MASTER OF SCIENCE EXAMINATION, 2017

(1^{st} Year, 1^{st} Semester)

PHYSICS

Quantum Mechanics

Paper – PHY/TG/103

Full Marks: 40 **Time: Two Hours**

Answer **Q. No. 1** and **any three (3)** questions *from the rest*

1. Answer **any four** ($2.5 \times 4 = 10$)

(a) Define basis vectors. Write down the conditions for position eigenvectors to be the continuous basis vectors.

(b) If $|\Psi(t)\rangle$ represents a state vector in LVS, then what do $\psi(x,t)$ and $\tilde{\psi}(p,t)$ mean physically? How are they related with each other?

(c) What does $\langle x|p\rangle$ mean? Find its value.

(d) Suppose a spin 1/2 particle is in the state $\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$. What are the probabilities of getting different eigen values of S_x, S_y.

(e) Show that if $|\Phi_i\rangle$'s form basis of a LVS then $|\Phi_i\rangle\langle\Phi_j|$ forms the basis of any operator in that LVS.

2. (a) Write down the Hamiltonian of L.H.O in terms of raising and lowering operator? Find out the dimension of each term of the raising operator?

(b) Find out the Eigen function of lowering operator 'a' in terms of the ground state ($|0\rangle$) of LHO and normalize it. **(5+5)**

3. (a) What does Clebsch-Gordan (C.G) coefficient mean? Prove the selection rule for C.G. coefficients relating the eigenvalue of J_{1z}, J_{2z}, J_z.

(b) Calculate from first principles the C.G coefficients for $j_1 = 1$ and $j_2 = ½$. **(4+6)**

4. Consider the three-dimensional infinite cubical well of side a and introduce a time independent perturbation

$$H' = \begin{cases} V_0, & if\ 0 < x < a/2, 0 < y < a/2\ and\ 0 < z < a \\ 0, & otherwise \end{cases}$$

(a) Calculate the first order correction to the ground state of unperturbed Hamiltonian.

(b) Show how does the perturbation H'_{z} remove the degeneracy of 1st excited state of unperturbed Hamiltonian. **(3+7)**

5. (a) If A be a hermitian operator that commutes with H^0 (unperturbed Hamiltonian having degenerate eigenstates) and H' (perturbation) then prove that the simultaneous eigen state of A and H^0 would be the "good" states.

(b)Suppose the Hamiltonian, in matrix form is

$$H = H_0 + H' = V_0 \begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}, \text{ where } \epsilon \ll 1.$$

(i) Write down the eigenvectors and eigenvalues of the unperturbed Hamiltonian ($\epsilon = 0$).

(ii) Solve for the exact eigenvalues of H directly upto second order of ϵ.

(iii) Use degenerate perturbation theory to find the first order correction to the two initially degenerate eigenvalues. Caompare it with the exact result. (3+7)

6. (a) Find out the radial probability density (D_{nl}) for the hydrogen atom in *1S* state. Draw the graphical representation of it as a function of ***r***. Find the value of r at which D_{nl} becomes maximum.

(b) Show how does the relativistic correction to the kinetic energy removes the ℓ degeneracy of the energy levels of hydrogen atom. (Given $\langle \frac{1}{r} \rangle = \frac{1}{n^2 a_0}$ and $\langle \frac{1}{r^2} \rangle = \frac{1}{(l+\frac{1}{2})n^3 a_0^2}$). (Symbols have their usual meanings). (5+5)