Ex/M.Sc/M/A1.8/35/2017

MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.3 (A1.8)

(Non-Relativistic Quantum Mechanics)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations have their usual meanings.)

Answer any *five* questions. 10×5

1. (a) State the postulates of quantum mechanics. 2

   (b) Derive the equation of motion in Heisenberg's approach. Also show the equivalence of it with the Schrödinger approach. 4+4

2. (a) Show that

$$\frac{d}{dt}\langle p_x \rangle = \left\langle -\frac{\partial v}{\partial x} \right\rangle$$ 5

[*Turn over*]

(b) If the Hamiltonian of a moving particle is given by

$$H = \frac{\vec{p}^2}{2\mu} + V(\vec{r}),$$

then show that

$$\sum_n (E_n - E_m)|x_{nm}|^2 = \hbar^2/2\mu$$

where summation is over all eigen states of H. 5

3. (a) For any two observables A and B if $[A, B] = 0$ then show that (i) A and B are simultaneously diagonalizable (ii) A and B have the same set of eigen states. 3+3

(b) Given $A^2 = 0$, $AA^+ + A^+A = 1$, $B = A^+A$. Show that $B^2 = B$. Express the matrix form of B in the basis of its eigen states. Can A be diagonalized in any representation ? 1+2+1

4. Define angular momentum operator. Find the eigen values of the angular momentum operator. In matrix representation, show that the raising operator is a lower diagonal matrix. 2+5+3

[*Turn over*]

5. (a) For harmonic oscillator show that

$$\frac{da}{dt} + iwa = 0$$

Hence prove that

$$q(t) = q(0)\cos wt + \frac{p(0)}{\mu w}\sin wt$$

and $p(t) = p(0)\cos wt - \mu w q(0)\sin wt$. 2+5

(b) Describe clearly the box normalization. 3

6. (a) Outline the stationary state perturbation theory and find the first order energy correction when the energy levels of the unperturbed state are g-fold degenerate. 5

(b) Find the ground state energy of the Helium atom using the perturbation method. 5

7. (a) Show how the degenerate energy levels of the $n = 2$ state of the hydrogen atom are split by the application of a magnetic field. What is Zeeman effect ? 5

(b) Explain Pauli's exclusion principle. 5

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