MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.1

(Numerical Analysis)

Full Marks : 30 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols have their usual meanings)

Part - I

(Marks - 15)

Answer any *one* question.

1. (a) State and prove the necessary and sufficient condition for convergence of an iterative method $\vec{x}^{(k+1)} = B\vec{x}^{(k)} + \vec{d}$, where B is the iterative matrix and $\vec{d}$ is the iterative vector. 8

(b) Use Newton's method to find the intersecting point of the circle $x^2 + y^2 = 1$ with the hyperbola $x^2 - y^2 = 1$

[*Turn over*]

near the point (1, 1). Show first three steps (correct upto four decimal places). 7

2. (a) Use SOR method to find solution of the following system of equations

$$\begin{aligned} 3x - y + z &= -1 \\ -x + 3y - z &= 7 \\ x - y + 3z &= -7 \end{aligned}$$

Show first three steps starting from (0, 0, 0) with the value of the relaxation parameter $\omega = 1.25$ (correct upto four decimal places). 9

(b) Let $f : \mathfrak{R}^n \to \mathfrak{R}$ be continuously differentiable function on $D \in \mathfrak{R}^n$ and $\vec{x}_0 \in D$. Let $t^* > 0$ be the minimizer of the function $\phi(t) = f\left(\vec{x}_0 - t\vec{\nabla}f\left(\vec{x}_0\right)\right)$, $t \geq 0$ and let $\vec{x}_1 = \vec{x}_0 - t^*\vec{\nabla}f\left(\vec{x}_0\right)$, then prove that $f\left(\vec{x}_1\right) < f\left(\vec{x}_0\right)$. 3

(c) Let $f : \mathfrak{R}^n \to \mathfrak{R}$ be continuously differentiable function on $D \in \mathfrak{R}^n$, and $\vec{x}_k$ and $\vec{x}_{k+1}$, for $k \geq 0$ be two consecutive iterates produced by the method of steepest decent. Show that $\vec{x}_k$ and $\vec{x}_{k+1}$ are orthogonal. 3

[*Turn over*]

Part - II

(Marks - 30)

Answer any *one* question.

3. (a) What do you mean buy consistency, convergence and stability of a finite difference method ?

(b) Describe the explicit finite difference method for solving the boundary-initial value problem $u_{xx} = \frac{1}{c} u_t$,

$u = u(x, t)$, $a < x < b$, $t > 0$, c = constant ;

$$u(a, t) = u(b, t) = 0$$

$$u(x, 0) = f(x).$$

Also derive the Von Neumann stability criterion for the scheme. 3+12

4. (a) Write down the finite difference analogue of the equation $u_{xx} + u_{yy} = 0$ defined on

$$R = \{(x,\ y) \mid a < x < b,\ c < y < d\}$$ with

$u(x, y) = f(x, y)$ for $(x, y) \in S$, where S denotes the boundary of R and f is a continuous function. Hence derive standard and diagonal five point formulae.

Also discuss how the initial guess values or previously computed values of u can be improved by (i) Jacobi iteration method and (ii) Gauss-Seidel iteration method.

(b) Show that the FTCS (forward difference in time and central difference in space) scheme for solving the first order linear wave equation $u_t + au_x = 0$, $a(>0)$ constant, is unconditionally stable. 8+7