MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.5 (B-1.33)

(Production and Inventory Control - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Notations/Symbols have their usual meanings.

Answer Question No. 6 and any *three* questions from the rest.

1. (a) Using a $(t,\ s_i)$ policy, examine an inventory system under the following assumptions :

(i) The system operates only for H units of time.

(ii) During the period H, there exists a total demand of D units.

(iii) The rate of demand R changes linearly with time t such that $R = \alpha t$, where $\alpha(>0)$ is a constant.

[*Turn over*]

(iv) c_1 and c_3 are the unit carrying cost and ordering cost, respectively.

Show that the optimum number of replenishments m^* satisfies the inequality

$$F\left(m^* - 1\right) \leq \frac{c_1 DH}{c_3} \leq F\left(m^*\right)$$

where $F\left(m\right) = \dfrac{m\left(m+1\right)}{\left(m+1\right)k\left(m\right) - mk\left(m+1\right)}$ and

$$k\left(m\right) = \frac{1}{2} + \frac{1}{6m} \cdot$$

(b) A certain product has demand of 25 units per month and the items are withdrawn uniformly. Each time a production run is made, the set up cost is Rs. 15. The production cost is Re 1 per item and inventory holding cost is Re. 0.30 per item per month.

(i) Assuming that shortages are not permitted, determine how often to make a production run and what size it should be ?

[*Turn over*]

(ii) If shortage cost is Rs. 1.50 per item per month, determine how often to make a production run and what size it should be ? 8+8

2. (a) Consider the inventory model in which the lot size is constant and shortages are allowed with a penalty cost for late deliveries. If c_1 is the holding cost per unit per unit time, c_2 is the penalty cost per unit time of failing to deliver one item on schedule and R is the demand rate, determine the optimum lot size and the minimum average cost of the inventory system.

(b) A manufacturer has to supply his customer with 600 units of product per year. Shortages are not allowed and storage cost amounts to 60 paisa per unit per year. The set up cost per set up is Rs. 80. Find (i) the EOQ (ii) the minimum average yearly cost (iii) the optimum number of orders per year (iv) the optimum period of supply per optimum order (v) the increase in the total cost associated with ordering 40% less than EOQ.

8+8

3. (a) What is an EOQ ? When the lot size q is restricted to discrete units, show that the optimum lot size q^* for the

Harris Wilson EOQ model can be derived from the following inequality :

$$q^*\left(q^* - u\right) \leq \frac{2Rc_3}{c_1} \leq q^*\left(q^* + u\right)$$

where the symbols have their usual meanings and the lot size q restricted to take the values u, $2u$, $3u$, ...

(b) Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	**Unit cost (Rs.)**
$0 \leq q_1 < 500$	25
$500 \leq q_2 < 1500$	24.8
$1500 \leq q_3 < 3000$	24.6
$3000 \leq q_4$	24.4

The annual demand for a product is 500 units. The cost of storage per unit per year is 10% of the unit cost. The ordering cost is Rs. 180 for each order. 8+8

4. (a) Describe an order level lot size system under the following assumptions :

[*Turn over*]

(i) Demand rate is deterministic and constant.

(ii) The replenishment rate is infinite.

(iii) Lead time is zero.

(iv) c_1, c_2 and c_3 are the unit carrying cost, shortage cost and replenishment cost, respectively.

Hence find the optimal values of the order level, lot size and minimum total cost.

(b) A compnay management has decided that the average stock level must not exceed 750 units of all types. The company makes three products and the following data are given.

Product *i*	**1**	**2**	**3**
$c_1^{(i)}$	0.05	0.02	0.04
$c_3^{(i)}$	50	40	60
$D^{(i)}$	100	120	75

Determine the optimal production quantities. 8+8

[*Turn over*]

5. (a) In a multi-item deterministic inventory model, find the economic lot size for each item separately which minimizes the total cost under limitation on the total number of orders that can be placed.

(b) In an inventory system, the cost of carrying an amount dQ of a perishable item for a period T is $dK = c_1 T^n \, dQ$ where n is the parameter larger than unity and c_1 is constant. If the demand rate R is uniform and no shortage is allowed then solve the system. 8+8

6. Explain the term 'measure of sensitivity' in Harris-Wilson lot size system. 2