = F(E_F) + \frac{\pi^2}{6}(k_B T)^2\left(\frac{\partial^2 F}{\partial E^2}\right)_{E_F},$$

show that the energy of Fermi level at room temperature T can be expressed as

$$E_F(T) \approx E_F(0)\left[1 - \frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2\right].$$

Symbols have their usual meaning. [4]

(b) Derive the expression of density of states for **two-dimensional** free electron system. [2]

(c) Now derive the expression of specific heat at low temperatures, $C_V(T)$ for the **two-dimensional** free electron system. And show that $C_V(T) \propto T$ at low temperatures. [4]

4. (a) Consider a thermodynamic system of fixed volume under the magnetic field $\vec{H}$ at finite temperature T. Show that in equilibrium, the entropy, S and the magnetic induction, $\vec{B}$ of this system can be obtained by using the expressions,

$$S = -\left(\frac{\partial G}{\partial T}\right)_{\vec{H}}, \quad \vec{B} = -\left(\frac{\partial G}{\partial \vec{H}}\right)_T,$$

respectively, where G is the Gibbs potential. [2]

(b) Show that the Gibbs potential for the superconducting state, G_s is related to that of the normal state, G_n by the equation,

$$G_s(T, \vec{H}) = G_n(T, \vec{H}) + \frac{1}{2}\mu_0 \left(H^2 - H_c^2(T)\right).$$

[3]

(c) Explain that the superconducting state to normal metallic state transition is second order at the points $T = 0$ and $T = T_c$, and otherwise first order, since the critical field of transition is

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right].$$

[2]

(d) In the phenomenological description of superconductivity, derive the first and the second London's equations. [2]

(e) By using the London's equations show that the canonical momentum vanishes in the superconducting state, *i. e.*, $\vec{p} + e\vec{A} = 0$. Symbols have their usual meaning. [1]

5. (a) Suppose (hkl) are the Miller Indices of a set of planes with respect to $\vec{a}$, $\vec{b}$, $\vec{c}$ the initial unit cell. Find the Miller indices $(h'k'l')$ when referred to another set of axes $\vec{a'}\vec{b'}\vec{c'}$ of the transformed unit cell.

(b) Find the matrix of transformation for the indices between the conventional hexagonal unit cell and the alternative orthorhombic unit cell. Draw the necessary diagram.

(c) A unit cell of diamond structure belongs to cubic crystal system and contain eight atoms of the same type and their positions are as follows

$(0\,0\,0)\ (\frac{1}{2}\frac{1}{2}0)\ (\frac{1}{2}0\frac{1}{2})\ (0\,\frac{1}{2}\frac{1}{2})\ (\frac{1}{4}\frac{1}{4}0)\ (\frac{3}{4}\frac{3}{4}\frac{1}{4})\ (\frac{3}{4}\frac{1}{4}\frac{3}{4})\ (\frac{1}{4}\frac{3}{4}\frac{3}{4})$

Compute the structure factor F_{hkl} and also $|F|^2$

(4 + 3 + 3)

6. (a) Write down Eulers' equations for finding angles between two rotational axes to generate the third rotational symmetry axes in a crystal. Hence find the feasibility of existence of the following combinations and draw their stereogram

(2 2 2); (3 3 3); (4 4 4); (4 2 2)

(b) Draw the stereogram of the following point groups showing intermediate projections in separate diagrams

$\frac{3}{m}$; $\bar{4}$; 2 m m; $\bar{6}$

(c) Defining atomic form factor for X-ray scattering obtain an expression for it.

(d) Explain screw axis and glide plane in a crystal with diagrams.

(3 + 2 + 3 + 2)