M.Sc. (Physics) 2nd Year, 1st Semester Examination, 2017.

PHY/TE/305: GENERAL RELATIVITY & ASTROPHYSICS

Max. Marks = 40. Max. Time = 2 hrs.

Answer any *four* questions 4x10=40

1. Let $g \equiv \det\{g_{\mu\nu}\}$, where g_{ab} denotes the metric tensor. Then using the standard definition of covariant derivative, in terms of the Christoffel connection $\Gamma^{\alpha}{}_{\beta\sigma}$ establish the any two of the following results:

(a) $g_{,\mu} = g^{\alpha\beta} g g_{\alpha\beta,\mu}$

(b) $\Gamma^{\alpha}{}_{\alpha\beta} = (\ln(\sqrt{-g}))_{,\beta}$

(c) $F^{\alpha\beta}{}_{;\beta} = \frac{1}{\sqrt{-g}}(\sqrt{-g}F^{\alpha\beta})_{,\beta}$ when $F^{\alpha\beta}$ is anti symmetric. 5+5=10

2. a. A static space-time has line element given by

$ds^2 = -e^{2\Phi}dt^2 + h_{ij}dx^i dx^j$, where ϕ and h_{ij} are functions of x. Show that $\Gamma^0{}_{0i} = \partial_i \phi$, and also evaluate the components $\Gamma^0{}_{ij}$.

b. Consider the case of a 3-space (with a positive definite metric). Show that the metric of such a 3-space can always be diagonalized. (3+3)+4=10

3. a. Show that the metric tensor g is symmetric and from the line element $ds^2 = g_{\mu\nu}(x)dx^{\mu}dx^{\nu}$, deduce the transformation law connecting $g(x')$ and $g_{\mu\nu}(x)$. 2+2

b. State the principal of equivalence. 2

c. Show that the principal of equivalence allows one to erect local inertial frame at any point X in the space time such that $g_{\mu\nu}(x) = \eta_{\mu\nu}(x)$ and $\frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} = 0$. 4

Or. Find out the number of independent components of the Riemann curvature tensor in N dimensional space. 4

(2+2)+(2+4)=10

4. a. Show that the geodesic of a freely falling massive particle in a gravitational field takes the form $\frac{d^2x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$=0, where τ is the proper time. What plays the role of τ for the mass-less particle.

b. Show that this equation represent an extremal proper time connecting two time like separated events by studying small variation about the world line. (4+1)+5=10

5. a. Write down the general form of the Schwarzschild metric tensor (for the vacuum solution to Einstein's equations) and discuss its geometric properties and symmetries. What are the conserved quantities in the Schwarzschild solution.

b. By considering the radial equation of motion (orbits), compare the Schwarzschild's case with the Newtonian/Keplerian scenario. Further, by considering the behaviour at infinity (the asymptotic limit), show that the Schwarzchild metric reduces to the Minkowski metric (in fact this is the only metric that does so)! (1+4+1)+(2+2)=10

6.a. Briefly give an outline (no need to derive all the steps) to linearize the vacuum Einstein's equation taking metric perturbation as $g_{\alpha\beta}(x) = \eta_{\alpha\beta}(x) + h_{\alpha\beta}(x)$ where $\eta_{\alpha\beta}(x) = \text{diag}(-1,1,1,1)$ and $h_{\alpha\beta}(x)$ are small quantities.

b. Discuss its Gauge freedom conditions and show how to solve it in general circumstances.

4+6=10