Master of Science, 1st Year, 1st Semester, Examination 2017

Classical Mechanics I (PHY/TG/101)

Time - Two hours

Full Marks : 40

Answer any **four** questions

1. (a) Show that the isotropy of space (rotational symmetry) gives rise to a conservation law. Identify the conserved physical quantity.

   (b) Show that a second conservation law follows from the homogeneity of space (translational symmetry). Identify the conserved physical quantity.

   (5+5)

2. Consider a double pendulum, with masses $m_1$ and $m_2$ hanging successively by strings of lengths $l_1$ and $l_2$.

   (a) Find the Lagrangian and Lagrange's equations for the system.

   (b) Find the generalised momenta.

   (c) Hence find the Hamiltonian and Hamilton's equations for the system.

   ((2+2)+2+(2+2))

3. Two masses, $m_1$ and $m_2$ are attached to fixed walls and to each other by identical springs, each of spring constant $k$. The masses execute one dimensional motion on a frictionless horizontal table.

   (a) Find the Lagrange's equations for the system.

   (b) Find the normal modes of motion. (4+6)

4. Consider a coordinate-momentum transformation $(q,p) \to (Q,P)$, which preserves the Hamilton's equation in the transformed variables.

   (a) Show how the original and transformed Hamiltonians are related in general.

(b) Show the existence of generating functions in such transformations. Identify the different types of such generating functions that are possible.

(c) Consider the generating function

$$F_1(q, Q) = \frac{1}{2} \sqrt{mk} \; q^2 \; \cot(Q).$$

How does the Hamiltonian of one-dimensional simple harmonic oscillator,

$$\mathcal{H}(q, p) = \frac{p^2}{2m} + \frac{1}{2}kq^2 \, ,$$

transform under the canonical transformation generated by this generating function.

(d) Solve the Hamilton's equations in transformed variables, and hence substituting properly, obtain expressions for $q(t)$ and $p(t)$. (1+2+3+4)

5. (a) The distances between any two points on a rigid body do not change with time. Clearly argue to justify the correct degrees of freedom for general motion of such a body.

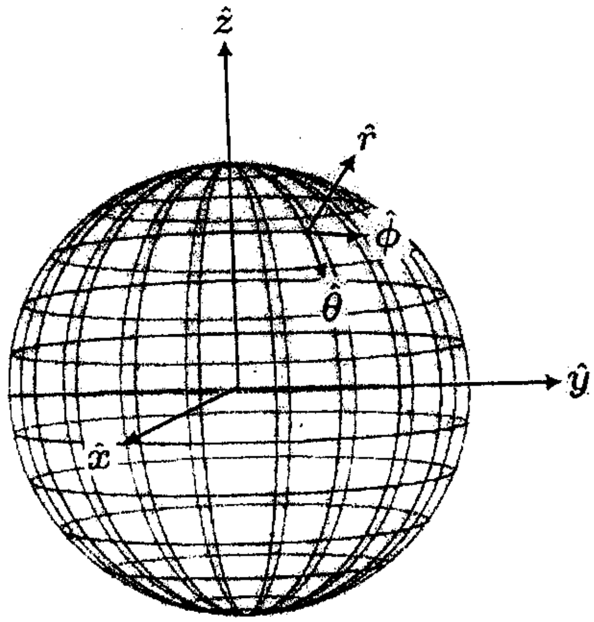
(b) For a rigid body moving with one point fixed, define Euler angles to describe its motion. Sketch diagram to explain the angles.

(c) Write down the rotation matrix $R_3(\psi)$ for rotation by angle $\psi$ about $z$-axis, and $R_1(\theta)$ for rotation by $\theta$ about $x$-axis.

(d) Obtain a general rotation matrix $R(\phi, \theta, \psi)$, corresponding to Euler rotations $(\phi, \theta, \psi)$.

(e) Can the resultalt of an arbitrary vector $\vec{\mathbf{A}}$ under the above rotation $R(\phi, \theta, \psi)$ be achieved by a single rotation about a suitable axis ? Justify your answer, particularly keeping in mind the importance of degrees of freedom. (2+2+2+2+2)

6. Earth provides a good example of a non-inertial rotating frame. Consider a fixed inertial frame, and orthonormal triad $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$. Consider a local orthonormal triad $\{\hat{\mathbf{r}}, \hat{\theta}, \hat{\phi}\}$ on the surface of the Earth.




(a) Find the transformation equations between the two triad systems.

(b) Find the acceleration for the position vector $\ddot{\mathbf{r}}$ of an arbitrary point in the local orthonormal triad $\{\hat{\mathbf{r}}, \hat{\theta}, \hat{\phi}\}$ frame.

(c) Find the equation of motion of a particle in the moving frame (acted upon by gravity of the Earth). Identify the various non-inertial terms. (3+3+4)