MASTER OF SCIENCE EXAMINATION, 2017.
(2nd Year. 1st Semester)

Subject: PHYSICS
Paper: QUANTUM FIELD THEORY
(Ref. No. *Ex/M.SC./P/II/203/20/2017*)

Time: Two Hours
Full Marks: 40

GROUP - A

Answer any ONE question

1. (a) In a fermionic many-particle system given by $|10011001\ldots\rangle$. what will be the result of creating a particle in the seventh place ? (Write the proper prefactor for the new ket that you obtain).

   (b) Show that for a Schrodinger field, the field number operator and the field Hamiltonian operator commute with each other, irrespective of the particle statistics.

   (c) A many-particle system is governed by a two-body interaction potential $U(x-x')$. and is given by the position-space Hamiltonian operator

$$\hat{H} = \int d^3x \hat{\Psi}^\dagger(x,t) H_0 \hat{\Psi}(x,t) + \frac{1}{2}\int d^3x \int d^3x' \hat{\Psi}^\dagger(x,t)\hat{\Psi}^\dagger(x',t)U(x-x')\hat{\Psi}(x,t)\hat{\Psi}(x',t).$$

   [here H_0 is the usual single particle Schrodinger Hamiltonian]. Show that the operation of this second quantized Hamiltonian operator on a many-particle Fock state leads to a many-particle Schrodinger equation in the first quantized form.

Marks: 3 + 7 + 10 = 20

2. (a) Show that the generator of an infinitesimal unitary transformation is a hermitian operator.

   (b) Show that in an isolated system, an operator which does not change under a unitary transformation, commutes with the generator of that unitary transformation.

   (c) A second-quantized Hamiltonian in momentum space is written as

$$\hat{H} = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m}\hat{a}^\dagger_{\vec{k}}\hat{a}_{\vec{k}} + \frac{1}{2}\sum_{\vec{k}_1}\sum_{\vec{k}_2}\sum_{\vec{q}} \tilde{U}(\vec{q})\hat{a}^\dagger_{\vec{k}_1+\vec{q}}\hat{a}^\dagger_{\vec{k}_2-\vec{q}}\hat{a}_{\vec{k}_1}\hat{a}_{\vec{k}_2}.$$

   Interpret the terms physically.

   (d) Now consider a condensate system with inter-particle *contact* interaction of strength g given by $U(x) = g\delta(x)$. The system has a *macroscopic occupation in its ground state*. Neglecting nonlinear powers of the number of particles in the excited states, show that for a N-particle system confined in a volume V with particle-density $\rho = \frac{N}{V}$,

$$\hat{H} = \frac{1}{2}g\rho^2 V + \frac{1}{2}\sum_{\vec{k}\neq 0}\left[(E_k + g\rho)(\hat{a}^\dagger_{\vec{k}}\hat{a}_{\vec{k}} + \hat{a}^\dagger_{-\vec{k}}\hat{a}_{-\vec{k}}) + g\rho(\hat{a}^\dagger_{\vec{k}}\hat{a}^\dagger_{-\vec{k}} + \hat{a}_{\vec{k}}\hat{a}_{-\vec{k}})\right]$$

   where $E_k = \frac{\hbar^2 k^2}{2m}\hat{a}^\dagger_{\vec{k}}\hat{a}_{\vec{k}}$.

Marks: 3 + 3 + 2 + 12 = 20

Group - B

Answer any four questions. **4x5=20**

1. The Lagrangian for an electromagnetic field with a current j^{μ} is given by $\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j_{\mu} A^{\mu}$, where, $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$. Treating A^{μ} as the fields find the Euler-Lagrange's equations.

2. The Hamiltonian for a free scalar field is given by $H = \frac{1}{2} \int d^3 p \, E_p [\, a^{\dagger}(p)\, a(p) + a(p) a^{\dagger}(p)]$. Write down the normal ordered Hamiltonian $:H:$. Then, show that $< 0| : H: |0 > = 0$. Why normal ordering is required?

3. What do you mean by time ordering? Using the step function replace the time order operator and expand the expression for propagator $\Delta_F (x - x') = < 0| \mathcal{T} [\, \phi(x)\phi(x')]|0 >$. Then, explain that it represents the propagation of a scalar particle from t to t' if $t' < t$ and vice versa. What is the expression of this propagator in momentum space?

4. The Yukawa interaction $\mathcal{L}_{int} = h \, \bar{\psi}\psi \, \phi$ describes the decay of a scalar B to electron and positron. Write the linear term in the S-matrix and then expand it in terms of creation and annihilation parts of each field and draw the Feynman diagram of the decay.

5. Write down the canonical quantization relation for a scalar field. Then write down all commutation relations for annihilation operator $a(p)$ and creation operator $a^{\dagger}(p)$ obtained from that relation.

6. The number operator for scalar field in Fock space is defined as $\mathcal{N} = \int d^3 p \; a^{\dagger}(p)\, a(p)$, show that $[\, \mathcal{N},\, a\,(p)] = a^{\dagger}(p)$ and $[\mathcal{N},\, a(p)] = -a(p)$. The Lagrangian for a complex scalar field is given by $\mathcal{L} = (\partial^{\mu}\phi^{\dagger})(\partial_{\mu}\phi) - m^2 \; \phi^{\dagger}\phi$. Find out the Noether's current for the transformations $\phi \rightarrow e^{iq\theta}\phi$ and $\phi^{\dagger} \rightarrow e^{-iq\theta}\phi^{\dagger}$.

7. What is the mass dimension of the action? Determine the mass dimension of Lagrangian density $\mathcal{L}$. Then find the mass dimension of the field ϕ from the Lagrangian $\mathcal{L} = (\partial^{\mu}\phi \;)(\partial_{\mu}\phi) - m^2 \; \phi^2$. Find the equation motion for ϕ.