Ref. No. Ex/M.Sc/INST./1/1/101/39/2017

M.Sc.(Instrumentation) Examination 2017
(1^{st} year 1^{st} semester)
Subject: Mathematical Methods

Full Marks: 100 Time-4 hrs

PAPER-I(T- 101)

Group – A

Answer any five questions

1. Classify and reduce the partial differential equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 4x^2$ to its canonical form. Also find the general solution. [10]

2. Use the method of separation of variables to solve the partial differential equation $u_{tt} = u_{xx}$ $(0 < x < 1)$ subject to the conditions
$$\begin{cases} u(0,t) = u(1,t) = 0 & t > 0 \\ u(x,0) = x(1-x),\ u_t(x,0) = 0 & 0 < x < 1 \end{cases}$$ [10]

3. Use the method of separation of variables to solve the partial differential equation $u_t = u_{xx}$ $(0 \le x \le 2\pi)$ subject to the conditions
$$\begin{cases} u_x(0,t) = u_x(2\pi,t) = 0 & t > 0 \\ u(x,0) = x & 0 \le x \le 2\pi \end{cases}$$ [10]

4. Use the method of separation of variables to solve the partial differential equation $u_{xx} + u_{yy} = 0$ $(0 \le x, y \le 1)$ subject to the conditions:
$$\begin{cases} u(x,0) = u_0 \ (constant) \\ u(x,1) = 0,\ u(0,y) = u(1,y) = 0 \end{cases}$$ [10]

5. Use the method of separation of variables to find a formal solution of the following partial differential equation
$u_{xx} - u_y = -Ae^{-\alpha x}$ $(A \ge 0, \alpha > 0)$ subject to the conditions
$$\begin{cases} u(0,y) = u(L,y) = 0 & y > 0 \\ u(x,0) = f(x) & 0 < x < L \end{cases}$$ [10]

6. (a) Give the formulas for the Fourier transform of a function f(x) and the inverse Fourier transform.
(b) Compute the Fourier transform of $f(x) = e^{-|x|}$.
(c) State Parseval's theorem, and use it to evaluate $\int_0^\infty \frac{1}{(1+k^2)^2} dk$. [2+3+5]

7. Solve the following heat equation with the help of complex Fourier transform
$u_t = ku_{xx}$ $(-\infty < x < \infty)$ subject to the initial conditions
$$u(x,0) = \begin{cases} 0 & x < 0 \\ 100 & x > 0 \end{cases}$$ [10]

8. (a) Find the Z- transformation of $f(k) = c^k \cos\alpha k$ $(k \ge 0)$ [4]
(b) Find the solution of the following difference equation by using Z-transformation, $y_{k+2} + 6y_{k+1} + 9y_k = 2^k,\ y_0 = y_1 = 0$ $(k \ge 0)$. [6]

Group - B

Section - I

Question No. 6 is compulsory and answer any three from the rest.

1.a) Prove that $\int_{-\pi}^{\pi} \cos mx \sin nx \, dx = 0, \quad \forall m,n.$

b) Obtain the Fourier series expansion of f(x) in $[-\pi,\pi]$

where $f(x) = 0 \quad -\pi \leq x < 0$

$(1/4)\pi x \quad 0 \leq x \leq \pi$

Hence show that $\pi^2/8 = 1 + (1/3^2) + (1/5^2) + \ldots\ldots\ldots\ldots$

3+5+2

2.a) If F(t) is a function of sectionally countinuous and if L{F(t)} = f(p) then prove that L{ t F(t) } = -f′(p).

b) Find Laplace Transform of t^n,when n is a positive integer

4+6

3.a) If L^{-1} {f(p)} = F(t) then prove that L^{-1}{ f (p-a) } = e^at L^{-1}{ f(p) }

b)Prove that L^{-1} { 3p+7/p^2-2p-3} = 4e^3t – e^-t

4+6

4.Using Laplace Transform to solve the differential equation

$(D+2)^2y = 4e^{-2t}$ where $D \equiv d/dt$

Given that y(0) = -1 and y′(0) = 4

10

5.a) Prove that Arithmetic mean is affected by change of both origin and scale.

b) Find the mean and median of heights from the following frequency distribution.

Height(cm)	135-140	140-145	145-150	150-155	155-160	160-165	165-170	170-175
No. of students:	4	9	18	28	24	10	5	2

3+3+4

Answer any one

6.a) Find the mean and variance of first 'n' natural numbers. 4

b) A student obtained the mean and standard deviation of 100 observations as 40.1 and 5.0 respectively. It was later found that he copied 50 wrongly instead of the correct value 40. Find the correct mean and correct standard deviation. 4

Section-II

Answer any two questions:

7. (a) Show that $f(z) = |z|^2$ is continuous everywhere but its derivative exists only at the origin. [4]

(b) For the function $(z) = \sqrt{|xy|}$, show that the Cauchy-Riemann equations are satisfied at $z = 0 + i0$ but the function is not differentiable at that point. [4]

8. (a) If $f(z)$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$ [4]

(b) Show that $u(x,y) = log\sqrt{x^2 + y^2}$ is harmonic, then find its harmonic conjugate function $v(x,y)$ so that $f = u(x,y) + i\, v(x,y)$ is analytic. [4]

9. (a) Evaluate the integral

$$\oint \frac{3z^2 + z}{z^2 - 1} dz \text{ around the circle } |z-1| = 1.$$ [2]

(b) Show that $\frac{1}{2\pi i} \oint \frac{e^{zt}}{z^2 + 1} dz = \sin t$, integral is over $|z| = \pi$, $(t > 0)$ [3]

(c) Use Cauchy's residue theorem to evaluate $\oint \frac{3z^2+z-1}{(z^2-1)(z-3)} dz$ around the Circle $|z| = 2$. [3]