MASTER OF SCIENCE EXAMINATION, 2017
(1st Year, 1st Semester)
PHYSICS
Mathematical Methods – I

Paper – PHY/TG/102

Time – Two hours | Full Marks : 40

Answer any **four** questions

1. a) Let V be the set of ordered pairs of real numbers: $V = \{(a, b) : a, b \in R\}$. Find whether V is a vector space over R with respect to the following operations of addition and scalar multiplication on V :

$$(a, b) + (c, d) = (a+c, b+d) \text{ and } k(a, b) = (k^2a, k^2b).$$

b) Show that the vectors $v = (1+i, 2i)$ and $w = (1, 1+i)$ in C^2 are linearly dependent over the complex field C but are linearly independent over the real field R.

c) Find whether the following mapping is linear

$$F : R^2 \rightarrow R^3 \text{ defined by } F(x, y) = (x+1, 2y, x+y)$$

d) Let V be the space of 2×2 matrices over R, and let $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Let T be the linear operator on V defined by T (A) = MA. Find the trace of T.

[Hint : take the basis $e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$; $e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$; $e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$; $e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$]

2+2+2+4

2. a) Using Fermat's principle derive laws of refraction in case of an arbitrary surface separating two media.

b) Frame a suitable isoperimetric problem and establish Schrödinger wave equation.

6+4

3. a) Define 'Conformal Mapping'.

b) Two coaxial infinite cylinders of radii r_1 and r_2 ($r_1 < r_2$) are maintained at potentials V_1 and V_2 respectively. Find the electrostatic potential at any point between the two cylinders.

c) Two infinite half cylinders of unit radius with the upper half maintained at potential $+V_0$ and the lower half maintained at $-V_0$. Find the potential at an inside point.

2+4+4

4. a) Show that $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta} = \frac{2\pi}{3}$

b) Show that $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$

5+5

5. a) Establish Cauchy-Riemann conditions for a function to be analytic.

b) Find the plane closed curve of fixed perimeter and maximum area.

c) Find whether 1, t, t^2 are linearly independent.

4+4+2