Ex./M.Sc./P/II/113/20/2017

MASTER OF SCIENCE EXAMINATION, 2017.
(2nd Year, 1st Semester)

Subject: PHYSICS
Paper: QUANTUM MECHANICS - III
PHY / TG / 113
(Ref. No. *EX/M.SC./P/II/113/20/2017*)

Time: Two Hours
Full Marks: 40

Answer any FOUR questions

1. For a hard sphere scatterer with radius r_0, use the formula $\tan\delta_l = \frac{j_l(kr_0)}{\eta_l(kr_0)}$ [where δ_l is the phase shift of the l-th partial wave and j_l and η_l are spherical Bessel and spherical Neumann functions respectively] along with the proper asymptotic forms of the functions to show that the total scattering cross section for low energy scattering is twice that for high energy.

Marks: 10

2. Consider a Coulomb scatterer of charge Ze placed at the origin. A charge q is incident on it from the negative z-direction. Here $(Ze, q) > 0$. In parabolic coordinates $\zeta = r - z$ and $\eta = r + z$, the Schrodinger equation for this problem can be written as

$$\left[\frac{\partial}{\partial\zeta}\left(\zeta\frac{\partial}{\partial\zeta}\right) + \frac{\partial}{\partial\eta}\left(\eta\frac{\partial}{\partial\eta}\right) + A + \frac{mE}{2\hbar^2}(\zeta+\eta)\right]\psi(\zeta,\eta) = 0$$

where $A = \frac{Zqme^2}{4\pi\epsilon_0\hbar^2}$. Use separation-of-variables to show that the solution of this equation is obtained as $\psi(\zeta,\eta) = e^{ikz}F(\frac{iA}{k}, 1; ik\zeta)$, where $F(a, c; x)$ is a standard solution of the confluent hypergeometric equation $xy'' + (c - x)y' - ay = 0$.

Marks: 10

3. A hydrogen atom is placed in a constant electric field $\vec{E} = E\hat{z}$. Use degenerate perturbation theory and the appropriate selection rules to find the first order correction to energy for the state with principal quantum number $n = 2$.

Marks: 10

4. (a) Show that you can write a general 2×2 spin matrix as $\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}$.

   (b) Find the eigenvalues and eigenvectors of this matrix.

   (c) Now if you rotate a general spinor χ through an angle $\theta = 2\pi$ about a direction $\hat{\mathbf{n}}$ to obtain a new spinor χ', then show that $\chi' = \pm\chi$, where the upper and lower signs hold for integral and half-integral spins respectively.

Marks: 1 + 4 + 5 = 10

5. Using the following unitary transformation that describes an infinitesimal rotation by an angle $\delta\theta$ about an arbitrary direction (along which the unit vector is $\hat{n}$)

$$U(\delta\theta)\psi(\vec{r}) = \left(\mathbf{I} - \frac{i}{\hbar}\hat{n}.\vec{L}\delta\theta\right)\psi(\vec{r}) = \psi[\mathbf{R}_n^{-1}(\delta\theta)\vec{r}]$$

derive the commutation relation $[L_x, L_y] = i\hbar L_z$. Here $\mathbf{I}$ is the identity matrix and $\mathbf{R}_n$ is the rotation operator (matrix) about the direction $\hat{n}$.

Marks: 10