MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.2

(Partial Differential Equations and Nonlinear Ordinary Differential Equations)

Full Marks : 50 Time : Two Hours

All questions carry equal marks.

Use a separate Answer-Script for each part.

(Symbols have their usual meanings)

Part - I

(Marks - 30)

Answer any *three* questions. 10×3

1. Discuss the nature of the equation

$$u_{xx} + y u_{yy} = 0$$

Hence reduce this equation to canonical form when $y < 0$.

2. A tanpura string of unit length is pulled upward at the middle so that it reaches a height h. Assume that the initial displacement of the string is

$$u(x,\ 0) = 2hx,\ 0 \le x < \tfrac{1}{2}$$
$$= 2h(1-x),\ \tfrac{1}{2} \le x < 1$$

Find the subsequent motion of the string if it is released suddenly.

3. Solve the exterior Dirichlet's problem for a circle given by

$$\nabla^2 u = 0,\quad a \le r < \infty,\quad 0 \le \theta \le 2\pi$$

$$u(a,\ \theta) = f(\theta),\quad 0 \le \theta \le 2\pi .$$

Here 'a' is the radius of the circle and $f(\theta)$ is known.

4. Suppose that u is a harmonic function in a region D and u and its normal derivative $\frac{\partial u}{\partial n}$ are known at every point of the boundary surface S of the finite region D. Using the fundamental solution of Laplaces equation fund u at any interior point of D.

[*Turn over*]

5. If $u(x, t)$ be a continuous function which is a solution of $u_t = u_{xx}$, in the rectangle R, $0 \le x \le l$, $0 \le t \le T$, then prove the maximum value of u are attained either on the boundary $t = 0$ or on the boundaries $x = 0$ and $x = l$.

Hence show that the solution of the following boundary value problem for $u(x, t)$ is unique.

$$u_t = u_{xx}, \quad 0 \le x \le l, \quad 0 \le t \le T.$$

$$u(0, t) = f_1(t)$$

$$u(l, t) = f_2(t)$$

$$u(x, 0) = \varphi(x)$$

where $f_1(t)$, $f_2(t)$, $\varphi(x)$ are continuous functions and $\varphi(0) = f_1(0)$, $\varphi(l) = f_2(l)$.

Part - II

(Marks - 20)

Answer question number 6 and any *three* from the rest.

6. Find all equilibrium points and their stabilities of the system

$$\dot{x} = \left|1 - x^2\right|. \quad 2$$

7. Use Poincare Bendixon theorem to show that the following system has a limit cycle.

$$\dot{x} = x - y - x^3$$
$$\dot{y} = x + y - y^3 \quad 6$$

8. Use the method of nullclines to draw the phase portrait of the system

$$\dot{x} = x - y$$
$$\dot{y} = 1 - e^x \quad 6$$

9. Classify a planar system in trace-determinant plane. Describe all possible qualitatively different solutions in that plane. 6

[*Turn over*]

10. Sketch all qualitatively different vector fields as r is varied in the system

$$\dot{x} = x\left(r - e^{x}\right)$$

Show that a transcritical bifurcation occurs in the above system. Find the critical value of r at which bifurcation occurs. Draw also the bifurcation diagram. 6
