Master of Science Examination, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.5 (B-1.19)

(Graph Theory - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols have their usual meanings, if not mentioned otherwise.)

Attempt the questions as follows.

1. Answer any *one* : 10

(a) Given a connected (p, q)-graph G with $p \geq 4$ and ω as the intersection number of G, prove that $\omega = q$ if and only if G has no triangle.

(b) If a (p, q)-graph G with $p \geq 3$ satisfies

$$\deg u + \deg v \geq p$$

[*Turn over*]

for every paid u and v of nonadjacent points, then, using Posa's Theorem, prove G is hamiltonian.

2. Answer any *two* : 10×2=20

(a) Write procedures for preorder, inorder and postorder traversals of a tree T. Prove that the preorder and inorder traversals of T can uniquely reconstruct the tree T. 6+4=10

(b) State and prove the Cayleys formula for counting the number of trees. 2+8=10

(c) Describe Kruskal's algorithm for finding a minimum spanning tree of an edge-weighted connected graph. Prove the correctness of the algorithm. 5+5=10

3. Answer any *two* : 10×2=20

(a) If $\Delta(G) = n \geq 3$ and 3-connected, then G is n-colorable unless K_{n+1} is a component of G. 10

(b) (i) If G is a uniquely n-colorable graph then prove that the subgraph induced by the union of any two color classes in the n-coloring of G is connected.

(ii) Define *chromatic polynomial* of graph. Find the chromatic polynomials for a complete graph K_p and a wheel graph W_n. 5+5=10

(c) (i) Define symmetric difference of two graphs. Prove that every component of the symmetric difference of two matchings of a graph is either a path or an even cycle.

(ii) Define *M*-augmenting path for a matching *M* in a graph *G*. Prove that a matching in a graph *G* is maximum if and only if *G* has no *M*-augmenting path. (1+3)+(1+5)=10