Ex/M.Sc/M/B-1.13/36/2017

MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.5 (B-1.13)

(Differential Geometry and its Applications - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols have their usual meanings.

Answer any *five* questions.

1. (a) Let M be a contact Riemannian manifold. Then prove that M is K-contact manifold iff $\nabla_X \xi = -\varphi X$, for any vector field X on M.

   (b) Prove that on an almost contact metric manifold,

   $$g(X,\ \varphi Y) + g(\varphi X,\ Y) = 0.$$ 7+3

2. (a) In a Kähler manifold, prove that

   $$S(\bar{X},\ \bar{Y}) = S(X,\ Y).$$

[*Turn over*]

(b) Define almost complex structure on an even dimensional differentiable manifold with an example. Check whether it is unique or not.

(c) Prove that $N\left(X,\ \overline{Y}\right) = -\overline{N\left(X,\ Y\right)}$. 4+3+3

3. (a) Let M be a K-contact manifold. Then prove that the sectional curvature of any plane section containing ξ is equal to –1. 5

(b) Prove that if a vector field V in an almost complex manifold is strictly almost analytic, then $N\left(V,\ X\right) = 0$, for every vector field X. What geometric implification can you draw from this result ? 4+1

4. (a) Define Riemannian metric on a differentiable manifold. Prove that every differentiable manifold which is Hausdorff and second countable has a Riemannian metric. 1+4

(b) Define Riemannian convection on a Riemannian manifold. Prove that in a Riemannian manifold, Riemannian metric is covariantly constant. 1+4

5. (a) Prove that $\grave{R}\left(X,\ Y,\ Z,\ W\right) = -\grave{R}\left(X,\ Y,\ W,\ Z\right)$.

[*Turn over*]

(b) Prove that a projectively flat Riemannian manifold is an Einstein manifold. 6+4

6. (a) If $\overline{\nabla}_X Y = \nabla_X Y - T(X, Y)$, then proe that $\overline{\nabla}$ is a linear connection and $\overline{T} = -T$.

(b) For any form *w*, prove that $d(f^* w) = f^*(dw)$. 5+5

7. (a) If *w* is an 1-form and μ is a 2-form, find the expression for $w \wedge \mu$. 2

(b) Define lie group with an example. 2

(c) Let $M = \mathbb{R}^2$ and let $\varphi : \mathbb{R} \times M \to M$ be defined by $\varphi(t, (x, y)) = (xe^{2t}, ye^{-2t})$. Check whether φ is an 1-parameter group of transformations or not. If so, find its generator.

Does every vector field induce an 1-parameter group of transformation ? Justify with example. 6
