*Ref. No.* Ex/M.Sc/P/I/104/17/2017

# MASTER OF SCIENCE EXAMINATION, 2017

**($1^{st}$ year, $1^{st}$ Semester)**

**PHYSICS**

**Statistical Mechanics- I**

**Paper - PHY/TG/104**

Time : Two hours | Full Marks: 40

Answer any four questions.

1. (a) Let the energy of a one dimensional simple harmonic oscillator is restricted in the range $E - \frac{\Delta}{2}$ and $E + \frac{\Delta}{2}$. What will be volume of the phase space trajectories. Estimate the number of eigen states within the allowed energy interval. 2 + 2

   (b) Show that the energy fluctuations in the canonical ensemble $< (\Delta E)^2 >= KT^2C_V$. Where $C_V$ specific heat at constant volume. 2

   (c) State Boltzmann H theorem. Prove it from the Boltzmann transport equation. 2 + 2

2. (a) Write down the basic postulates of quantum statistics. 2

   (b) Show that if the basis functions are the eigen functions of Hamiltonian itself, the density matrix $\rho$ in a quantum mechanical ensemble is diagonal and for any other representation $\rho$ is symmetric. 2 + 2

   (c) Consider a single electron which possesses an intrinsic spin $\frac{1}{2}\hbar\hat{\sigma}$ and a magnetic moment $\mu_B$, where $\hat{\sigma}$ is Pauli's spin operator. What is the configurational Hamiltonian of the spin when the applied magnetic filed is in the $z$ direction. Using the following Pauli's spin matrices calculate the density matrix in the canonical ensemble. Hence calculate the $< \sigma_z >$. 4

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

3. (a) Derive the equation of state for an non-ideal gas using mean field approximation. Assume the following interaction potential.

$$\begin{aligned} u(r) &= \infty, \text{ for } 0 \leq V_m \leq V_{ex} \\ &= -u, \text{ for } V_{ex} < V_m \leq V \end{aligned}$$

   Where, $V_{ex}$ is the excluded volume and V is the size of the system. 7

   (b) The universal equation of state for a van der Waal gas is given by

$$(P_r + \frac{3}{v_r})(3v_r - 1) = 8T_r.$$

   where, $P_r = \frac{P}{P_C}$, $v_r = \frac{v}{v_C}$ and $T_r = \frac{T}{T_C}$. Establish the "degree of flatness" of the critical isotherm at the critical point. 3

4. (a) A chain of $N$ spins interacting with its two nearest neighbours and with the external magnetic field can be modeled as one dimensional Ising system specified by $\{\sigma_1, \sigma_2, ...\sigma_n\}$. Configurational Hamiltonian is given by

$$E\{\sigma_i\} = -J \sum_{<ij>} \sigma_i \sigma_j - \mu B \sum_i \sigma_i$$

Using transfer matrix method, calculate the Helmholtz free energy. where symbols have their usual meaning. 6

(b) What is Bragg-Williams approximation in the context of phase transition using Ising model. Define order parameter in this approximation. 2+1

5. (a) An ideal Fermi gas satisfies the following equation

$$\frac{PV}{KT} = lnZ = \sum_{\epsilon} ln(1 + ze^{-\beta\epsilon})$$

$$N = \sum_{\epsilon} < n_\epsilon >= \sum_{\epsilon} \frac{1}{1 + z^{-1}e^{\beta\epsilon}}$$

where, symbols have their usual meaning. Calculate the internal energy $(U)$ of the system. Show that the Fermi system, in general, satisfies the relationship $P = \frac{2}{3}(\frac{U}{V})$. 4 + 2

(b) Derive the relation between Fermi energy and zero point energy (at $T = 0$). Hence establish the ground state pressure of the system. 3+1

6. (a) What is the origin of the magnetic properties of a physical substance? 2

(b) Show that the low field susceptibility of a Pauli's paramagnetism at very low temperature is independent of temperature. 5

(c) What is De Haas-Van Alphen effect? 3