# Master of Science Examination, 2017 

## (2nd Year, 1st Semester)

## MATHEMATICS

Unit - 3.3 (A1.7)

## (Principles of Operations Research)

The figures in the margin indicate full marks.
(Symbols and Notations have their usual meanings.)

Answer any two from Group A and any three from Group B.

## Group - A

1. (a) Formulate an assignment problem mathematically and hence establish the following theorem :
"In an assignment problem, if we add or subtract a constant to every element of any row or column of the cost matrix, then an assignment that minimize the total cost on one matrix will also minimize the total cost on the other matrix."$2+5$

## [ 2 ]

(b) What is the necessary and sufficient condition for the existence of a feasible solution to a transportation problem?
(c) A manufacturer wants to ship 22 loads of his product as shown below. The matrix gives the distances in kilometres from sources of supply to the destinations. The shipping cost is Rs. 10 per load per km.

Destination

Source

|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ | $\mathrm{D}_{5}$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{S}_{1}$ | 5 | 8 | 6 | 6 | 3 | 8 |
| $\mathrm{~S}_{2}$ | 4 | 7 | 7 | 6 | 5 | 5 |
| $\mathrm{~S}_{3}$ | 8 | 4 | 6 | 6 | 4 | 9 |
| Demand | 4 | 4 | 5 | 4 | 8 |  |

What shipping schedule should be used in order to minimise the total transportation cost ?
2. (a) Establish the equivalence between a LPP and a two person zero sum game.

## [ 3 ]

(b) Solve the following game by simplex method of LPP whose pay-off is given as follows :

Player B

|  |  | $\mathrm{B}_{1}$ | $\mathrm{B}_{2}$ | $\mathrm{B}_{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| Player A | $\mathrm{A}_{1}$ | 1 | -1 | 3 |
|  | $\mathrm{A}_{2}$ | 3 | 5 | -3 |
|  | $\mathrm{A}_{3}$ | 6 | 2 | -2 |

3. (a) Derive and solve the steady state difference equation governing the queuing model $(\mathrm{M} / \mathrm{M} / 1):(\mathrm{N} / \mathrm{FIFO})$.
(b) A supermarket has two girls as sales persons at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the counters at the rate of 10 per hour, calculate the probability that an arrival will have to wait for service.

## [ 4 ]

## Group - B

4. A marketing manager has five salesmen and five sales districts. Considering the capabilities of the salesmen and the nature of the districts, the marketing manager estimates that the sales per month (in hundred rupees) for each salesman in each district would be as follows. Find the assignment of salesmen to districts that will result in maximum sales.

Districts

|  |  | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 32 | 38 | 40 | 28 | 40 |
|  | 2 | 40 | 24 | 28 | 21 | 36 |
| Salesman | 3 | 41 | 27 | 33 | 30 | 37 |
|  | 4 | 22 | 38 | 41 | 36 | 36 |
|  | 5 | 29 | 33 | 40 | 35 | 39 |

5. A salesman has to visit five cities $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E . The distances (in 100 kms ) between the five cities are as follows :

|  | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
| A | - | 1 | 6 | 8 | 4 |
| B | 7 | - | 8 | 5 | 6 |
| C | 6 | 8 | - | 9 | 7 |
| D | 8 | 5 | 9 | - | 8 |
| E | 4 | 6 | 7 | 8 | - |
|  |  |  |  |  |  |

If the salesman starts from city A and has to come back to city $A$, visiting each city once and only once, which route should he select so that the total distance travelled is minimum ?
6. Use the graphical method for solving the following game and find the value of the game.

Player B

| Player A | $\mathrm{B}_{1}$ | $\mathrm{~B}_{2}$ | $\mathrm{~B}_{3}$ | $\mathrm{~B}_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~A}_{1}$ | 2 | 2 | 3 | -2 |
| $\mathrm{~A}_{2}$ | 4 | 3 | 2 | 6 |
|  |  |  |  |  |

7. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Calculate :
(i) Expected line length
(ii) Probability that the queue size exceeds 10 .

If the input increases to 33 trains per day what will the change in (i)?

