Ex/M.Sc/M/A1.4/35/2017

MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.3 (A1.4)

(Fluid Mechanics - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Notations and Symbols have their usual meanings.

Answer question no. 1 and any *three* from the rest.

10×5=50

1. Give the physical significance of stream function. 2

2. (a) Derive Euler's equation of motion for an inviscid fluid.

   (b) A long pipe is of length l and has slowly tapering cross-section. It is inclined at an angle α to the horizontal and water flows steadily through it from the upper to the lower end. The section at the upper end has twice the radius of the lower end. At the lower end, the water is delivered at atmospheric pressure. If the pressure at the

[*Turn over*]

upper end is twice atmospheric pressure then find the velocity of delivery at the lower end. 8+8

3. (a) Find the velocity potential φ, the stream function ψ and the complex potential W for the motion of a circular cylinder moving in an infinite mass of liquid at rest at infinity with velocity U in the direction of x-axis.

(b) Find the complex potential for an elliptic cylinder moving in an infinite mass of liquid with velocity U parallel to axis plane through the major axis of any cross-section of the cylinder. 8+8

4. (a) State and prove Kutta-Joukowski's theorem.

(b) If a source and a sink of equal strength are placed at the points $\left(\pm\frac{a}{2},\ 0\right)$ within a fixed circular boundary $x^2+y^2=a^2$, then show that the streamlines are given by $\left(r^2-\frac{1}{4}a^2\right)\left(r^2-4a^2\right)-4a^2y^2=Ky\left(r^2-a^2\right)$, where K is constant and $r^2=x^2+y^2$. 8+8

5. (a) Define Karman vortex street.

[*Turn over*]

(b) Three parallel rectilinear vortices of the same strength K and in the same sense meet any plane perpendicular to them in an equilateral triangle of side a. Prove that all the vortices move round the same cylinder with uniform speed in time $\frac{2\pi a^2}{3K}$.

(c) Find the velocity potential φ and the stream function ψ for a vortex doublet of strength μ. 2+8+6

6. (a) Find the velocity components of the fluid particles for the progressive waves on the surface of a canal of uniform depth.

(b) Define group velocity. Prove that the group velocity of deep sea water is half of the wave velocity and also show that the group velocity of shallow water is equal to the wave velocity. 8+8